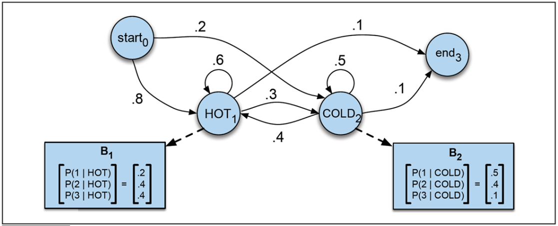
Exercise: The Forward Algorithm & Viterbi Algorithm

Today's exercise is to:

1. Implement the Forward Algorithm for the Hidden Markov Model (shown on the next slide) to compute the probability of the observation sequence *3 1 3*.
2. Implement the Viterbi Algorithm to compute the most likely weather sequence for the observation sequence *3 1 3*.

Use the file `hmm_template.py` posted on blackboard, it contains the incomplete functions `compute_forward` and `compute_viterbi`.

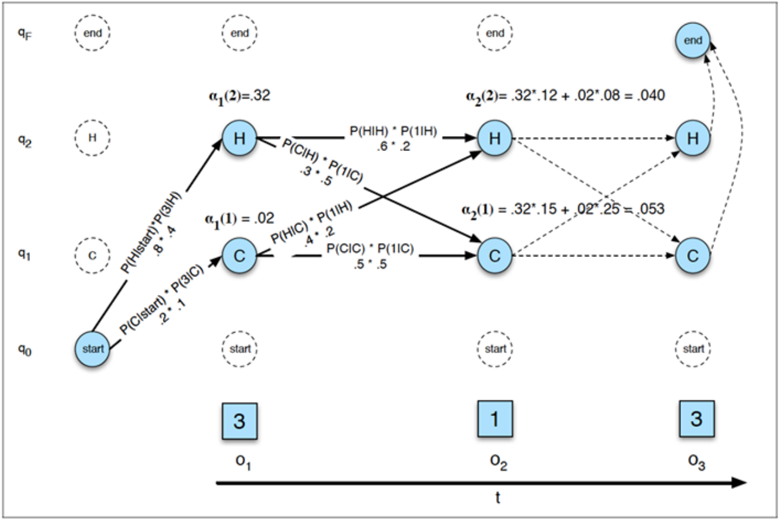
The Hidden Markov Model Used in the Exercise



The above shows a Hidden Markov Model for relating numbers of ice creams eaten by Jason (the observations) to the weather (H or C, the hidden variables).

You can also check https://web.stanford.edu/~jurafsky/slp3/9.pdf for further details.

Visual Representation of the Forward Algorithm.



Pseudocode for the Forward Algorithm.

```
function FORWARD(observations of len T, state-graph of len N) returns forward-prob

  create a probability matrix forward[N+2,T]
  for each state s from 1 to N do                          ; initialization step
      forward[s,1] ← a_{0,s} * b_s(o_1)
  for each time step t from 2 to T do                      ; recursion step
    for each state s from 1 to N do
```

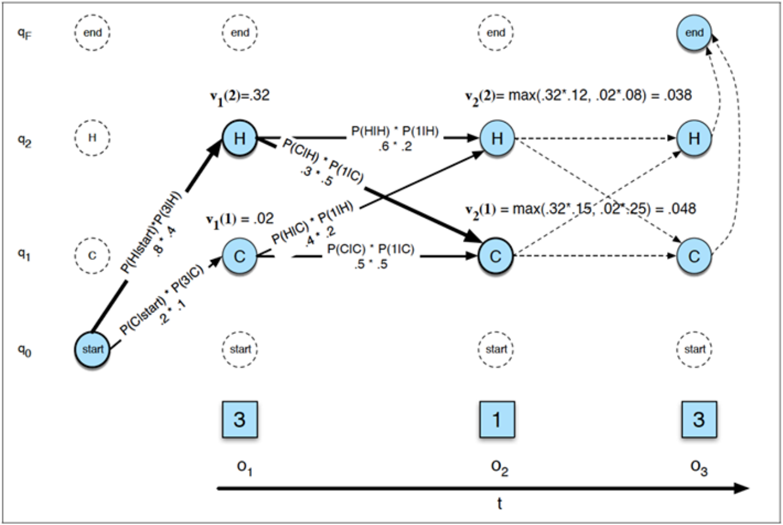
$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,\mathrm{T}] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F} \qquad \text{; termination step}$$

return *forward*$[q_F, T]$

Note that in the code, the transition matrix corresponds to `a`, whereas the emissions matrix corresponds to `b` .

Visual Representation of the Viterbi Algorithm.



Pseudocode for the Viterbi Algorithm.

function VITERBI(*observations* of len T, *state-graph* of len N) **returns** *best-path*

create a path probability matrix *viterbi[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
$\quad viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$
$\quad backpointer[s,1] \leftarrow 0$
for each time step t **from** 2 **to** T **do** ; recursion step
$\quad$ **for** each state s **from** 1 **to** N **do**
$$viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$$
$$backpointer[s,t] \leftarrow \operatorname{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$$
$$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step
$$backpointer[q_F,T] \leftarrow \operatorname{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step
return the backtrace path by following backpointers to states back in time from $backpointer[q_F,T]$

Note that in the code, the transition matrix corresponds to a, whereas the emissions matrix corresponds to b.

Homework

Find the probability of the following observation sequences:

- *3, 3, 1, 1, 2, 2, 3, 1, 3.*
- *3, 3, 1, 1, 2, 3, 3, 1, 2.*

Also find the most likely weather sequences for the two observation sequences.