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Problem Statement and Experiment Setup: To evaluate the performance of Kruskal's algorithm, we have implemented our own version and compared its execution time with a built-in algorithm. The objective is to assess the efficiency and effectiveness of our implementation in finding minimum spanning trees compared to established algorithms.

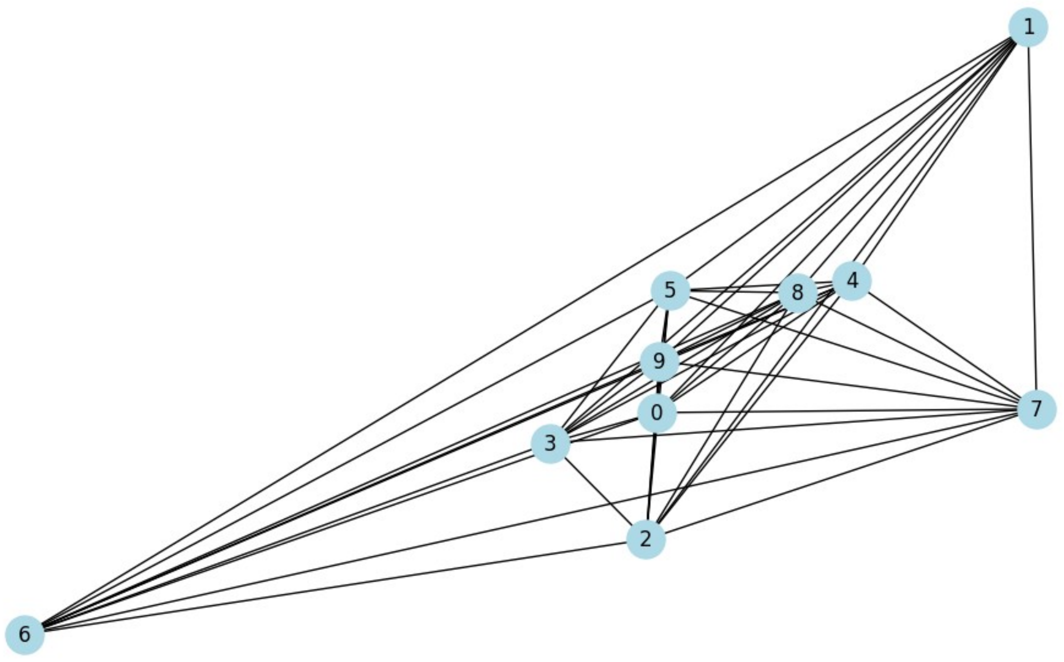
```
!pip install networkx
!pip install matplotlib
!pip install tqdm
```

```
Requirement already satisfied: networkx in
./algorithms_lab/venv/lib/python3.12/site-packages (3.2.1)

[notice] A new release of pip is available: 23.2.1 -> 24.0
[notice] To update, run: pip install --upgrade pip
Requirement already satisfied: matplotlib in
./algorithms_lab/venv/lib/python3.12/site-packages (3.8.2)
Requirement already satisfied: contourpy>=1.0.1 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(1.2.0)
Requirement already satisfied: cycler>=0.10 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(4.48.1)
Requirement already satisfied: kiwisolver>=1.3.1 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(1.4.5)
Requirement already satisfied: numpy<2,>=1.21 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(1.26.4)
Requirement already satisfied: packaging>=20.0 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(23.2)
Requirement already satisfied: pillow>=8 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(10.2.0)
Requirement already satisfied: pyparsing>=2.3.1 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(3.1.1)
Requirement already satisfied: python-dateutil>=2.7 in
./algorithms_lab/venv/lib/python3.12/site-packages (from matplotlib)
(2.8.2)
Requirement already satisfied: six>=1.5 in
./algorithms_lab/venv/lib/python3.12/site-packages (from python-
dateutil>=2.7->matplotlib) (1.16.0)
```

```
[notice] A new release of pip is available: 23.2.1 -> 24.0
[notice] To update, run: pip install --upgrade pip
Requirement already satisfied: tqdm in
./algorithms_lab/venv/lib/python3.12/site-packages (4.66.2)

[notice] A new release of pip is available: 23.2.1 -> 24.0
[notice] To update, run: pip install --upgrade pip

import random
import networkx as nx
import matplotlib.pyplot as plt
from itertools import combinations, groupby
from copy import deepcopy
```

Generating graph

```
# You can use this function to generate a random graph with
'num_of_nodes' nodes
# and 'completeness' probability of an edge between any two nodes
# If 'directed' is True, the graph will be directed
# If 'draw' is True, the graph will be drawn
def gnp_random_connected_graph(num_of_nodes: int,
                               completeness: int,
                               directed: bool = False,
                               draw: bool = False):
    """
    Generates a random graph, similarly to an Erdős-Rényi
    graph, but enforcing that the resulting graph is conneted (in case
of undirected graphs)
    """


    if directed:
        G = nx.DiGraph()
    else:
        G = nx.Graph()
    edges = combinations(range(num_of_nodes), 2)
    G.add_nodes_from(range(num_of_nodes))

    for _, node_edges in groupby(edges, key = lambda x: x[0]):
        node_edges = list(node_edges)
        random_edge = random.choice(node_edges)
        if random.random() < 0.5:
            random_edge = random_edge[::-1]
        G.add_edge(*random_edge)
        for e in node_edges:
            if random.random() < completeness:
                G.add_edge(*e)
    for (u,v,w) in G.edges(data=True):
        w['weight'] = random.randint(-5, 20)

    if draw:
        plt.figure(figsize=(10,6))
        if directed:
            # draw with edge weights
            pos = nx.arf_layout(G)
            nx.draw(G,pos, node_color='lightblue',
                    with_labels=True,
                    node_size=500,
                    arrowsize=20,
                    arrows=True)
            labels = nx.get_edge_attributes(G,'weight')
            nx.draw_networkx_edge_labels(G, pos,edge_labels=labels)

        else:
            nx.draw(G, node_color='lightblue',
                with_labels=True,
                node_size=500)

    return G

G = gnp_random_connected_graph(10, 1, False, True)
```



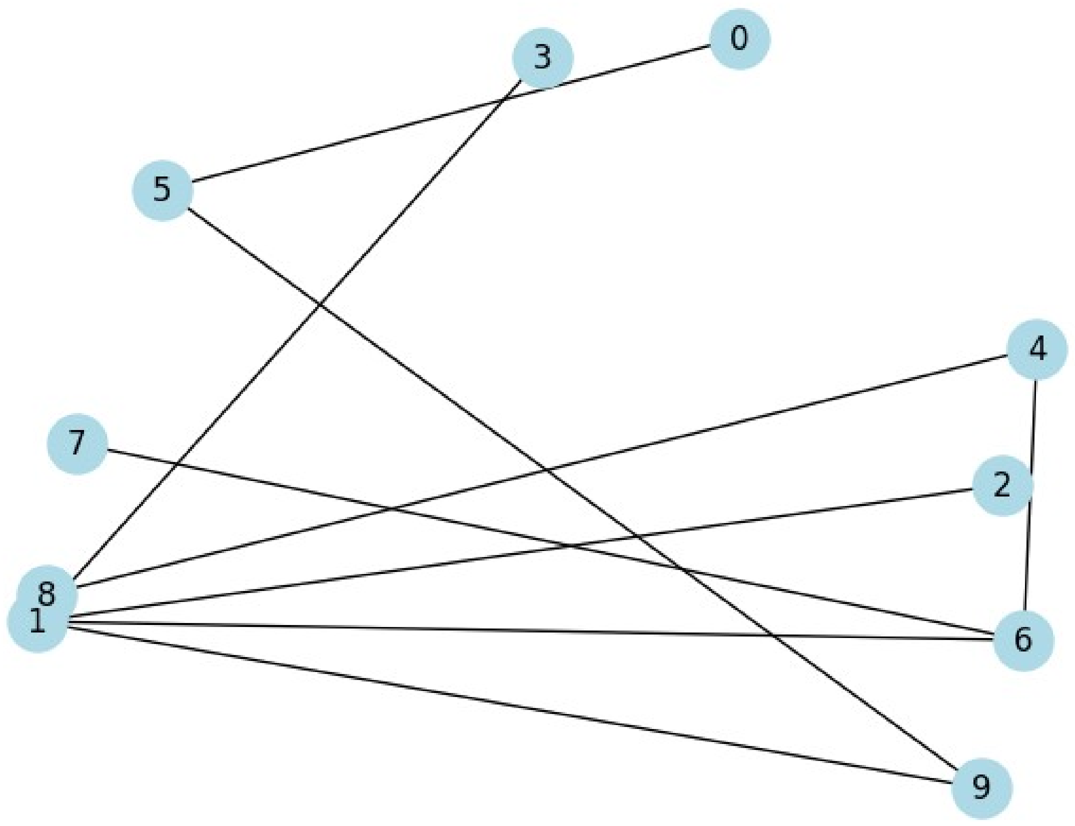
For Task 1

Kruskal's algorithm

```
from networkx.algorithms import tree

mstk = tree.minimum_spanning_tree(G, algorithm="kruskal")
```

The function weight_build_in_algorithm calculates the weight of the obtained framework built using the built-in algorithm. It iterates through each edge and adds its weight to the output.

```
def weight_build_in_algorithm(mstp: object) -> int:
    """
    Weight of build in algorithm.
    """
    output_weight = 0
    conections = dict(mstp.adjacency())
    edges = mstp.edges()
    for edge in edges:
        for node_num, conection in conections.items():
            if node_num == edge[0]:
                for connected_node, weight in conection.items():
                    if connected_node == edge[1]:
                        output_weight += weight['weight']
                        break
    return output_weight

nx.draw(mstk, node_color='lightblue',
        with_labels=True,
        node_size=500)
```



```
mstk.edges(), len(mstk.edges())

(EdgeView([(0, 5), (1, 2), (1, 3), (1, 6), (1, 9), (4, 8), (4, 6), (5,
9), (6, 7)]),
 9)
```

The function min_weight searches for the edge with the smallest weight among the dictionary obtained using the adjacency function. It iterates through the edges, and if the weight of an edge is less than the previous one, it assigns this weight to that edge.

The functions has_cycle and has_cycle_dfs, which search for cycles using Depth-First Search (DFS), operate by visiting each node of the graph and checking if it has already been visited during the current traversal. During DFS traversal, nodes of the graph are visited in depth until a terminal node is reached or a cycle is found.

The main function serves as the core of our algorithm. It operates until the length of the obtained spanning tree equals the number of vertices minus 1. The function begins by searching for the edge with the smallest weight. Next, it verifies, using additional functions, whether adding this edge would create a cycle in our spanning tree. If the edge passes this check, it is added to the spanning tree. Regardless of the outcome of the cycle check, the edge is removed because including it would create a cycle in the spanning tree. The function repeats these steps until the spanning tree reaches the desired size. In summary, the main function iteratively constructs a spanning tree by finding the minimum weight edge, checking for potential cycles, and then adding the edge to the spanning tree if it passes the cycle check. However, the edge is always removed afterward to prevent cycles in the resulting spanning tree.

```python
def min_weight(conections: dict) -> tuple:
    """
    >>> min_weight({0: {1: {'weight': -4}, 2: {'weight': 9}}, 1: {0:
{'weight': -4}, \
2: {'weight': 0}}, 2: {0: {'weight': 9}, 1: {'weight': 0}}})
    ((0, 1), -4)
    """
    min_start_point = 0
    min_end_point = 0
    output_weight = float("inf")
    for node_num, conection in conections.items():
        for connected_node, weight in conection.items():
            if weight['weight'] < output_weight:
                output_weight = weight['weight']
                min_start_point = node_num
                min_end_point = connected_node
    return (min_start_point, min_end_point), output_weight

def has_cycle_dfs(edges: list, current: int, visited: int, parent:
int) -> bool:
    """
    Depth-first search.
    """
    visited.add(current)
    for edge in edges:
        node, neighbor = edge
        if node == current:
            if neighbor not in visited:
                if has_cycle_dfs(edges, neighbor, visited, current):
                    return True
            elif neighbor != parent:
                return True
        elif neighbor == current:
            if node not in visited:
                if has_cycle_dfs(edges, node, visited, current):
                    return True
            elif node != parent:
                return True
    return False

def has_cycle(edges: list) -> bool:
    """
    Checks if the graph contains any cycles.
    >>> has_cycle([(1, 2), (2, 3), (1, 3)])
    True
    """
    visited = set()
    for edge in edges:
        node, neighbor = edge
        if node not in visited:
            if has_cycle_dfs(edges, node, visited, None):
                return True
        if neighbor not in visited:
            if has_cycle_dfs(edges, neighbor, visited, None):
                return True
        visited.add(node)
        visited.add(neighbor)
    return False

def main(G:object) -> int:
    """
    >>> main(G) == weight_build_in_algorithm(mstk)
    True
    """
    graph = deepcopy(G)
    conections = dict(graph.adjacency())
    length = len(conections) - 1
    weight = 0
    visited = set()
    kruskal = []
    while len(kruskal) < length:
        try_conection, edge_weight = min_weight(conections)
        if (try_conection[1], try_conection[0]) not in kruskal:
            if has_cycle(kruskal + [try_conection]) is False:
                weight += edge_weight
                kruskal.append(try_conection)
                visited.add(try_conection[0])
                visited.add(try_conection[1])
        del conections[try_conection[0]][try_conection[1]]
    return weight
if __name__ == '__main__':
    import doctest
    print(doctest.testmod())
print(weight_build_in_algorithm(mstk))
print(main(G))
```

```
TestResults(failed=0, attempted=3)
-26
-26
```

Example on time measuring

Read more on this: https://realpython.com/python-timer/

Recall that you should measure times for 5, 10, 20, 50, 100, 200, 500 nodes 1000 times (and take mean of time taken for each node amount).

Then you should build the plot for two algorithms (x - data size, y - mean time of execution).

```
import time
from tqdm import tqdm

NUM_OF_ITERATIONS = 1000
time_taken = 0
for i in tqdm(range(NUM_OF_ITERATIONS)):

    # note that we should not measure time of graph creation
    G = gnp_random_connected_graph(100, 0.4, False)

    start = time.time()
    tree.minimum_spanning_tree(G, algorithm="prim")
    end = time.time()

    time_taken += end - start

time_taken / NUM_OF_ITERATIONS
```

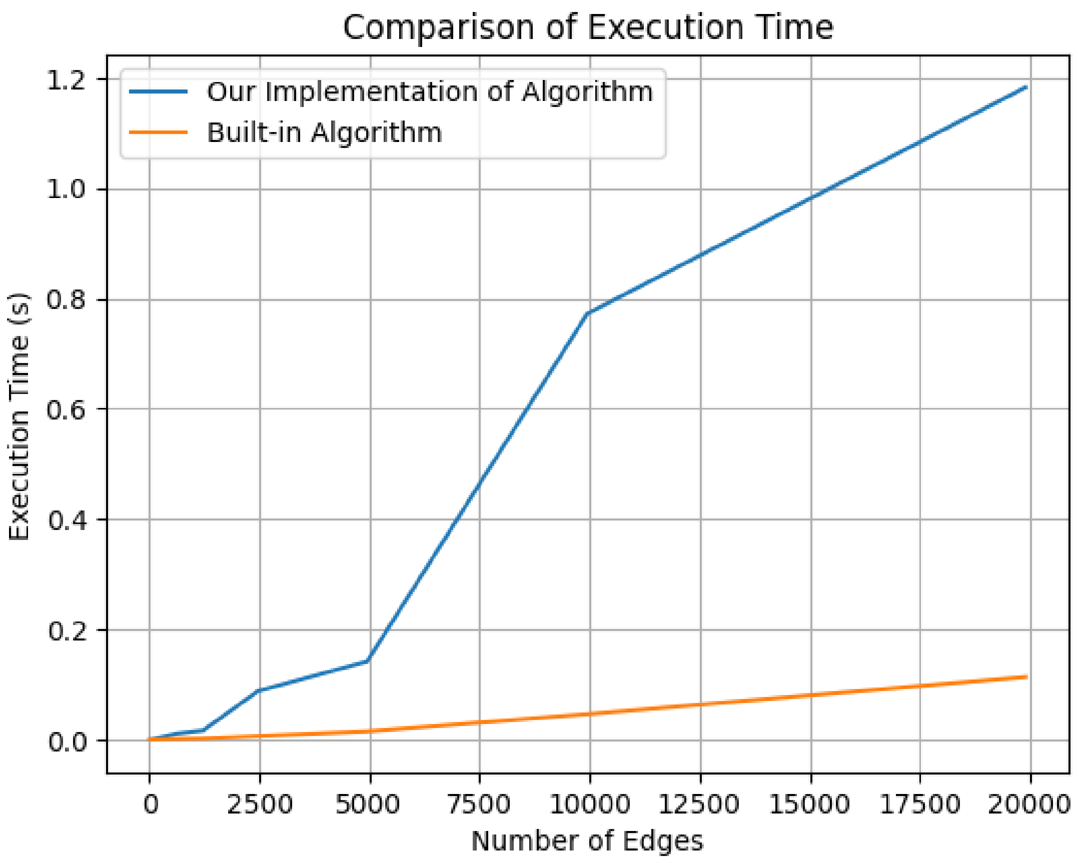
```
100%|██████████| 1000/1000 [00:02<00:00, 342.56it/s]

0.0008487312793731689
```

We ran the algorithm for graphs with 10, 50, and 100 vertices and densities of 0.5 and 1. For each parameter, we executed the algorithm 1000 times and calculated the average value for both our algorithm and the built-in one. We plotted the graph based on the time taken against the number of edges. We chose to analyze the number of edges because for the same number of vertices, we executed the algorithm twice with different densities, making it the most representative graph in our opinion.

```
import time
import matplotlib.pyplot as plt
from tqdm import tqdm

NUM_OF_ITERATIONS = 1000
graph_sizes = [10, 50, 100, 200]
fullness_levels = [0.5, 1]

main_execution_times = []
built_in_algorithm_execution_times = []
num_edges = []

for size in graph_sizes:
    for fullness in fullness_levels:
        my_time = 0
        built_in_time = 0
        for i in tqdm(range(NUM_OF_ITERATIONS)):
            G = gnp_random_connected_graph(size, fullness, False,
False)
```

```
            start_main_execution = time.time()
            main_execution_time = main(G)
            end_main_execution = time.time()
            my_time += end_main_execution - start_main_execution

            start_built_in_execution = time.time()
            build_in_algorithm_execution_time =
weight_build_in_algorithm(G)
            end_built_in_execution = time.time()
            built_in_time += end_built_in_execution -
start_built_in_execution

        main_execution_times.append(my_time / 1000)
        built_in_algorithm_execution_times.append(built_in_time /
1000)
        num_edges.append(size * (size - 1) / 2 * fullness)

plt.plot(num_edges, main_execution_times, label='Our Implementation of
Algorithm')
plt.plot(num_edges, built_in_algorithm_execution_times, label='Built-
in Algorithm')
plt.xlabel('Number of Edges')
plt.ylabel('Execution Time (s)')
plt.title('Comparison of Execution Time')
plt.legend()
plt.grid(True)
plt.show()
```

```
100%|██████████| 1000/1000 [00:00<00:00, 5535.69it/s]
100%|██████████| 1000/1000 [00:00<00:00, 4101.39it/s]
100%|██████████| 1000/1000 [00:12<00:00, 82.02it/s]
100%|██████████| 1000/1000 [00:19<00:00, 51.68it/s]
100%|██████████| 1000/1000 [01:36<00:00, 10.33it/s]
100%|██████████| 1000/1000 [02:40<00:00,  6.21it/s]
100%|██████████| 1000/1000 [13:49<00:00,  1.21it/s]
100%|██████████| 1000/1000 [21:58<00:00,  1.32s/it]
```



1) With 25 edges, both algorithms perform quickly due to the small number of edges and the simplicity of the graph. In this case, the execution time is nearly identical because there is no significant complexity in edge selection or cycle checking. 2) Similarly to the previous case, with 45 edges, both algorithms work efficiently as the graph is still small and not complex. The difference in execution time is minimal. 3) With 615 edges, a slight difference in execution time emerges as the graph becomes slightly more complex. Although both algorithms still work quickly, it may be noticeable that our implementation starts to lag slightly behind the built-in algorithms. 4) Analogously to the previous scenario, with 1225 edges, the difference in execution time is small as the graph is not very large, and both algorithms can efficiently handle it. 5) With 2475 edges, the difference in execution time is 0.1 seconds, which may be attributed to the increased complexity of edge selection and cycle checking. Our algorithm lags in speed compared to the built-in algorithms due to less optimization. 6) With 4950 edges, the situation is similar to the previous one. The increase in edge count leads to further growth in the difference in execution time. 7) With 9950 edges, the difference in execution time is 0.7 seconds, indicating that our algorithm begins to lag in speed compared to the built-in algorithms. 8) With 19900 edges, the difference in execution time is 1.1 seconds, confirming the hypothesis that our algorithm significantly lags in speed compared to the built-in algorithms due to the increasing computational complexity.

Based on the experimental results, it is evident that for graphs with a small number of edges (up to 2000), the difference in execution time between our implementation of the Kruskal's algorithm and the built-in algorithms for minimum spanning trees is almost negligible. However, as the number of edges increases to 20000, there is a noticeable increase in the execution time of our algorithm compared to the built-in algorithms. This can be attributed to the increasing complexity of edge selection and cycle checking in the graph. The difference is likely due to the use of different libraries and optimized data structures and algorithms in the built-in algorithms, leading to their better efficiency.