

A, B卷答案

一 填空题(每小题4分, 共32分)

1. 答案: $\frac{1}{2} - \frac{1}{2} \ln 2$

2. 答案:

X	1	2	3	4	5	6
p	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

3. 答案: $\frac{9}{64}$

4. 答案: $z_{0.95}$

5. 答案: $\frac{1}{\sqrt{2\pi \cdot 3}} e^{-\frac{(x-1)^2}{2 \cdot 3}}$

6. 答案: 0

7. 答案: $F(1, 1)$

8. 答案: $\frac{1}{3}$

二 选择题(每小题4分, 共32分)

1. 答案: A

2. 答案: D

3. 答案: C

4. 答案: C

5. 答案: A

6. 答案: D

7. 答案: B

8. 答案: C

三 (本题12分)

解答:(1) $\int_0^2 \int_0^2 k(x+y)dydx = 1, k = \frac{1}{8};$ (2分)

(2) $f_X(x) = \begin{cases} \int_0^2 \frac{1}{8}(x+y)dy = \frac{1}{4}(x+1), & 0 \leq x \leq 2 \\ 0, & \text{其他} \end{cases}$ (4分)

$f_Y(y) = \begin{cases} \int_0^2 \frac{1}{8}(x+y)dx = \frac{1}{4}(y+1), & 0 \leq y \leq 2 \\ 0, & \text{其他} \end{cases}$ (6分)

(3) $f(x, y) \neq f_X(x)f_Y(y)$ 不独立;..... (8分)

(4) $P(X+Y \leq 2) = \int_0^2 \int_0^{2-x} \frac{1}{8}(x+y)dydx$
 $= \frac{1}{8} \int_0^2 \left(-\frac{x^2}{2} + 2\right) dx = \frac{1}{8} \left(-\frac{x^3}{6} + 2x\right) \Big|_0^2 = \frac{1}{3}.$ (12分)

[四] (本题学《概率统计A》的学生做,学《概率统计B》的学生不做,本题12分)

解答:(1)状态空间 $\{0, 1, 2, \dots, n, \dots\}$ (2分)

$$P\{X_{n+1} = j | X_n = i\} = \begin{cases} 0, & j < i \\ 0.96, & j = i \\ 0.04, & j = i + 1 \\ 0, & j = i + 1 \end{cases} \dots\dots\dots (6分)$$

(2)是齐次马尔科夫链..... (8分)

$$(3)P\{X_{n+2} = 3 | X_n = 2\} = \sum_{i=2}^3 P\{X_{n+1} = i | X_n = 2\}P\{X_{n+2} = 3 | X_{n+1} = i\} = 0.96 \cdot 0.04 + 0.04 \cdot 0.96 = 0.0768 \dots\dots\dots (12分)$$

四 (本题学《概率统计B》的学生做,学《概率统计A》的学生不做,本题12分)

$$\text{解: } f_X(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases} \dots\dots\dots (2分)$$

$$f_Y(y) = \begin{cases} 1, & 1 < y < 2 \\ 0, & \text{其他} \end{cases} \dots\dots\dots (4分)$$

$$f(x, y) = f_X(x)f_Y(y) = \begin{cases} 2e^{-2x}, & x \geq 0, 1 < y < 2 \\ 0, & \text{其他} \end{cases} \dots\dots\dots (6分)$$

当 $z < 1$ 时, $f_Z(z) = 0$;..... (8分)

$$\text{当 } 1 \leq z < 2 \text{ 时, } f_Z(z) = \int_0^{z-1} 2e^{-2t} dt = 1 - e^{-2(z-1)}; \dots\dots\dots (10分)$$

$$\text{当 } z > 2 \text{ 时, } f_Z(z) = \int_{z-2}^{z-1} 2e^{-2t} dt = e^{-2(z-2)} - e^{-2(z-1)}; \dots\dots\dots (12分)$$

[五] (本题学《概率统计A》的学生做, 学《概率统计B》的学生不做, 本题12分)

解答:(1) $f(x, y) = \begin{cases} 1, & 0 \leq x, y \leq 1 \\ 0, & \text{其他} \end{cases}$ (2分)

$$P\{Z \leq z\} = P\{X + tY \leq z\}$$

$$= \begin{cases} 0, & z < 0 \\ \int_0^z \int_0^{\frac{z-x}{t}} 1 dy dx = \frac{z^2}{2t}, & 0 \leq z < 1 \\ \int_0^1 \int_0^{\frac{z-x}{t}} 1 dy dx = \frac{z}{t} - \frac{1}{2t}, & 1 \leq z < t \\ \int_0^1 \int_{\frac{z-x}{t}}^1 1 dy dx = -\frac{z^2}{2t} + (1 + \frac{1}{t})z - \frac{t^2 + 1}{2t}, & t \leq z < t + 1 \\ 1, & z \geq t + 1 \end{cases} \quad \text{..... (10分)}$$

(2) 非严平稳过程 (12分)

五 (本题学《概率统计B》的学生做, 学《概率统计A》的学生不做, 本题12分)

解答:(1) 结果 $\left\{ \bar{x} : \left| \frac{\bar{x} - \mu_0}{S/\sqrt{n}} \right| > t_{1-\frac{\alpha}{2}}(n-1) \right\}$

$= \left\{ \bar{x} : \bar{x} < \mu_0 - \frac{S}{\sqrt{n}} t_{1-\frac{\alpha}{2}}(n-1) \text{ or } \bar{x} > \mu_0 + \frac{S}{\sqrt{n}} t_{1-\frac{\alpha}{2}}(n-1) \right\}$, 过程略... (6分)

$$(2) \left| \frac{\bar{x} - \mu_0}{S/\sqrt{n}} \right| = \frac{5 - 4}{2/\sqrt{25}} = 2.5, t_{1-\frac{\alpha}{2}}(n-1) = t_{0.975}(24) = 2.07,$$

故拒绝原假设 (12分)