

A卷答案

一 填空题(每小题4分, 共32分)

1. 答案: $1 - \frac{A_N^n}{N^n} = 1 - \frac{N!}{(N-n)!N^n}$

2. 答案:

| | | | |
|-----|-----|-----|-----|
| X | 0 | 1 | 2 |
| p | 0.3 | 0.4 | 0.3 |

3. 答案: 0.15

4. 答案: $\frac{1}{2}$

5. 答案: $\frac{1}{2} \max\{X_1, X_2, \dots, X_n\}$

6. 答案: $\frac{\pi}{12}(a^2 + ab + b^2)$

7. 答案: $f_Y(y) = \begin{cases} \frac{1}{y^2}, & y \geq 1 \\ 0, & y < 1 \end{cases}$

8. 答案: $\frac{1}{50}$

二 选择题(每小题4分, 共32分)

1. 答案: B

2. 答案: D

3. 答案: C

4. 答案: C

5. 答案: A

6. 答案: B

7. 答案: D

8. 答案: D

三 (本题12分)

$$\text{解答: (1) } f_X(x) = \begin{cases} \int_0^2 \left(x^2 + \frac{1}{3}xy\right) dy = 2x^2 + \frac{2}{3}x, & 0 \leq x \leq 1 \\ 0, & \text{其他} \end{cases} \dots\dots\dots (3\text{分})$$

$$f_Y(y) = \begin{cases} \int_0^1 \left(x^2 + \frac{1}{3}xy\right) dx = \frac{1}{3} + \frac{1}{6}y, & 0 \leq y \leq 2 \\ 0, & \text{其他} \end{cases} \dots\dots\dots (6\text{分})$$

$$\begin{aligned} (2) P(X + Y \leq 2) &= \int_0^1 \int_0^{2-x} \left(x^2 + \frac{1}{3}xy\right) dy dx \\ &= \int_0^1 \left(-\frac{5}{6}x^3 + \frac{4}{3}x^2 + \frac{2}{3}x\right) dx = \frac{41}{72} \dots\dots\dots (12\text{分}) \end{aligned}$$

[四] (本题学《概率统计A》的学生做,学《概率统计B》的学生不做,本题12分)

解答:(1) $X(t) = \frac{1}{4}\cos(\omega t), X(t) = \frac{1}{2}\cos(\omega t), -\infty < t < +\infty,$ (4分)

$$(2)X\left(\frac{\pi}{3\omega}\right) = \frac{1}{2}V,$$

$$F_X(x) = P(X \leq x) = P\left(\frac{1}{2}V \leq x\right) = P(V \leq 2x)$$

$$= \begin{cases} 0, & 2x < 0 \\ 2x, & 0 \leq 2x < 1 \\ 1, & 2x \geq 1 \end{cases} = \begin{cases} 0, & x < 0 \\ 2x, & 0 \leq x < \frac{1}{2} \\ 1, & x \geq \frac{1}{2} \end{cases} \dots\dots\dots (8分)$$

$$f_X(x) = \begin{cases} 2, & 0 \leq x < \frac{1}{2} \\ 0, & \text{其他} \end{cases} \dots\dots\dots (12分)$$

四 (本题学《概率统计B》的学生做,学《概率统计A》的学生不做,本题12分)

解: $F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y})$

$$= \begin{cases} 0, & y < 0 \\ \frac{\sqrt{y}}{3}, & 0 \leq \sqrt{y} < 3 \\ 1, & \sqrt{y} \geq 3 \end{cases} = \begin{cases} 0, & y < 0 \\ \frac{\sqrt{y}}{3}, & 0 \leq x < 9 \\ 1, & x \geq 9 \end{cases} \dots\dots\dots (8分)$$

$$f_Y(y) = \begin{cases} \frac{1}{6\sqrt{y}}, & 0 \leq x < 9 \\ 0, & \text{其他} \end{cases} \dots\dots\dots (12分)$$

[五] (本题学《概率统计A》的学生做,学《概率统计B》的学生不做,本题12分)

解答:(1) $\mu_Z(t) = E(X\cos\omega t + Y\sin\omega t) = 0,$

(2) $R_Z(t_1, t_2) = E[(X\cos\omega t_1 + Y\sin\omega t_1)(X\cos\omega t_2 + Y\sin\omega t_2)] = \cos\omega(t_2 - t_1)$ 或 $R_Z(t, t + \tau) = \cos\omega\tau$ (6分)

(3) $E[Z^2(t)] = 1$,是广义平稳过程..... (12分)

五 (本题学《概率统计B》的学生做,学《概率统计A》的学生不做,本题12分)

解答:(1) $EX = \int_0^\theta x \cdot \frac{6x}{\theta^3}(\theta - x) = \frac{\theta}{2}$ (3分)

$$EX^2 = \int_0^\theta x^2 \cdot \frac{6x}{\theta^3}(\theta - x) = \frac{3}{10}\theta^2$$

$DX = \frac{1}{20}\theta^2$ (6分)

(2) $EX = \frac{\theta}{2} = \bar{X}, \hat{\theta} = 2\bar{X}$ (9分)

(3) $E\hat{\theta} = 2E\bar{X} = \theta$,无偏估计..... (12分)