Chapter 32

String Matching

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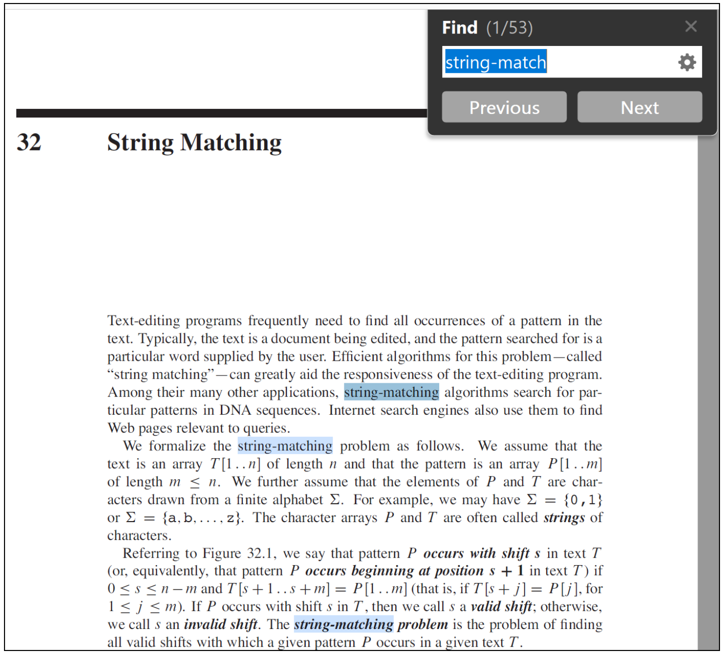
VII Selected Topics

32 String Matching

char *strstr(char *text, char *pattern);

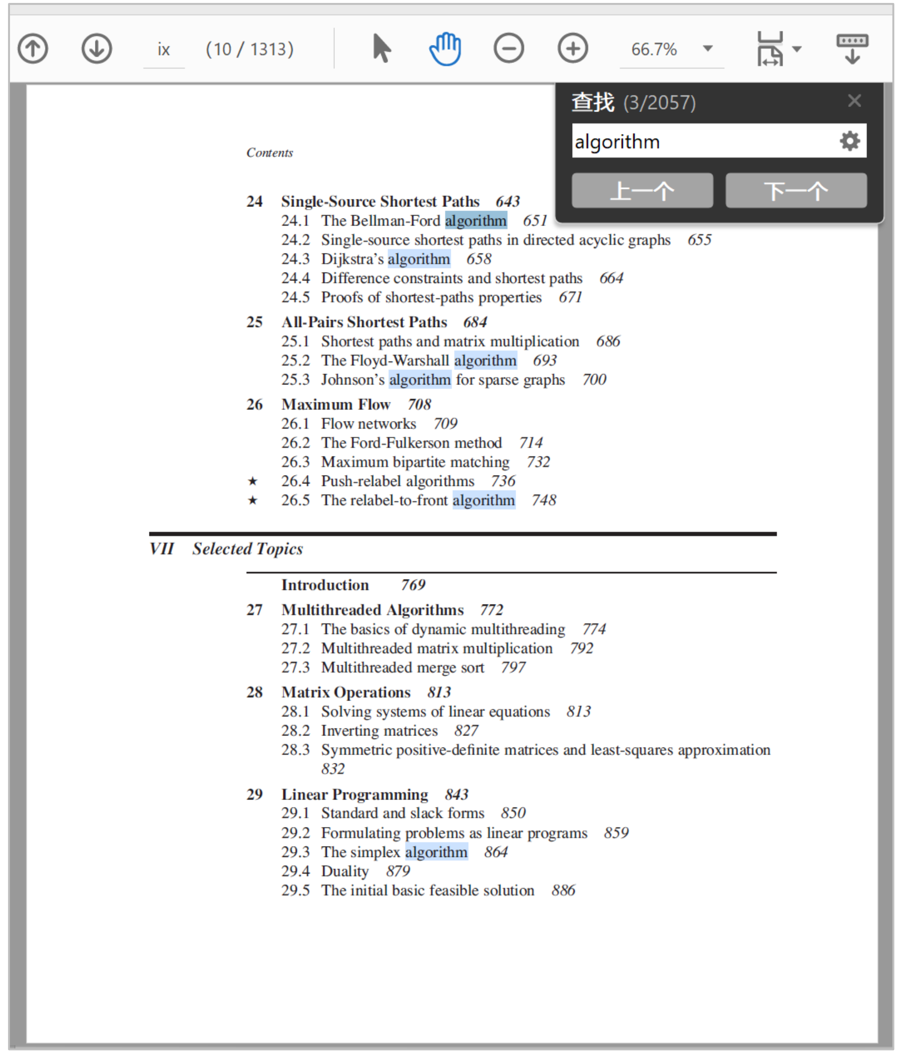


32 **String Matching**

Text-editing programs frequently need to find all occurrences of a pattern in the text. Typically, the text is a document being edited, and the pattern searched for is a particular word supplied by the user. Efficient algorithms for this problem—called "string matching"—can greatly aid the responsiveness of the text-editing program. Among their many other applications, string-matching algorithms search for particular patterns in DNA sequences. Internet search engines also use them to find Web pages relevant to queries.

We formalize the string-matching problem as follows. We assume that the text is an array $T[1 \mathinner{..} n]$ of length n and that the pattern is an array $P[1 \mathinner{..} m]$ of length $m \le n$. We further assume that the elements of P and T are characters drawn from a finite alphabet Σ. For example, we may have $\Sigma = \{0, 1\}$ or $\Sigma = \{a, b, \ldots, z\}$. The character arrays P and T are often called ***strings*** of characters.

Referring to Figure 32.1, we say that pattern P ***occurs with shift*** s in text T (or, equivalently, that pattern P ***occurs beginning at position*** $s+1$ in text T) if $0 \le s \le n-m$ and $T[s+1 \mathinner{..} s+m] = P[1 \mathinner{..} m]$ (that is, if $T[s+j] = P[j]$, for $1 \le j \le m$). If P occurs with shift s in T, then we call s a ***valid shift***; otherwise, we call s an ***invalid shift***. The ***string-matching problem*** is the problem of finding all valid shifts with which a given pattern P occurs in a given text T.



Contents

Finding all occurrences of a pattern in a text is a problem that arises frequently in text-editing programs.

32 String Matching

char *strstr(char *text, char *pattern);

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创建快讯

Fast pattern matching in strings
DE Knuth, JH Morris, Jr, VR Pratt - SIAM journal on computing, 1977 - SIAM
An algorithm is presented which finds all occurrences of one given string within another, in running time proportional to the sum of the lengths of the strings. The constant of proportionality is low enough to make this algorithm of practical use, and the procedure can …
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[HTML] Fast pattern-matching on indeterminate strings
J Holub, WF Smyth, S Wang - Journal of Discrete Algorithms, 2008 - Elsevier
In a string x on an alphabet Σ, a position i is said to be indeterminate iff x [i] may be any one of a specified subset {λ 1, λ 2,…, λ j} of Σ, 2⩽ j⩽| Σ|. A string x containing indeterminate positions is therefore also said to be indeterminate. Indeterminate strings can arise in DNA …
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Fastest pattern matching in strings
L Colussi - Journal of Algorithms, 1994 - Elsevier
An algorithm is presented that substantially improves the algorithm of Boyer and Moore for pattern matching in strings, both in the worst case and in the average. Both the Boyer and Moore algorithm and the new algorithm assume that the characters in the pattern and in the …
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Pattern matching in strings
AV Aho - Formal Language Theory, 1980 - Elsevier
… In this way an algorithm can construct from the pattern whatever Pattern Matching in

32 String Matching

- **Efficient algorithms for this problem can greatly aid the responsiveness(响应性) of the text-editing program.**

- **Applications: String-matching algorithms are also used, for example, to search for particular patterns in DNA sequences.**

char *strstr(char *text, char *pattern);

Find (1/53)

string-match

Previous Next

32 String Matching

Text-editing programs frequently need to find all occurrences of a pattern in the text. Typically, the text is a document being edited, and the pattern searched for is a particular word supplied by the user. Efficient algorithms for this problem—called "string matching"—can greatly aid the responsiveness of the text-editing program. Among their many other applications, string-matching algorithms search for particular patterns in DNA sequences. Internet search engines also use them to find Web pages relevant to queries.

We formalize the string-matching problem as follows. We assume that the text is an array $T[1..n]$ of length n and that the pattern is an array $P[1..m]$ of length $m \le n$. We further assume that the elements of P and T are characters drawn from a finite alphabet Σ. For example, we may have $\Sigma = \{0,1\}$ or $\Sigma = \{a, b, \ldots, z\}$. The character arrays P and T are often called ***strings*** of characters.

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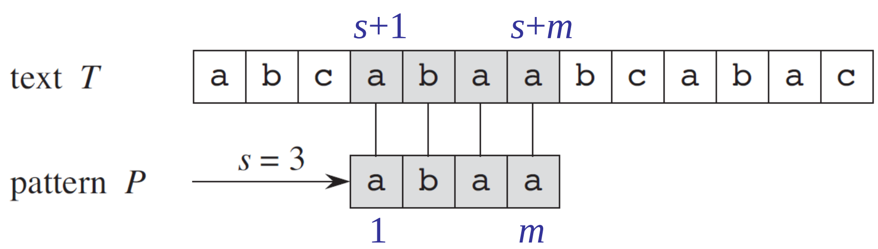
32 String Matching



The pattern occurs only once in the text, at shift $s = 3$. The shift $s = 3$ is said to be a valid shift.

String-matching problem:

- **Text: $T[1 \,..\, n]$, Pattern: $P[1 \,..\, m]$, $m \le n$.**
- **Finite alphabet: Σ , for example, $\Sigma = \{0, 1\}$ or $\Sigma = \{a, b, \ldots, z\}$.**
- **$P_i \in \Sigma$, $T_i \in \Sigma$.**
- **P occurs with shift s in T if $0 \le s \le n\text{-}m$ and $T[s+1 \,..\, s+m] = P[1 \,..\, m]$ (that is, if $T[s+j] = P[j]$, for $1 \le j \le m$). (or, equivalently, that P occurs beginning at position $s+1$ in T).**
- **Valid shift s: if P occurs with shift s in T; otherwise, s is an invalid shift.**
- **Finding all valid shifts with which a given P occurs in a given T .**

Notation and terminology

- **Σ^* : the set of all finite-length strings formed using characters from the alphabet Σ. Example:**

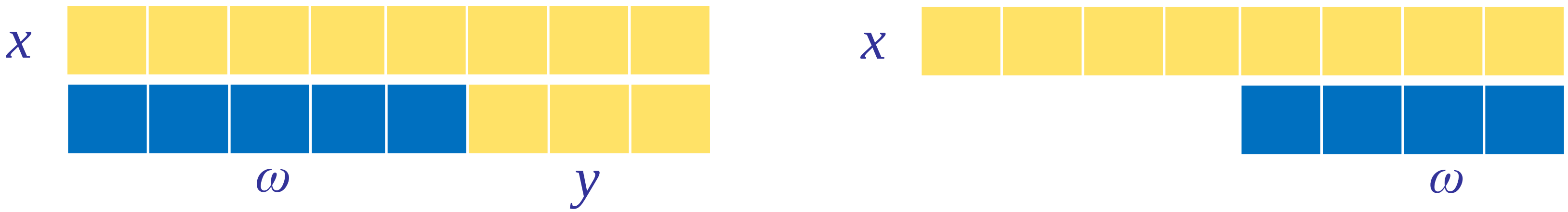
  $\Sigma = \{a, b, c\}$

  $\Sigma^* = \{\varepsilon, a, b, c, ab, bc, ac, abc, acb, aabbc, \ldots\ldots\}$

- **ε : The zero-length empty string, also belongs to Σ^*.**
- **$|x|$: The length of x.**
- **The concatenation of two strings x and y, denoted xy, has length $|x| + |y|$ and consists of the characters from x followed by the characters from y.**

Notation and terminology

- $\omega \sqsubset x$: string ω is a **prefix** of x, if $x = \omega y$ for some $y \in \Sigma^*$.



prefix **suffix**

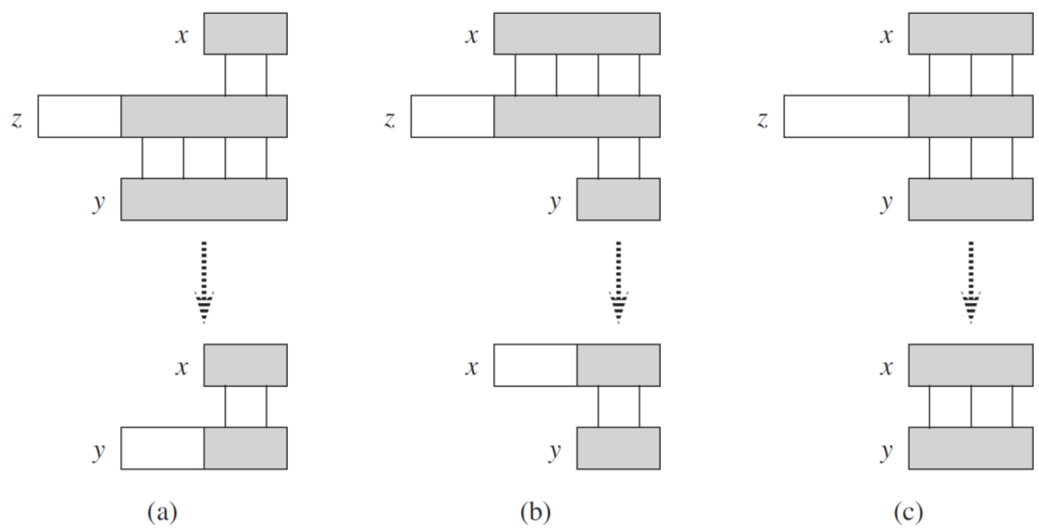
- $\omega \sqsupset x$: ω is a **suffix** of x, if $x = y\omega$ for some $y \in \Sigma^*$.
  - If $\omega \sqsubset x$ or $\omega \sqsupset x$, then $|\omega| \leq |x|$.
  - The empty string ε is both a suffix and a prefix of every string.
  - For example, we have ab $\sqsubset$ abcca and cca $\sqsupset$ abcca.
  - For any strings x and y and any character a, we have $x \sqsupset y$ if and only if $xa \sqsupset ya$.
  - $\sqsubset$ and $\sqsupset$ are transitive relations.

Notation and terminology

Lemma 32.1: (Overlapping-suffix lemma)

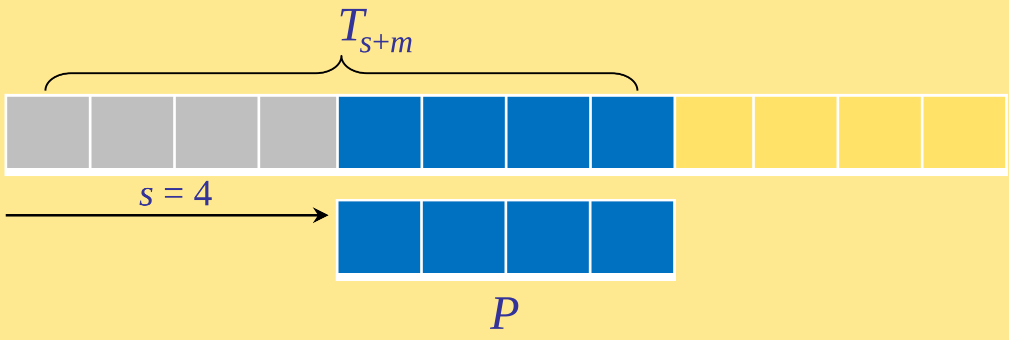
Suppose that x, y, and z are strings such that $x \sqsupset z$ and $y \sqsupset z$. If $|x| \le |y|$, then $x \sqsupset y$. If $|x| \ge |y|$, then $y \sqsupset x$. If $|x| = |y|$, then $x = y$.

Proof See Fig for a graphical proof.



Notation and terminology

- **For brevity of notation, we denote the *k*-character prefix $P[1..k]$ of the pattern $P[1..m]$ by P_k**
  - **Thus, $P_0 = \varepsilon$ and $P_m = P = P[1..m]$**
- **Similarly, we denote the *k*-character prefix of the text T as T_k**
- **string-matching problem:**
  finding all shifts s in the range $0 \le s \le n\text{-}m$ such that $P \sqsupset T_{s+m}$



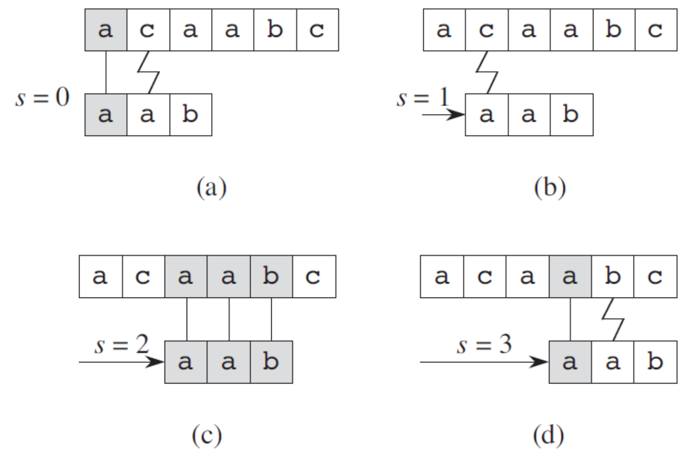
字符串匹配过程：从头到尾依序扫描文本 T，扫描到的字符串都是 T 的前缀 T_x，若有 P 匹配，则此时 P 为 T_x 的一个后缀。也就是，求文本 T 的前缀的 P 后缀。

- **Primitive operation: comparing characters**

32.1 The naive string-matching algorithm

The naive algorithm finds all valid shifts using a loop that checks the condition $P[1 .. m] = T[s+1 .. s+m]$ for each of the $n-m+1$ possible values of s.

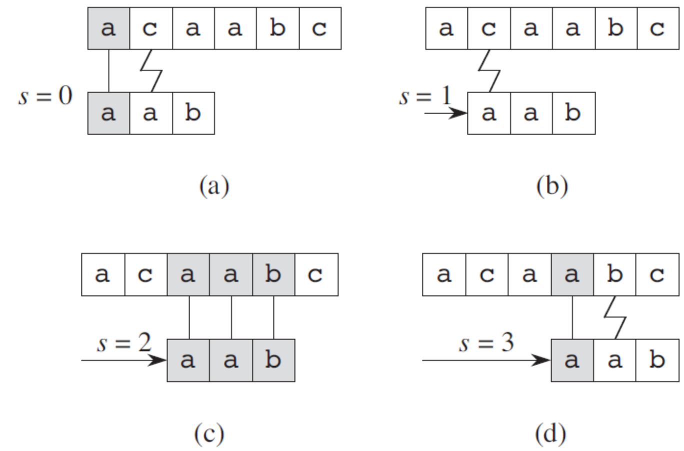
```
NAIVE-STRING-MATCHER(T, P)
1 n ← length[T]
2 m ← length[P]
3 for s ← 0 to n-m
4       if P[1 .. m] = T[s+1 .. s+m]
5             print "Pattern occurs with shift" s
```



- **The procedure can be interpreted graphically as sliding a "template" containing the pattern over the text.**
- **Line 3 considers each possible shift explicitly.**
- **The test on line 4 determines whether the current shift is valid or not; this test involves an implicit loop**（第4行包括一个隐式的循环）**.**

32.1 The naive string-matching algorithm

```
NAIVE-STRING-MATCHER(T, P)
n ← length[T]
m ← length[P]
for s ← 0 to n-m
    if P[1 .. m] = T[s+1 .. s+m]
        print "Pattern occurs with shift" s
```



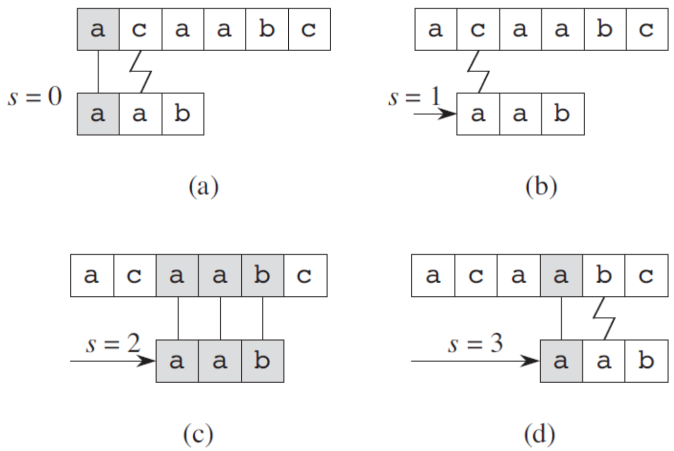

```
// 返回首次匹配位置，用库函数实现
strstr(T, P);
```

思考题：

- 所有匹配（按伪代码规则）都找出来，怎么实现？
- 怎么实现strrstr？（最后一次匹配的位置）

```
// 返回首次匹配位置，自定义函数实现
char * _strstr(const char *T, const char *P)
{
    if(T == NULL)
        return NULL;
    int n = strlen(T), m = strlen(P), s, i;
    for(s=0; s<=n-m; s++)
    {
        for(i=0; i<m; i++)
            if(P[i] != T[s+i]) break;
        if(i == m) return T+s;
    }
    return NULL;
}
```

32.1 The naive string-matching algorithm



Running time ?

NAIVE-STRING-MATCHER(T, P)

```
n ← length[T]
m ← length[P]
for s ← 0 to n-m
    if P[1 .. m] = T[s+1 .. s+m]
        print "Pattern occurs with shift" s
```

Exercise 32.1-2, 32.1-4

*32.2 The Rabin-Karp algorithm

T

a	b	c	a	d	d	b	c
	b	c	a	d			

P

R-K algorithm performs well in practice and that also generalizes to other algorithms for related problems, such as two-dimensional pattern matching.
用了Hash和简单数论的方法。对 T_s 计算 p 时，还有更有效的方法（后一个 m 个 T_s 的 p 值跟前面计算的结果有关系，可充分利用前面的计算信息，加快计算速度）。计算 $p(P)$，可以用Horner's rule.

coding rule: {a, b, c, d} ⇨ {0, 1, 2, 3},

then $p(P) = 1*4^3 + 2*4^2 + 0*4^1 + 3*4^0 = 99$

$((12345)_{10} = 1*10^4 + 2*10^3 + 3*10^2 + 4*10^1 + 5*10^0)$

if $p(P) == p(T_{s+m})$ ($T_{s+m} = T[s+1 .. s+m]$, s = 0, 1, ..), then match.

or, if $(p(P) \bmod q) == (p(T_s) \bmod q)$, *check if $P == T_s$*

扩展阅读chapter31
Number-Theoretic Algorithms

preprocessing time: $\Theta(m)$
worst-case running time: $\Theta((n-m+1)m)$

*32.2 The Rabin-Karp algorithm

T

a	b	c	a	d	d	b	c

	b	c	a	d

P

coding rule: {a, b, c, d} ⇨ {0, 1, 2, 3},

then $p(P) = 1*4^3 + 2*4^2 + 0*4^1 + 3*4^0 = 99$

$((12345)_{10} = 1*10^4 + 2*10^3 + 3*10^2 + 4*10^1 + 5*10^0)$

Efficient randomized pattern-matching algorithms

RM Karp, MO Rabin - IBM journal of research and development, 1987 - ieeexplore.ieee.org

We present randomized algorithms to solve the following string-matching problem and some of its generalizations: Given a string X of length n (the pattern) and a string Y (the text), find the first occurrence of X as a consecutive block within Y. The algorithms represent strings of …

Naive vs RK vs FA vs KMP

T

b a c a c a b a c a

a c a c

P

Naive

右移+1（依序扫描），再一一匹配判断

b a c a c a b a c a

a c a c

与 P “无关”
（无需预处理）

RK

a c a c

P

$p(P)$

FA

b a c a c a b a c a

a c a c

根据$\delta(4, a) = 3$，即状态4时输入a得到状态3（3个匹配），接着求$\delta(3, b)$?

$\delta(P\!: Q, \Sigma)$，输入字符不匹配时，快速移动 P

KMP

b a c a c a b a c a

a c a c

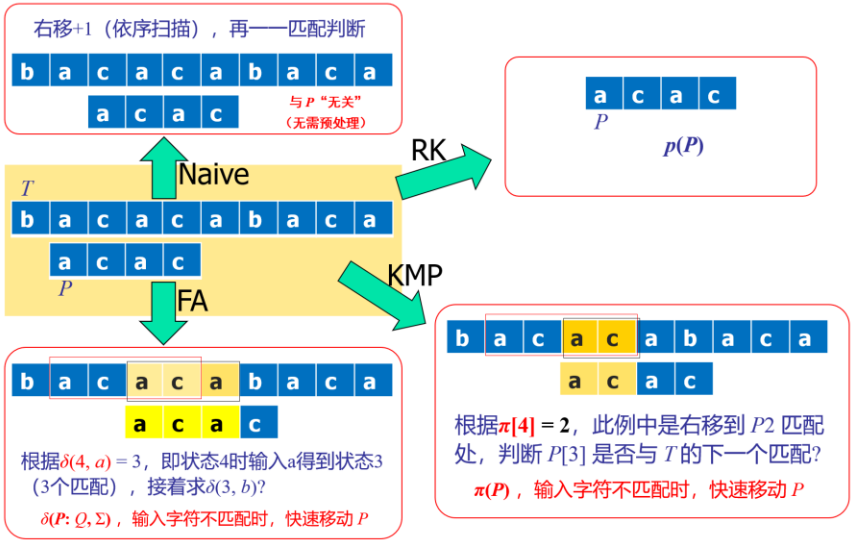
根据$\pi[4] = 2$，此例中是右移到 $P2$ 匹配处，判断 $P[3]$ 是否与 T 的下一个匹配?

$\pi(P)$，输入字符不匹配时，快速移动 P

Naive vs RK vs FA vs KMP

Algorithm	Preprocessing time	Matching time
Naive	0	$O((n-m+1)m)$
Rabin-Karp	$\Theta(m)$	$O((n-m+1)m)$
Finite automaton	$O(m\lvert\Sigma\rvert)$	$\Theta(n)$
Knuth-Morris-Pratt	$\Theta(m)$	$\Theta(n)$

	关键	特征
FA	求 $\delta(P: Q, \Sigma)$	输入字符$T[i]$不匹配时，快速移动 P，每个$T[i]$匹配一次
KMP	求 $\pi(P)$	输入字符$T[i]$不匹配时，快速移动 P，每个$T[i]$匹配一次



32.3 String matching with finite automata（有限自动机）

[引用] The design and analysis of computer algorithms
AV Aho, JE Hopcroft - 1974 - Pearson Education India
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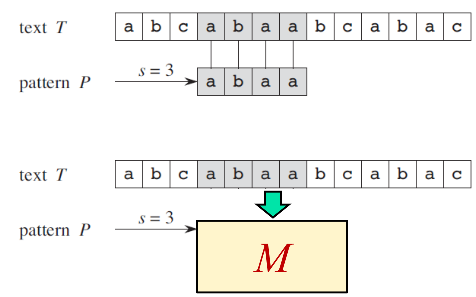
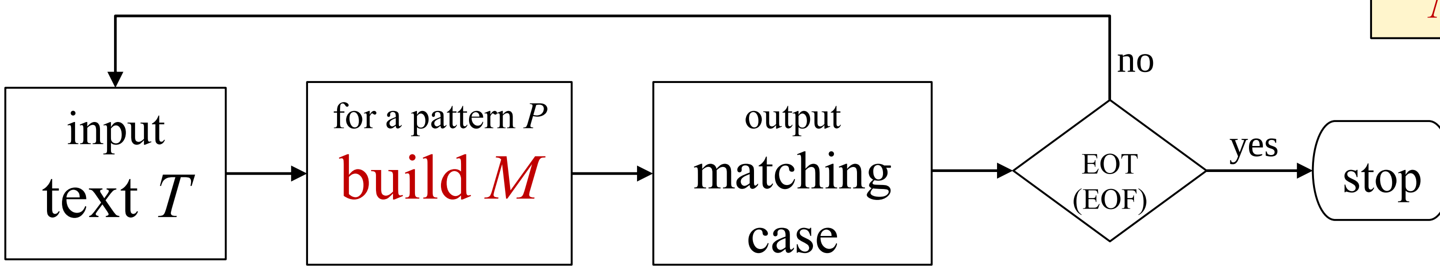
- **Many string-matching algorithms build a finite automaton (Machine) that scans the text *T* for all occurrences of the pattern *P*.**
- **These string-matching automata are very efficient:**
  - **they examine each text character *exactly once* ;**
  - **taking constant time per text character.**





32.3 String matching with finite automata（有限自动机）

- **These string-matching automata (Machine) are very efficient:**
  - **they examine each text character *exactly once* ;**
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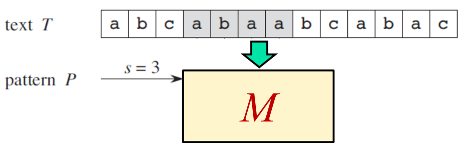
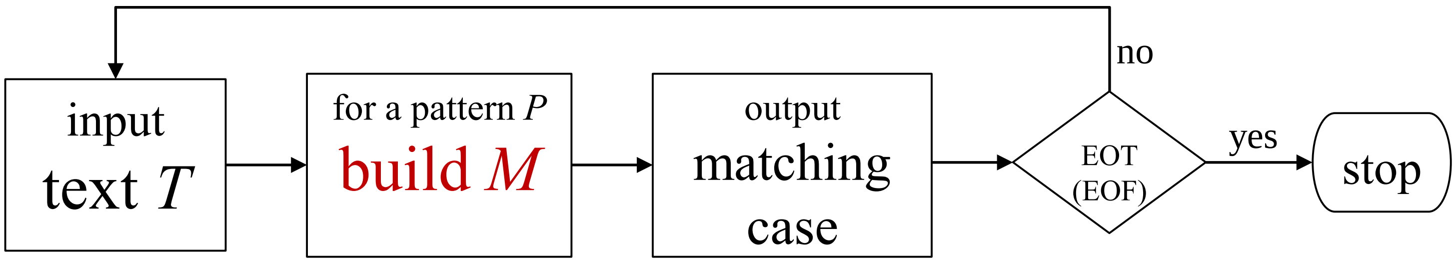
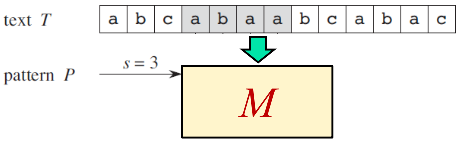


- **The matching time is $\Theta(n)$.**
  - **The preprocessing time (to build the automaton by pattern) can be large if Σ is large.** (对西文文本来说，小写26，大写26，数字10，共62， 再加上各种标点符号、或特殊符号、或希腊字母等，Σ约百余个字符，不算大；若中文，Σ可以很大)
  - **Section 32.4 describes a clever way around this problem.**

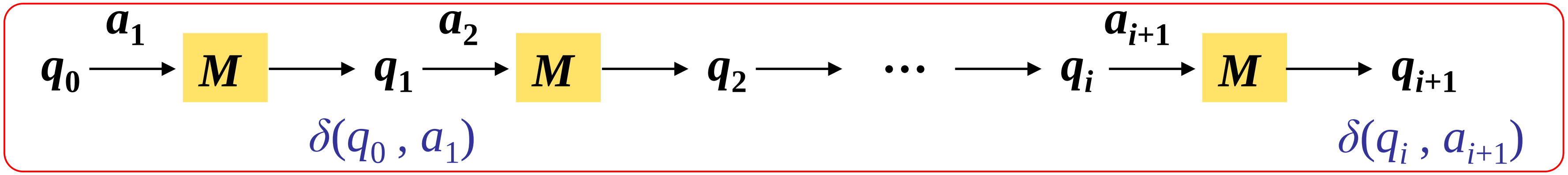


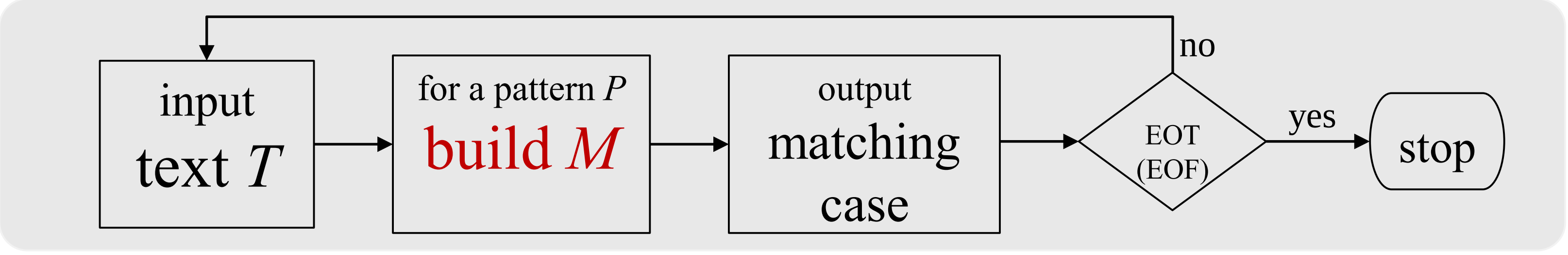
Finite automata **(Machine)**

- **A *finite automaton M* is a 5-tuple** $M = (Q, q_0, A, \Sigma, \delta)$ **, where**
  - Q is a finite set of ***states***,
  - $q_0 \in Q$ is the ***start state***,
  - $A \subseteq Q$ is a distinguished set of ***accepting states***,
  - Σ is a finite ***input alphabet***,
  - δ is a function from $Q \times \Sigma$ into Q, called the ***transition function*** of M.

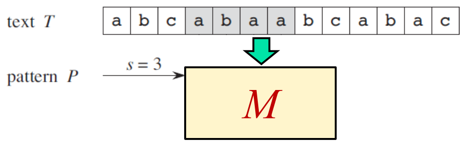


- The ***M*** begins in state q_0 and reads the characters of its input string one at a time. If the ***M*** is in state q and reads input a, it moves ("makes a transition") from state q to $\delta(q, a)$. Whenever its current state q is a member of A, the ***M*** is said to have ***accepted*** the string read so far. An input that is not accepted is said to be ***rejected***.





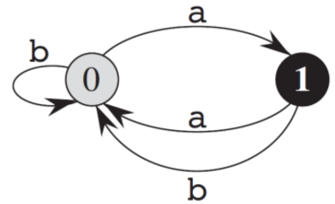
Finite automata: an example



- Figure 32.6: A simple two-state finite automaton with state set $\boldsymbol{Q = \{0, 1\}}$, start state $\boldsymbol{q_0 = 0}$, and input alphabet $\boldsymbol{\Sigma = \{a, b\}}$.

	input	
state	a	b
0	1	0
1	0	0

(a)



(b)

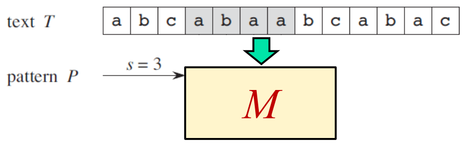
(a) A tabular representation of the transition function $\boldsymbol{\delta}$.

(b) An equivalent state-transition diagram.

a b a a a
<0, 1, 0, 1, 0, 1>

- **State 1 is the only accepting state** (shown blackened). Directed edges represent transitions. For example, the edge from state 1 to 0 labeled b indicates $\delta(1, b) = 0$. This automaton accepts those strings that end in an odd number of a's. For example, the sequence of states this automaton enters for input abaaa (including the start state) is <0, 1, 0, 1, 0, 1>, so it accepts this input. For input abbaa, the sequence of states is <0, 1, 0, 0, 1, 0>, so it rejects this input.

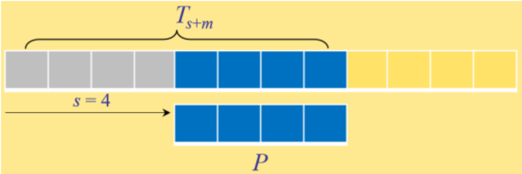
Finite automata: *final-state function*



- A finite automaton ***M*** induces a function Φ, called the ***final-state function***, from Σ^* to Q such that $\Phi(\omega)$ is the state ***M*** ends up in after scanning the string ω. 终态函数：***M*** 读入字符串 ω 后得到状态$\Phi(\omega)$
- Thus, ***M*** accepts a string ω if and only if $\Phi(\omega) \in A$.
- The function Φ is defined by the recursive relation

$$\Phi(\varepsilon) = q_0 ,$$

$$\Phi(\omega a) = \delta(\Phi(\omega), a) \text{ for } w \in \Sigma^*, a \in \Sigma .$$



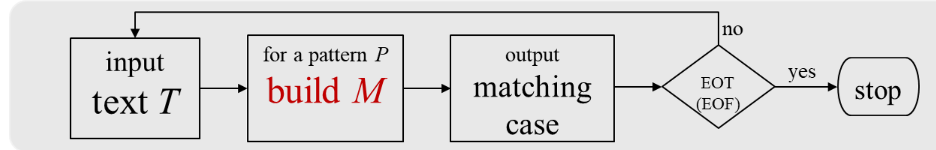
字符串匹配过程：*M* 从头到尾依序扫描文本 *T*，扫描到的字符串都是 *T* 的前缀 T_x，若有 *P* 匹配，则此时 *P* 为 T_x 的一个后缀。也就是，求文本 *T* 的前缀的 *P* 后缀。

$$\xrightarrow{\boldsymbol{\varepsilon}} \boxed{\mathbf{M}} \longrightarrow \mathbf{q_0} \xrightarrow{\boldsymbol{\omega}} \cdots \longrightarrow \boxed{\mathbf{M}} \longrightarrow \mathbf{q} \xrightarrow{\boldsymbol{a}} \boxed{\mathbf{M}} \longrightarrow \mathbf{q'}$$

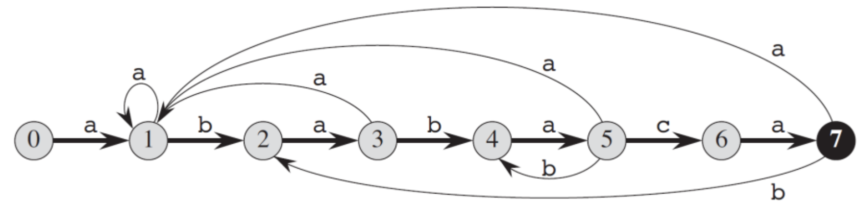
$q_0 = \Phi(\varepsilon)$ $\qquad$ $q = \Phi(\omega)$ $\qquad$ $q' = \Phi(\omega a) = \delta(\Phi(\omega), a)$

$$= \delta(q, a)$$

String-matching automata



- There is a string-matching automaton**(Machine)** for every pattern P
- This automaton must be constructed from the pattern in a preprocessing step before it can be used to search the text string.
- Figure illustrates this construction for the pattern P = ababaca.



(a)

state	input a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

$M = (Q, q_0, A, \Sigma, \delta)$

i	—	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	—	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

(c)

ω ω

aba abaa

| | | |

P : ababaca ababaca

$\delta(2, a) = 3$ $\delta(3, a) = 1$

$q' = \Phi(\omega a) = \delta(\Phi(\omega), a) = \delta(q, a)$

含义：读入ω时状态（与 P 中字符匹配个数）为q，再读入a时的状态是什么？

应用：读入T_k，看T_k的后缀跟P的前缀P_p的匹配情况，匹配数为m，则字符串匹配。

String-matching automata: an example

(a) A state-transition diagram for the string-matching automaton that accepts all strings ending in the string ababaca. State 0 is the start state, and state 7 (shown blackened) is the only accepting state. A directed edge from state i to state j labeled α represents $\delta(i, \alpha) = j$. The right-going edges forming the "spine" of the automaton, shown heavy, correspond to successful matches between pattern and input characters.

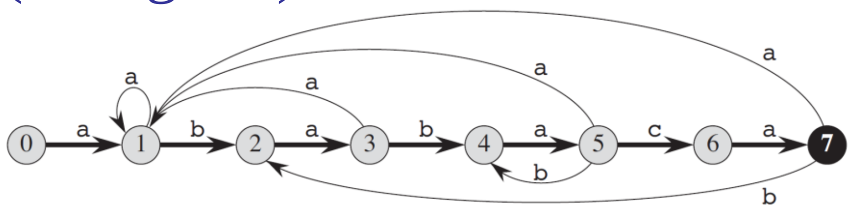
The left-going edges correspond to failing matches.

$\boldsymbol{M} = (Q, q_0, A, \Sigma, \delta)$

Some edges corresponding to failing matches are not shown; by convention, if a state i has no outgoing edge labeled α for some $\alpha \in \Sigma$, then $\delta(i, \alpha) = 0$.

M *of* P = ababaca

(a diagram)



(a)

Figure 32.7

String-matching automata: an example

(b) The corresponding transition function δ (a table), and the pattern string P = ababaca. The entries corresponding to successful matches between pattern and input characters are shown shaded.

(c) The operation of the automaton on the text T = abababacaba.

自动机 M 处理T_i后，其状态为 $\Phi(T_i)$

One occurrence of the pattern is found, ending in position 9.

$\boldsymbol{M} = (Q, q_0, A, \Sigma, \delta)$

M *of* P = ababaca
(a table)

	input			
state	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

i	—	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	—	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

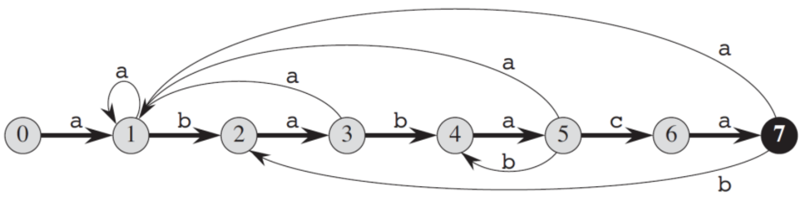
(c)

Kore: how to build $\delta(q, a)$?

String-matching automata: an example

The operation of the automaton on the text T = ababababacaba.

$M = (Q, q_0, A, \Sigma, \delta)$



(a)

自动机处理T_i后，其状态为$\Phi(T_i)$

$\delta(0, a) = 1, \delta(3, b) = 4, \delta(4, c) = 0, \delta(5, b) = 4$?

T a b a b a b a c a b a …

P a b a b a c a

state	input a	input b	input c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

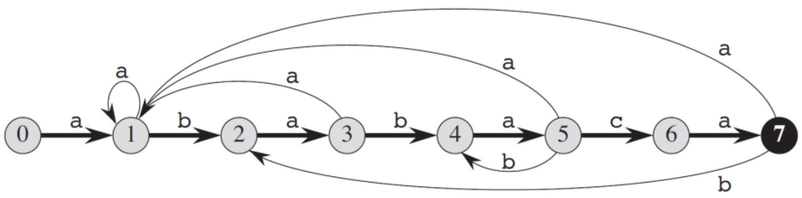
i	—	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	—	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

(c)

String-matching automata：an example

The operation of the automaton on the text T = ababababacaba.

$M = (Q, q_0, A, \Sigma, \delta)$



(a)

自动机处理T_i后，其状态为$\Phi(T_i)$

δ(0, a) = 1 ?

		input			
state	a	b	c		P
0	1	0	0		a
1	1	2	0		b
2	3	0	0		a
3	1	4	0		b
4	5	0	0		a
5	1	4	6		c
6	7	0	0		a
7	1	2	0		

(b)

T a b a b a b a c a b a …

P a b a b a c a

i	—	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	—	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

(c)

…

T a …

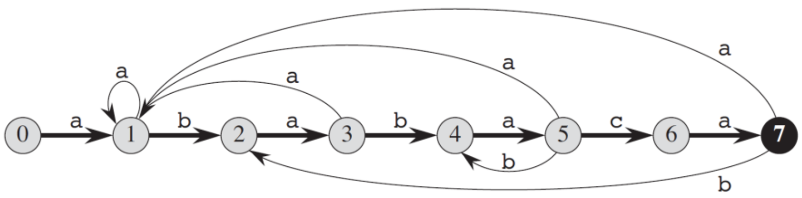
P a b a b a c a

从空字符开始，从文本串 T 的第一字符依序扫描，输入 a…时（…表示还有很多字符），P 的第1个跟其匹配，即 δ(0, a) = 1；【*从T一个一个地扫描，跟KMP有相似性】

String-matching automata：an example

The operation of the automaton on the text T = ababababacaba.

$M = (Q, q_0, A, \Sigma, \delta)$



(a)

自动机处理T_i后，其状态为$\Phi(T_i)$

$\delta(0, \text{a}) = 1$, $\boldsymbol{\delta(3, \text{b}) = 4}$?

	input			
state	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

T a b a b a b a c a b a …

P a b a b a c a

a b a …

T a b a b …

P a b a b

状态3时（有3个匹配），输入b，即文本串 T 为abab…时（…表示还有很多字符），P 的前4个跟其匹配，即 $\delta(3, \text{b}) = 4$

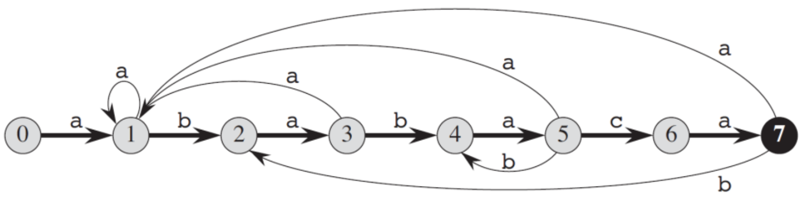
i	—	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	—	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

(c)

String-matching automata: an example

The operation of the automaton on the text T = ababababacaba.

$M = (Q, q_0, A, \Sigma, \delta)$



(a)

自动机处理T_i后，其状态为$\Phi(T_i)$

$\delta(0, a) = 1$, $\delta(3, b) = 4$, $\boldsymbol{\delta(4, c) = 0}$?

	input			
state	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

T a b a b a b a c a b a …

P a b a b a c a

a b a b …

T **a b a b c …**

P **a b a b a**

a b a b

a b a

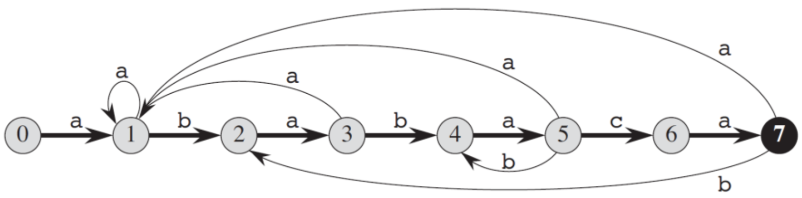
……

Naive: 状态4时（有4个匹配），输入c，即文本串 T 为ababc…时，P 的前5个跟其不匹配，即$\delta(4, c) \mathrel{!}= 5$；把 P 按字符右移1位（寻找新的可能匹配），P 的前4个跟其不匹配，即$\delta(4, c) \mathrel{!}= 4$；把 P 按字符右移2位，P 的前3个跟其不匹配，即$\delta(4, c) \mathrel{!}= 3$；以此类推。

String-matching automata: an example

The operation of the automaton on the text T = abababacaba.

$M = (Q, q_0, A, \Sigma, \delta)$



(a)

自动机处理T_i后，其状态为$\Phi(T_i)$

δ(0, a) = 1, δ(3, b) = 4, δ(4, c) = 0, **δ(5, b) = 4** ?

T a b a b a b a c a b a …

P a b a b a c a

	input			
state	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

a b a b a …

T **a b a b a b …**

P **a b a b a c**

a b a b a

a b a b

每次把 P 右移1位后，都从 P 的第一个字符开始匹配，跟naive方法一样?

肯定不用这样做!

快速右移到 $\delta(q, x)$ 处!

求 $\delta(q, x)$ 是关键!

i	—	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	—	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

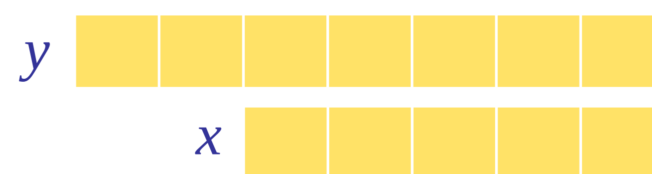
(c)

String-matching automata: ***suffix function***

- Suffix function σ corresponding to P :
  A mapping from Σ^* to $\{0, 1, . . . , m\}$ such that $\sigma(x)$ is the length of the longest prefix of P that is a suffix of x （x 的后缀且是P的最长前缀的长度）：

  $$\sigma(x) = \max \{k : P_k \sqsupset x\}.$$

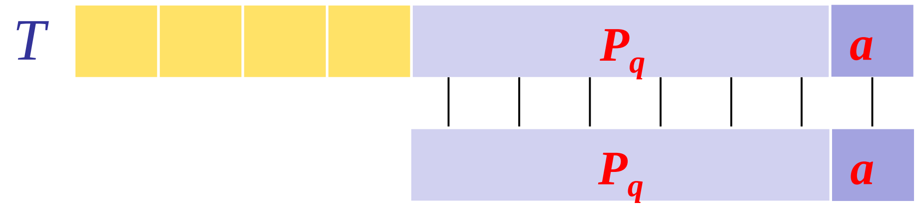
- The suffix function σ is well defined since the empty string $P_0 = \varepsilon$ is a suffix of every string. As examples,
  - for the pattern P = ab, we have $\sigma(\varepsilon) = 0$, $\sigma(\text{ccaca}) = 1$, and $\sigma(\text{ccab}) = 2$.
- For a pattern P of length m, we have $\sigma(x) = m$ if and only if $P \sqsupset x$.
- From the definition of the suffix function, if $x \sqsupset y$, then $\sigma(x) \le \sigma(y)$.



String-matching automata

$M = (Q, q_0, A, \Sigma, \delta)$

$\sigma(x) = \max\{k : P_k \sqsupset x\}.$



We define the ***string-matching automaton*** that corresponds to a given pattern $P[1 .. m]$ as follows.

- The transition function δ is defined by the following equation, for any state q and character a:

$$\boldsymbol{\delta(q, a) = \sigma(P_q a)} \qquad \textbf{(32.3)}$$

- where, the state set Q is $\{0, 1, \ldots, m\}$, the start state q_0 is state 0, and state m is the only accepting state A.

$\delta(q, a) = \sigma(P_q a)$ 的定义合理，后面将证明，$\delta(q, a) = \sigma(P_q a) = \sigma(T_i a)$, 即，扫描 T_i 后，匹配为 P_q，接着读入a，对 $T_i a$ 的匹配与对 $P_q a$ 的匹配是一样的(**Lemma 32.1**)。 $P_q a$ 的长度比 $T_i a$ 短，处理起来（求δ）就简单得多。

String-matching automata

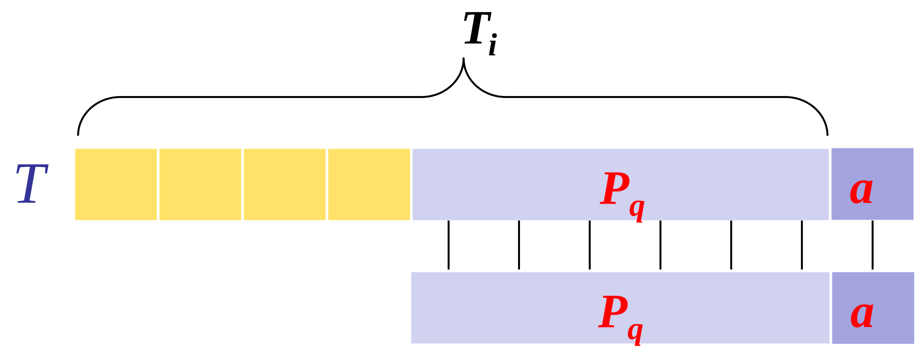
$M = (Q, q_0, A, \Sigma, \delta)$

$\sigma(x) = \max \{k : P_k \sqsupset x\}.$

We define the *machine* $\boldsymbol{M}$:

$Q = \{0, 1, \ldots, m\}$; $q_0 = 0$; $A = \{m\}$;

$$\boldsymbol{\delta(q, a) = \sigma(P_q a)} \qquad \textbf{32.3)}$$



Intuitively, the machine ***M*** maintains an invariant:

$$\boldsymbol{\Phi(T_i) = \sigma(T_i)}\ , \qquad \textbf{(where, } \boldsymbol{\Phi(T_i) = q = \sigma(T_i)}\ \textbf{).} \qquad \textbf{(32.4)}$$

自动机 M 扫描字符串 T 的过程中，扫描到前缀子串 T_i 时状态为 q（为 T_i 的后缀函数$\sigma(T_i)$），接着扫描下一个字符 $T[i+1]$（记为a），状态转移到$\delta(q, a) = \sigma(P_q a)$，这就是扫描到前缀子串 T_{i+1} 时状态（为 T_{i+1} 的后缀函数$\sigma(T_{i+1})$）

$$\boldsymbol{\Phi(T_{i+1}) = \Phi(T_i a) = \delta(\Phi(T_i), a) = \delta(q, a) = \sigma(P_q a) \overset{?}{=} \sigma(T_i a) = \sigma(T_{i+1})} \qquad \textbf{(32.4)*}$$

[(32.3) maintains the invariant, or, it is rationale for defining (32.3).]

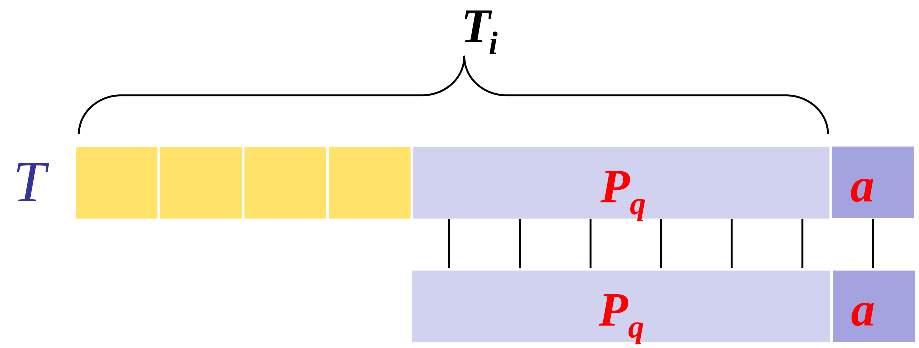
String-matching automata

$\sigma(x) = \max\{k : P_k \sqsupset x\}.$

We define the *machine* $\boldsymbol{M}$:

$Q = \{0, 1, \ldots, m\}$; $q_0 = 0$; $A = \{m\}$; Σ;

$$\boldsymbol{\delta(q, a) = \sigma(P_q a)} \qquad \textbf{(32.3)}$$



- $\boldsymbol{\Phi(T_{i+1}) = \Phi(T_i a) = \delta(\Phi(T_i), a) = \delta(q, a) = \sigma(P_q a) \overset{?}{=} \sigma(T_i a) = \sigma(T_{i+1})}$ **(32.4)**
  [(32.3) maintains the invariant, or, it is rationale for defining (32.3).]

- *Lemma 32.3*: $\boldsymbol{\sigma(T_i a) = \sigma(P_q a)}$ (32.A)

  this lemma means definition (32.3) maintains the desired invariant (32.4).

- **Compute: With (32.A), to compute $\boldsymbol{\sigma(T_i a)}$, we can compute $\boldsymbol{\sigma(P_q a)}$.**

String-matching automata

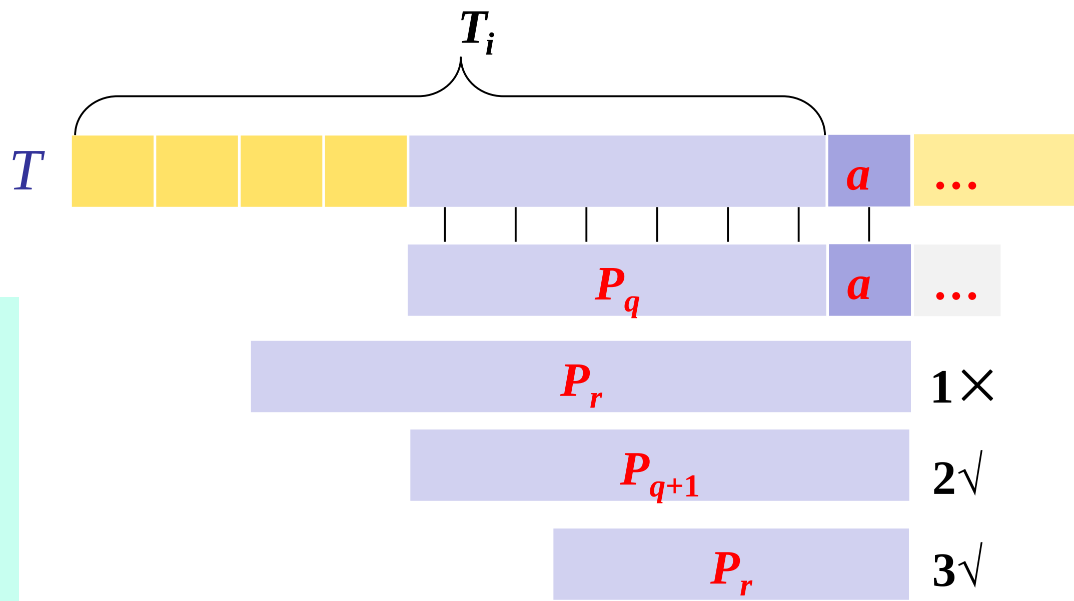
$\sigma(x) = \max \{k : P_k \sqsupset x\}$.

We define the *machine* $\boldsymbol{M}$:

$Q = \{0, 1, \ldots, m\}$;

$q_0 = 0$; $A = \{m\}$; Σ;

$\boldsymbol{\delta(q, a) = \sigma(P_q a)}$. **(32.3)**

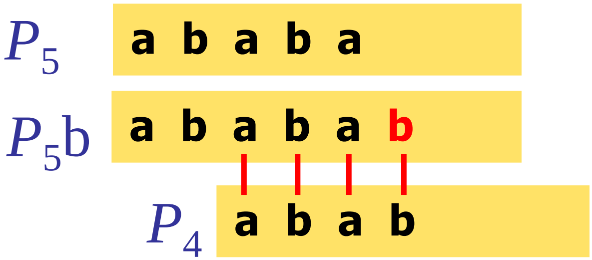


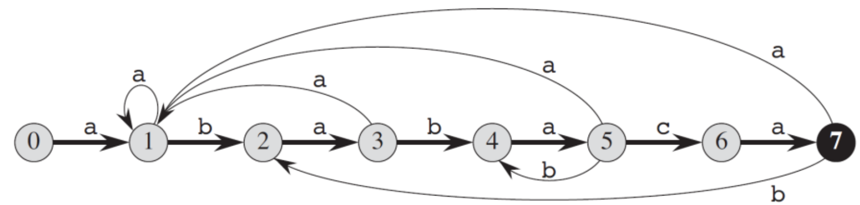
- $\boldsymbol{\Phi(T_{i+1}) = \Phi(T_i a) = \delta(\Phi(T_i), a) = \delta(q, a) = \sigma(P_q a) = \sigma(T_i a) = \sigma(T_{i+1})}$ **(32.4)**
  [(32.3) maintains the invariant, or, it is rationale for defining (32.3).]
- ***Lemma 32.3*****: If $\boldsymbol{\sigma(T_i) = \sigma(P_q) = q}$, then $\boldsymbol{\sigma(T_i a) = \sigma(P_q a)}$.** **(32.A)**
  Proof
  Situation 1 is impossible（$\boldsymbol{\sigma(T_i a) > q+1}$不可能）. if 1 满足， $\boldsymbol{\sigma(T_i) > q}$, 与假设矛盾。
  Apparently, if $\boldsymbol{P[q+1] = a}$, it is situation 2, $\boldsymbol{\sigma(T_i a) = \sigma(P_{q+1}) = q+1}$; else , situation 3.
- ***Form 32.3*** **and** ***32.A*** **shows the automaton is in state $\boldsymbol{\sigma(T_i)}$ after scanning character $\boldsymbol{T[i]}$. Since $\boldsymbol{\sigma(T_i) = m}$ if and only if $\boldsymbol{P \sqsupset T_i}$, the machine is in the accepting state $\boldsymbol{m}$ if and only if the pattern $\boldsymbol{P}$ has just been scanned.**

String-matching automata

For example, in the string-matching automaton of Figure 32.7, $\delta(5, \mathbf{b}) = 4$.

We make this transition because if the automaton reads a **b** in state $q = 5$, then $P_q\mathbf{b} = \mathbf{ababab}$, and then, $\delta(5, \mathbf{b}) = \sigma(\mathbf{ababab}) = 4$.





(a)

$\sigma(x) = \max\{k : P_k \sqsupset x\}$.

We define the *machine* $\boldsymbol{M}$:

$Q = \{0, 1, \ldots, m\}$;

$q_0 = 0$; $A = \{m\}$; Σ;

$$\boldsymbol{\delta(q, a) = \sigma(P_q a)}. \qquad \mathbf{(32.3)}$$

	input			
state	a	b	c	*P*
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

Figure 32.7

i	—	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	—	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

(c)

String-matching automata: *program*

$\sigma(x) = \max \{k : P_k \sqsupset x\}.$

A string-matching automaton

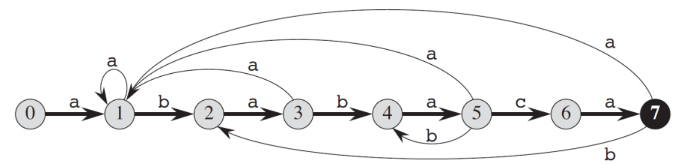
$\boldsymbol{M} = (Q, q_0, A, \Sigma, \delta)$:

$Q = \{0, 1, \ldots, m\}; \;\; q_0 = 0; \;\; A = \{m\};$

$$\boldsymbol{\delta(q, a) = \sigma(P_q a) = \sigma(T_{i\text{-}1} a)} \; . \qquad \textbf{(32.3)}$$

FINITE-AUTOMATON-MATCHER(*T, δ, m*)

$n \leftarrow length[T]$
$q \leftarrow 0$
for *i* ← 1 to *n*   **// scan *T***
    ***a* ← *T*[*i*]**
    ***q* ← *δ*(*q*, *a*)**
    if *q* == *m*
        print "Pattern occurs with shift" *i* - *m*



(a)

	input			
state	a	b	c	*P*
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

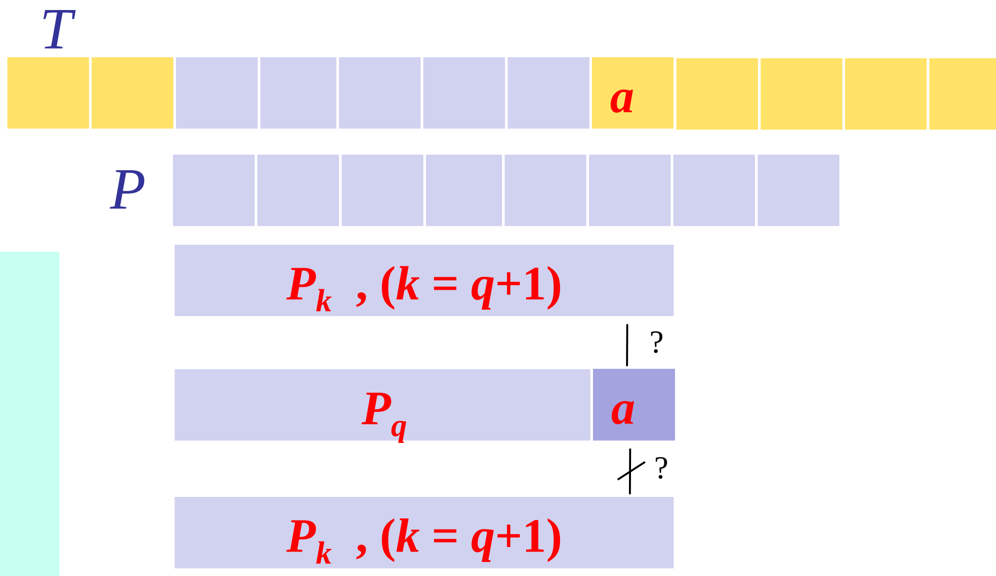
i	—	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	—	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

(c)

Running time ?

- **The matching time is Θ(*n*).**
- **However, it does not include the preprocessing time required to compute the transition function *δ*.**

Computing the transition function



$\sigma(x) = \max\{k : P_k \sqsupset x\}$.

A string-matching automaton

$M = (Q, q_0, A, \Sigma, \delta)$:

$Q = \{0, 1, \ldots, m\}$; $q_0 = 0$; $A = \{m\}$;

$$\boldsymbol{\delta(q, a) = \sigma(P_q a)} \;. \qquad \mathbf{(32.3)}$$

Computing δ from a given pattern $P[1 \,..\, m]$:

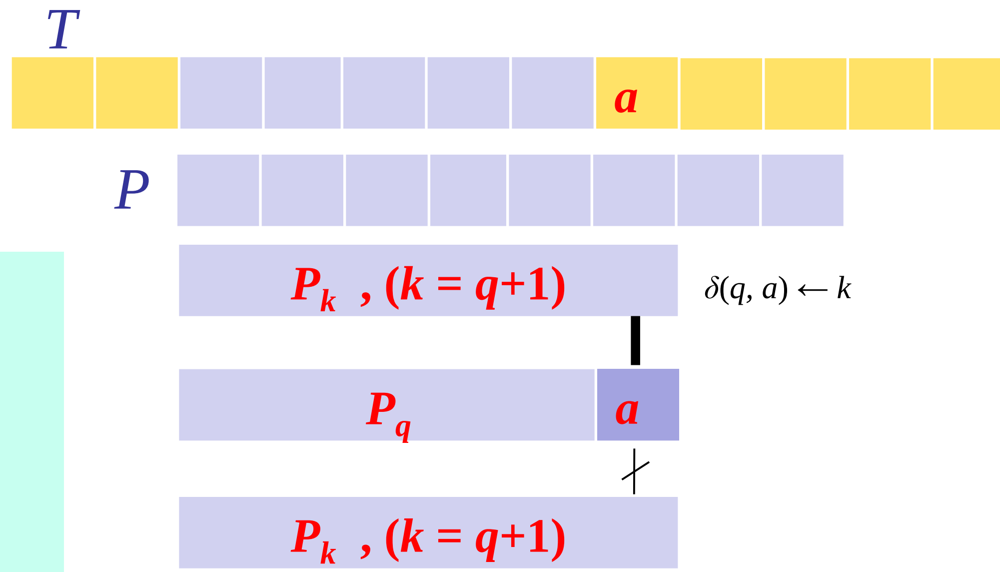
```
COMPUTE-TRANSITION-FUNCTION(P, Σ)
m ← length[P]
for q ← 0 to m
    for each character a ∈ Σ
        k ← min(m, q + 1)
        while P_k !⊐ P_q a
            k--
        δ(q, a) ← k
return δ
```

P_q 时（即T与P的前q个字符匹配时），输入第 q+1（即第k个）字符 a 时:

1. $k = q+1$ （超过m时，取m，匹配数不大于m）

2. $P_k \sqsupset P_q a$ ？

Computing the transition function



$\sigma(x) = \max\ \{k : P_k \sqsupset x\}$.

A string-matching automaton

$M = (Q, q_0, A, \Sigma, \delta)$:

$Q = \{0, 1, \ldots, m\}$; $q_0 = 0$; $A = \{m\}$;

$$\delta(q, a) = \sigma(P_q a)\ . \qquad (32.3)$$

Computing δ from a given pattern $P[1 .. m]$:

```
COMPUTE-TRANSITION-FUNCTION(P, Σ)
m ← length[P]
for q ← 0 to m
    for each character a ∈ Σ
        k ← min(m, q + 1)
        while P_k !⊐ P_q a
            k--
        δ(q, a) ← k
return δ
```

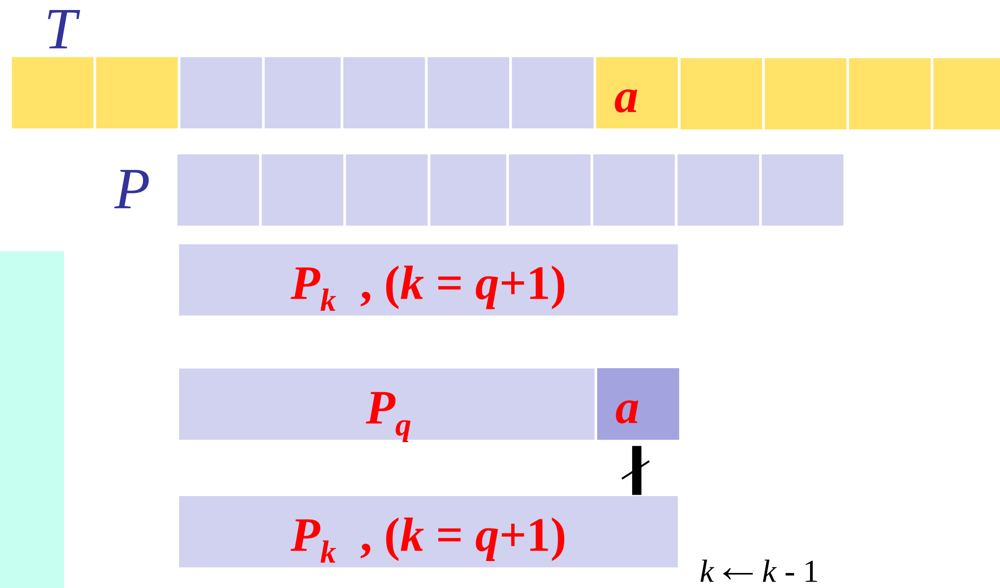
P_q 时（即T与P的前q个字符匹配时），输入第 q+1（即第k个）字符 a 时：

1. $k = q+1$ （超过m时，取m，匹配数不大于m）

2. $P_k \sqsupset P_q a$ ？

3. 若2成立，则 $P_q a == P_k$，即，对在 T 的继续扫描过程中，若扫描的下一个字符 $a{==}P[k]$，则匹配字符增加1

Computing the transition function



$\sigma(x) = \max\ \{k : P_k \sqsupset x\}$.

A string-matching automaton

$M = (Q, q_0, A, \Sigma, \delta)$:

$Q = \{0, 1, . . . , m\}$; $q_0 = 0$; $A = \{m\}$;

$\delta(q, a) = \sigma(P_q a)$. (32.3)

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1. $k = q+1$ （超过m时，取m，匹配数不大于m）
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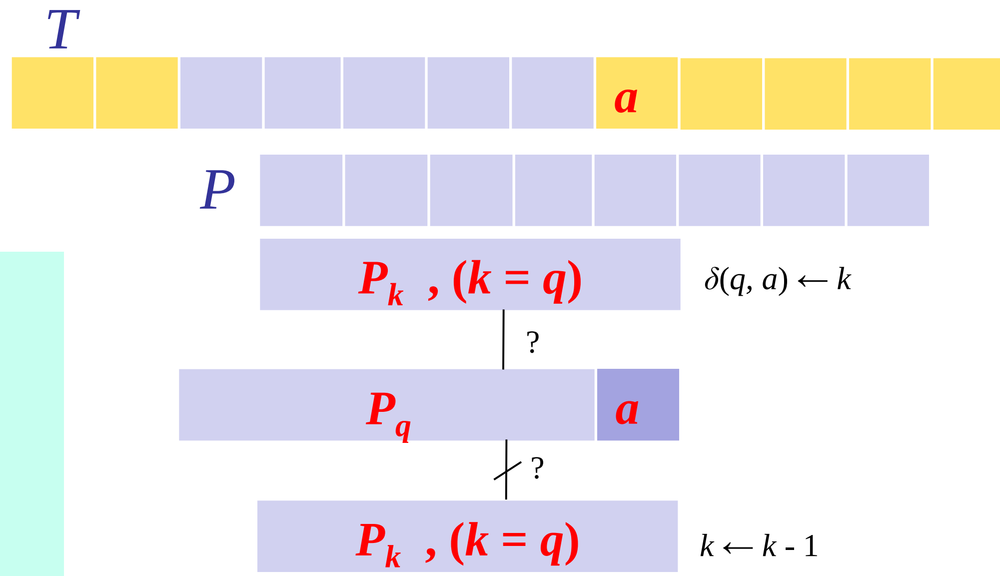
3. 若2成立，则 $P_q a == P_k$ ，即，对在 T 的继续扫描过程中，若扫描的下一个字符 a==$P[k]$，则匹配字符增加1
4. 若2不成立，即 $P_q a$!= P_k ，即，对在T 的继续扫描过程中，若扫描的下一个字符a != $P[k]$，模版右移(k--)，goto step 2

Computing the transition function



$\sigma(x) = \max\ \{k : P_k \sqsupset x\}.$

A string-matching automaton

$\boldsymbol{M} = (Q, q_0, A, \Sigma, \delta)$:

$Q = \{0, 1, \ldots, m\};\ \ q_0 = 0;\ \ A = \{m\};$

$$\boldsymbol{\delta(q, a) = \sigma(P_q a)}\ . \qquad \textbf{(32.3)}$$

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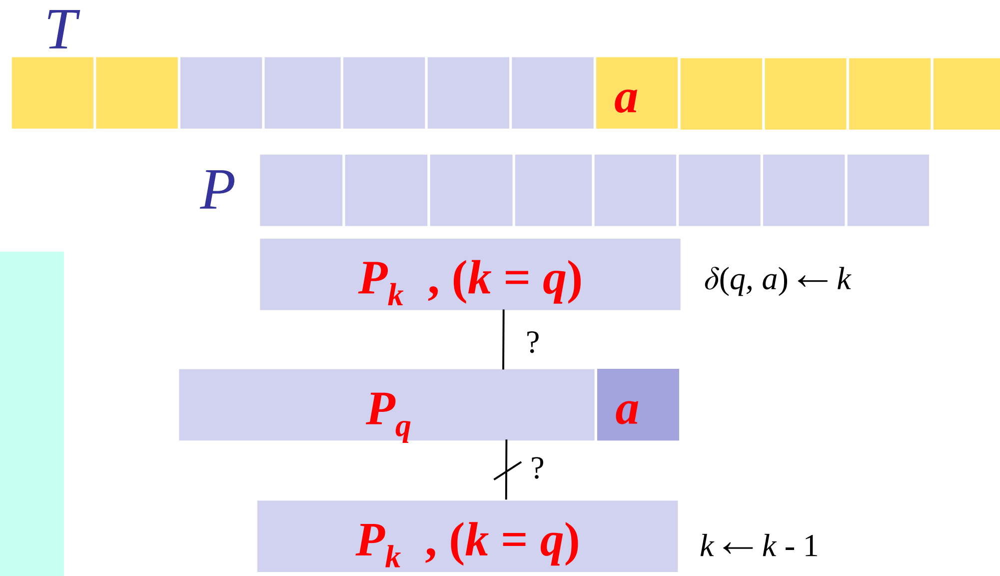
            k--
        δ(q, a) ← k
return δ
```

P_q 时（即T与P的前q个字符匹配时），输入第 q+1（即第k个）字符 a 时:

1. $k = q+1$ （超过m时，取m，匹配数不大于m）
2. $P_k \sqsupset P_q a$ ？
3. 若2成立，则 $P_q a == P_k$ ，即，对在 T 的继续扫描过程中，若扫描的下一个字符 a==$P[k]$，则匹配字符增加1
4. 若2不成立，即 $P_q a\ != P_k$ ，即，对在T 的继续扫描过程中，若扫描的下一个字符a != $P[k]$，模版右移(k--)，goto step 2

Computing the transition function



$\sigma(x) = \max \{k : P_k \sqsupset x\}$.

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        while P_k !⊐ P_q a
            k--
        δ(q, a) ← k
return δ
```
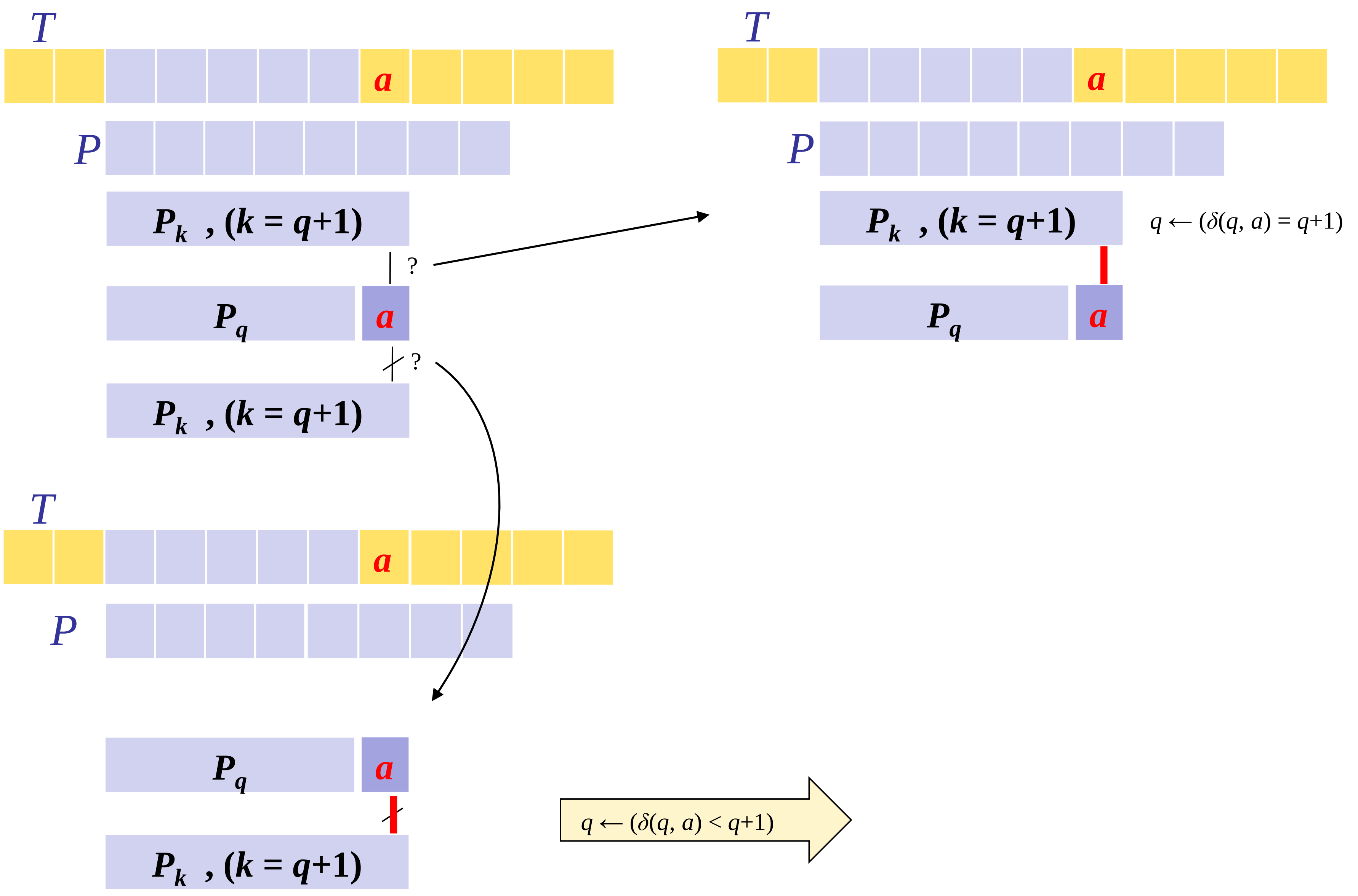
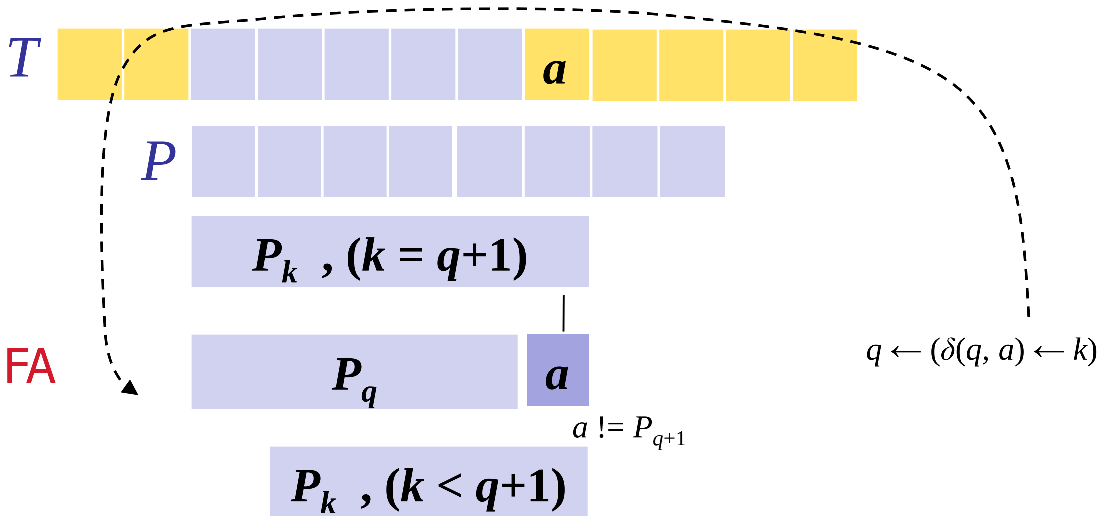

- **Running time ?**

32.3 String matching with finite automata



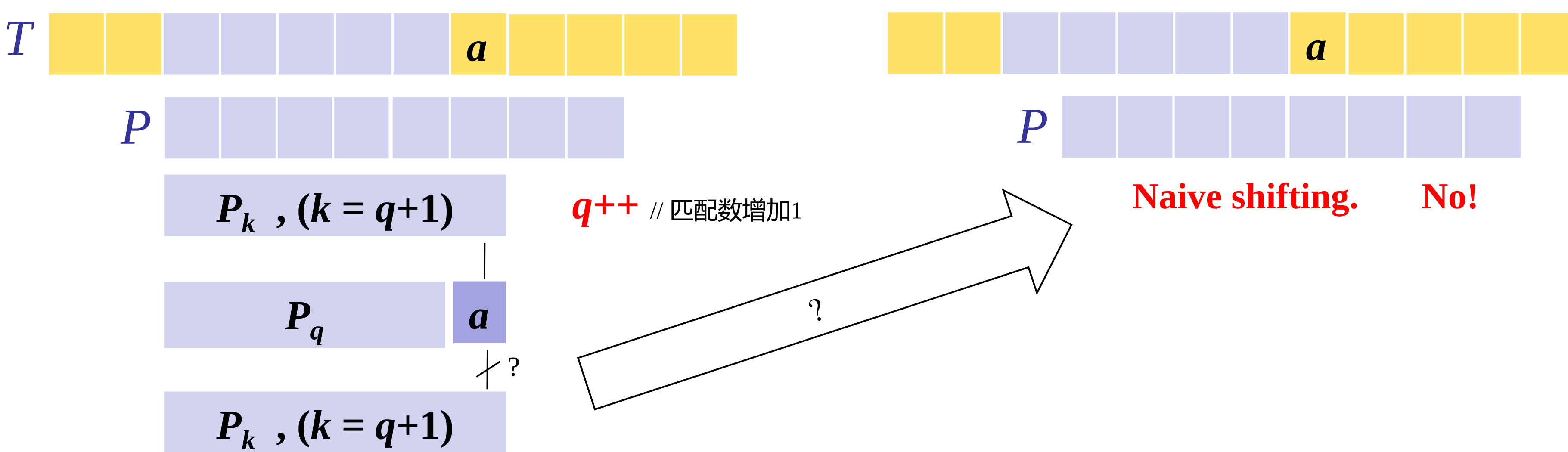
*32.4 The Knuth-Morris-Pratt algorithm



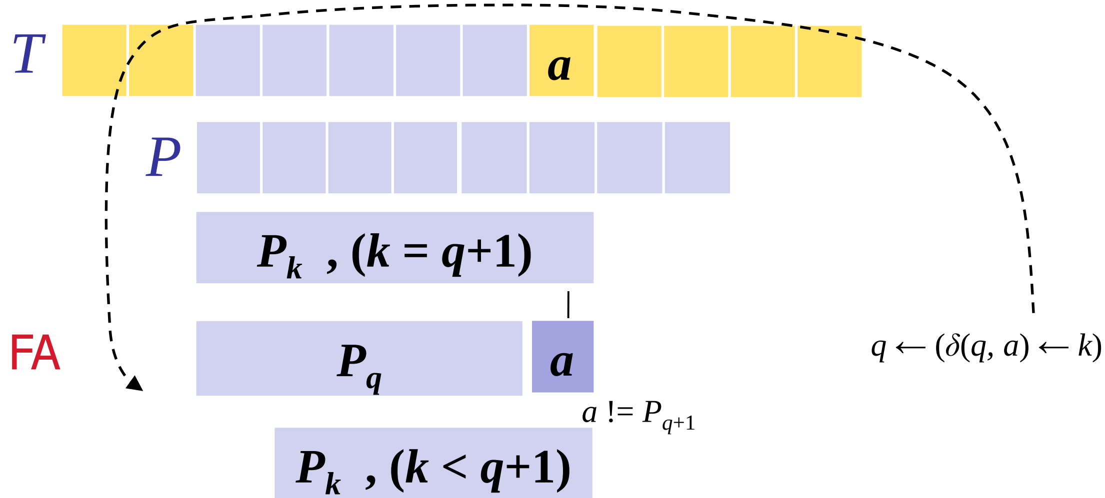
根据输入的 a，然后查表 δ 知道需要转移的位置。

自动机构造完后，从 δ 已经知道输入 a 后 P 应快速右移多少（这种思想跟 KMP 相似，但 FA 的核心在于求 δ 有额外计算开销）。

T 中一个字符扫描一次。



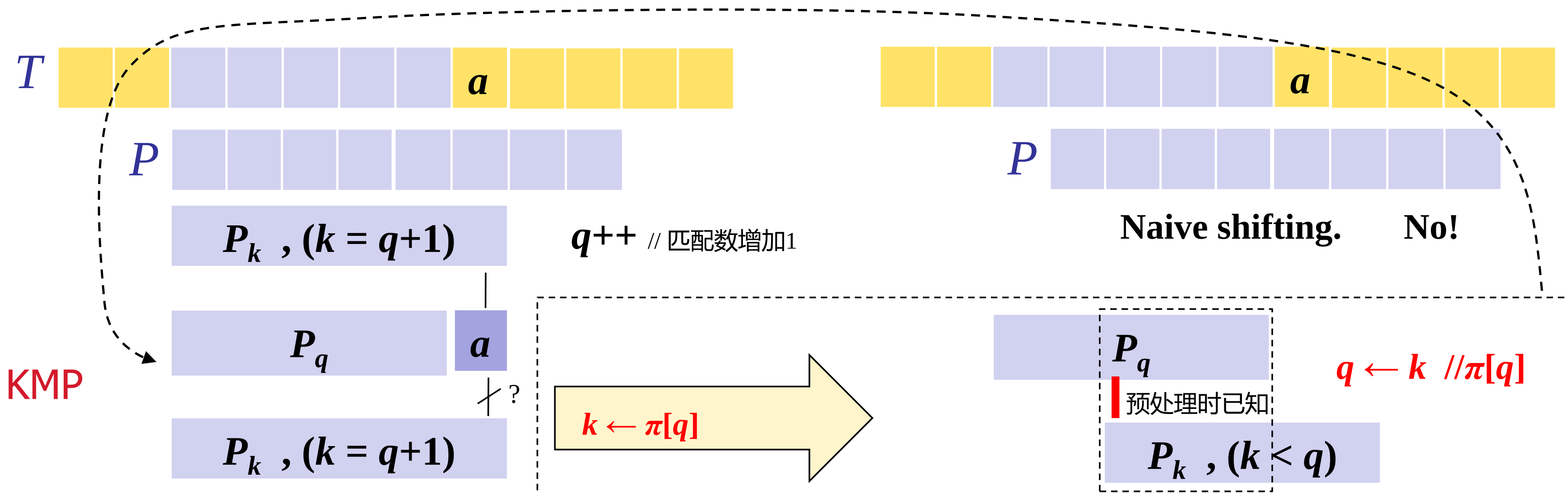
*32.4 The Knuth-Morris-Pratt algorithm



根据输入的 a，然后查表 δ 知道需要转移的位置。

自动机构造完后，从 δ 已经知道输入 a 后 P 应快速右移多少（这种思想跟 KMP 相似，但 FA 的核心在于求 δ 有额外计算开销）。

T 中一个字符扫描一次。

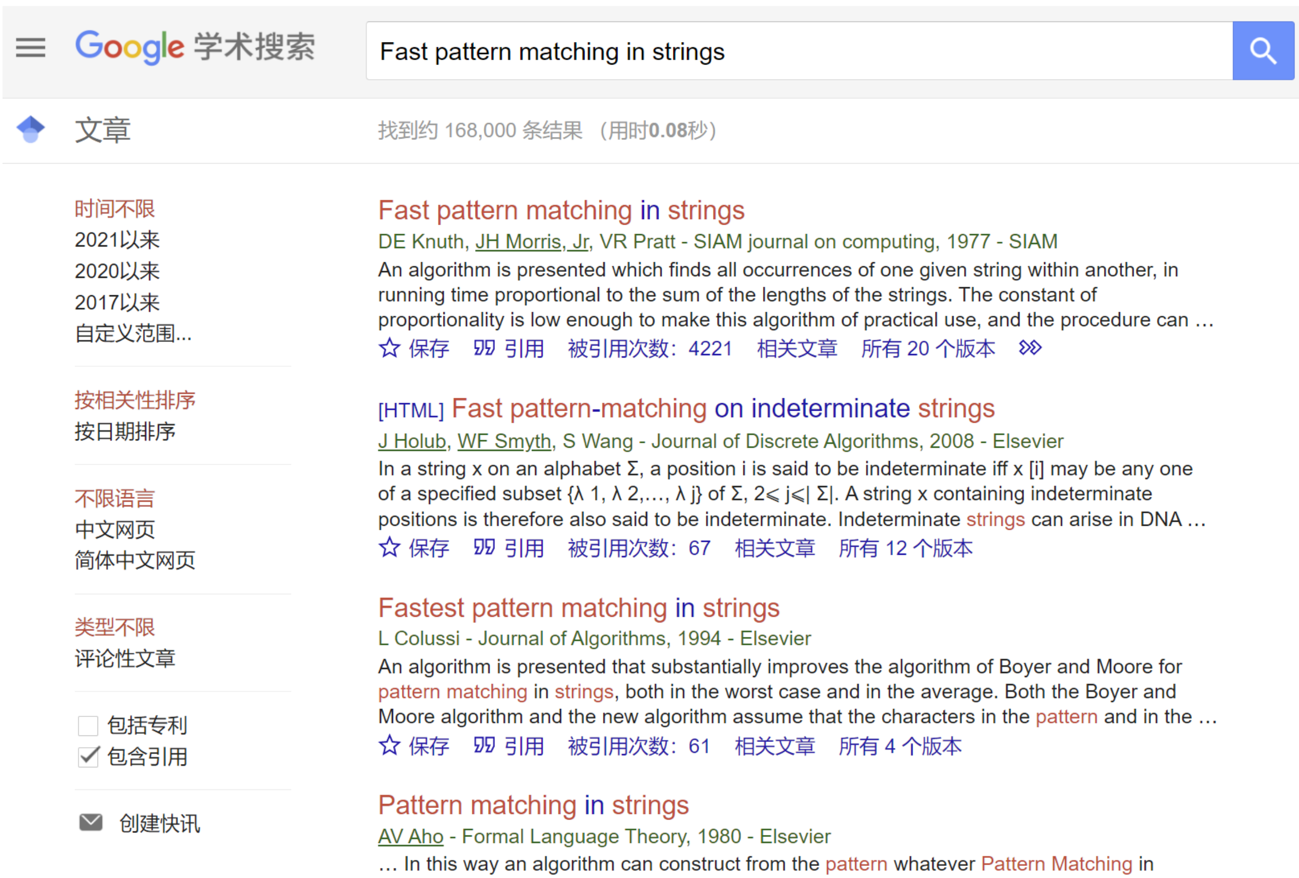


KMP本质：P的前缀P_q与T的匹配；P_k ($k<q$)是 P_q 的后缀（也是 P_q 的前缀），最大的k是多少？**$q \leftarrow \pi(q)$ 后，重新看是否 a == P[q+1]** （重复执行此过程）

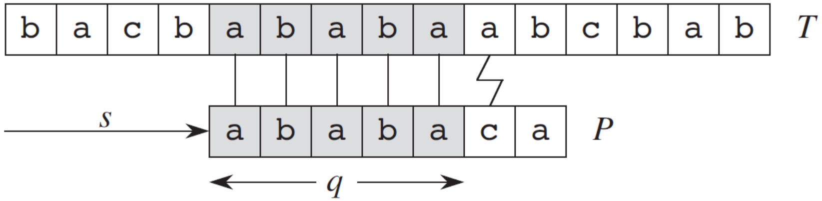
KMP 的前缀函数构造完后，不需要输入下一个 a 就知道移动多少位置（从 P_q 移动到 P_k）。T 中一个字符可能扫描多次。

*32.4 The Knuth-Morris-Pratt algorithm

KMP is a linear-time string-matching algorithm due to Knuth, Morris, and Pratt.



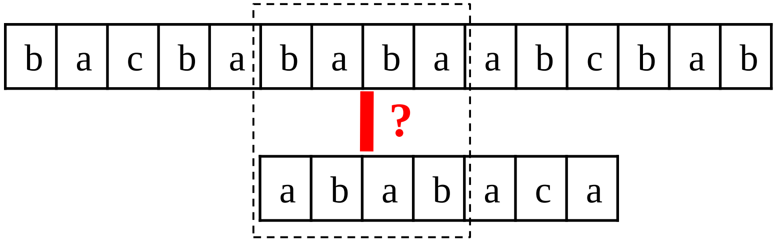
*32.4 The Knuth-Morris-Pratt algorithm



(a)

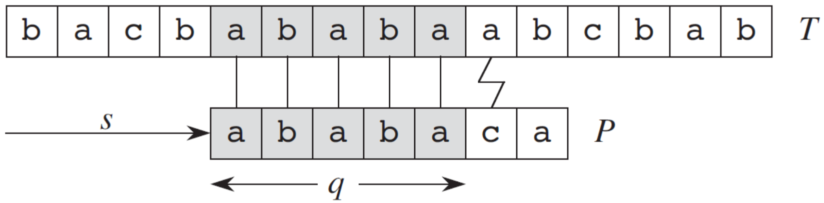
$P_5 \sqsupset T_{s+5}$, but,

$T[s+5+1] \mathrel{!=} P[5+1]$ ($q = 5$)



Naive shifting, $P_4 \sqsupset T_{s+1+4}$? No!

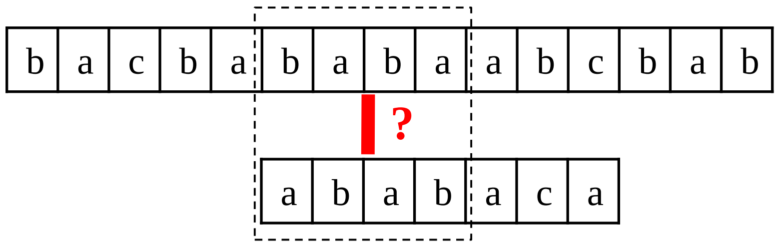
*32.4 The Knuth-Morris-Pratt algorithm



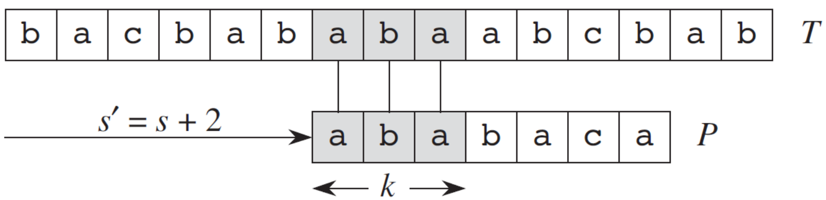
(a)

$P_5 \sqsupset T_{s+5}$, but,

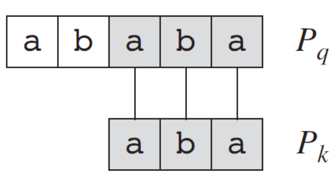
$T[s+5+1] \mathrel{!=} P[5+1]$ ($q = 5$)



Naive shifting, $P_4 \sqsupset T_{s+1+4}$? No!



(b)



(c)

We have already known that P_k is the maximum suffix of P_q, that is, $P_{k'} \sqsupset P_q$ ($k' < q$, and $k = \max(k')$). For this example, q is 5, k is 3. So, we have $P_3 \sqsupset P_5 \sqsupset T_{s+5}$, we just check whether ..

$T[s+5+1] \mathrel{!=} P[3+1]$ ($q \leftarrow k = 3$)

*32.4 The Knuth-Morris-Pratt algorithm

prefix function for the pattern P is the function..

π : {1, 2, …, m} → {0, 1, …, m-1} such that

$\boldsymbol{\pi[q] = \max\{k : k < q \text{ and } P_k \sqsupset P_q\}}$

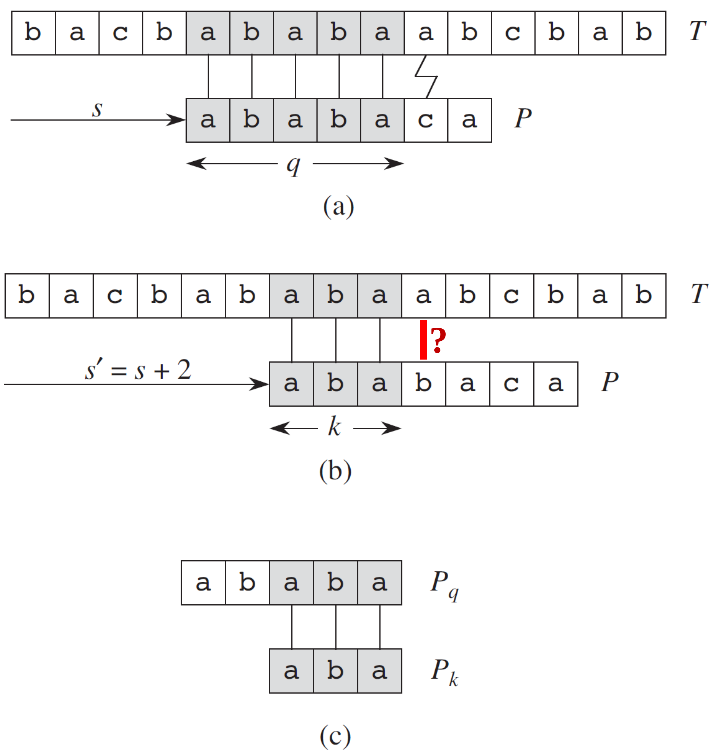
P_q是 P 的前缀，
P_k是 P_q的前缀，且 P_k是 P_q的后缀，
最大的 k 即为 $\pi[q]$

P = ababaca
P_5 = ababa
P_3 = aba

$\boldsymbol{\pi[5] = 3}$

P 的 前缀P_q的后缀是P 的 前缀P_k （最长的）

✔ P 的 前缀P_q的 前缀P_k是 前缀P_q的后缀（最长的）



$P_5 \sqsupset T_{s+5}$, but,

$T[s+5+1]$!= $P[5+1]$ (q = 5)

We have already known that P_k is the maximum suffix of P_q , that is, $P_{k'} \sqsupset P_q$ ($k' < q$, and $k = \max(k')$). For this example, q is 5, k is 3. So, we have $P_3 \sqsupset P_5 \sqsupset T_{s+5}$, we just check whether ..

$T[s+5+1]$!= $P[3+1]$ ($q \leftarrow k = 3$)

*32.4 The Knuth-Morris-Pratt algorithm

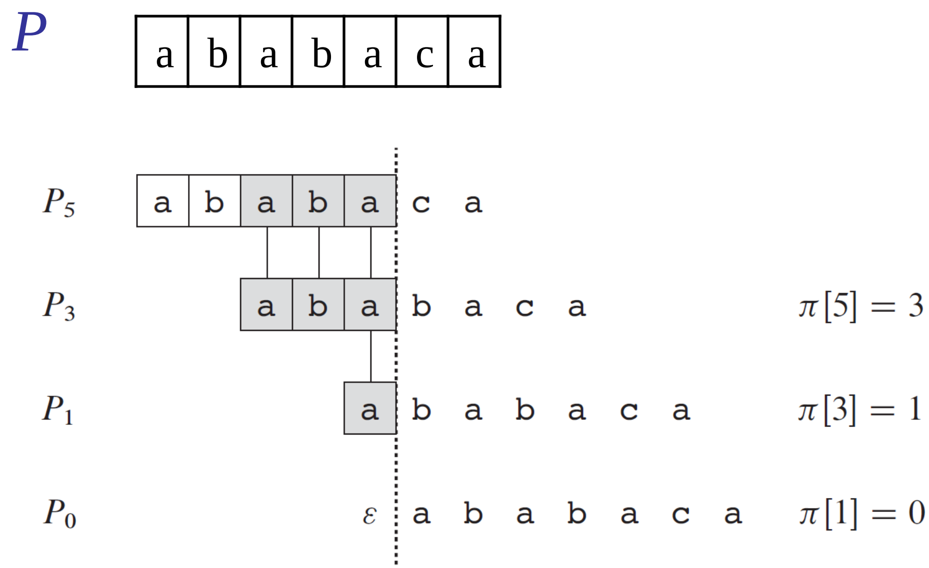
prefix function for the pattern P is the function..

π : {1, 2, …, m} → {0, 1, …, m-1} such that

$$\pi[q] = \max\{k : k < q \text{ and } P_k \sqsupset P_q\}.$$

i	1	2	3	4	5	6	7
$P[i]$	a	b	a	b	a	c	a
$\pi[i]$	0	0	1	2	3	0	1

(a)



(b)

P 的 前缀P_q 的 前缀P_k 是 前缀P_q 的后缀（最长的）

*32.4 The Knuth-Morris-Pratt algorithm

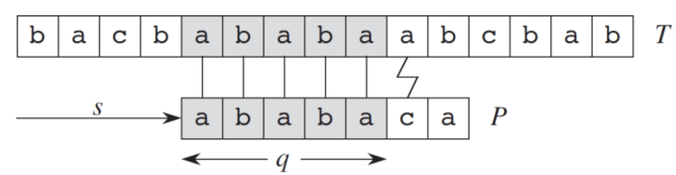
```
KMP-MATCHER(T, P)
n = T.length
m = P.length
π = COMPUTE-PREFIX-FUNCTION(P)
q = 0                                        // number of characters matched
for i = 1 to n                               // scan the text from left to right
    while q > 0 and P[q + 1] ≠ T[i]
        q = π[q]                             // next character does not match
    if P[q + 1] == T[i]
        q = q + 1                            // next character matches
    if q == m                                // is all of P matched?
        print "Pattern occurs with shift" i − m
        q = π[q]                             // look for the next match
```

?

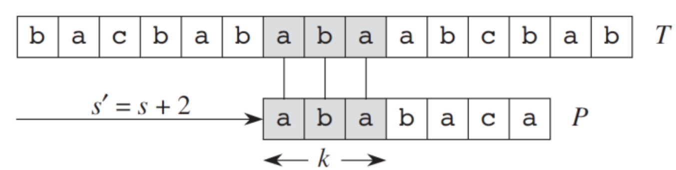
we have $P_q \sqsupset T[i\text{-}1]$,

check whether $P[q+1] \neq T[i]$

i	1	2	3	4	5	6	7
$P[i]$	a	b	a	b	a	c	a
$\pi[i]$	0	0	1	2	3	0	1



(a)



(b)

*32.4 The Knuth-Morris-Pratt algorithm

```
KMP-MATCHER(T, P)
n = T.length
m = P.length
π = COMPUTE-PREFIX-FUNCTION(P)
q = 0
for i = 1 to n
    while q > 0 and P[q + 1] ≠ T[i]
        q = π[q]
    if P[q + 1] == T[i]
        q = q + 1
    if q == m
        print "Pattern occurs with shift" i − m
        q = π[q]
```

```
COMPUTE-PREFIX-FUNCTION(P)
m = P.length
let π[1 . . m] be a new array
π[1] = 0
k = 0
for q = 2 to m
    while k > 0 and P[k + 1] ≠ P[q]
        k = π[k]
    if P[k + 1] == P[q]
        k = k + 1
    π[q] = k
return π
```

we have $P_k \sqsupset P[q\text{-}1]$,
check if $P[k+1] \neq P[q]$

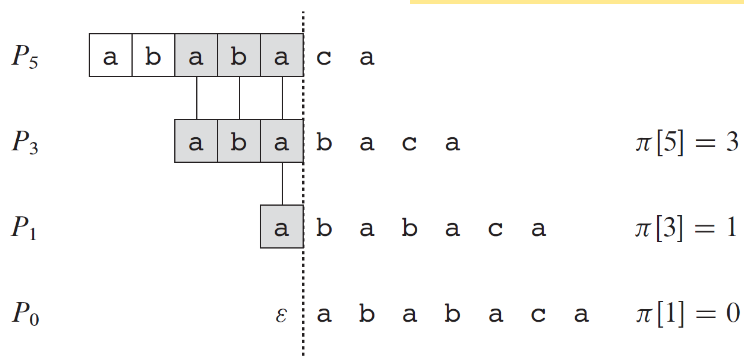
prefix function for the pattern P is the function..

$\pi : \{1, 2, \ldots, m\} \rightarrow \{0, 1, \ldots, m\text{-}1\}$ such that

$\pi[q] = \max\{k : k < q \text{ and } P_k \sqsupset P_q \}$.

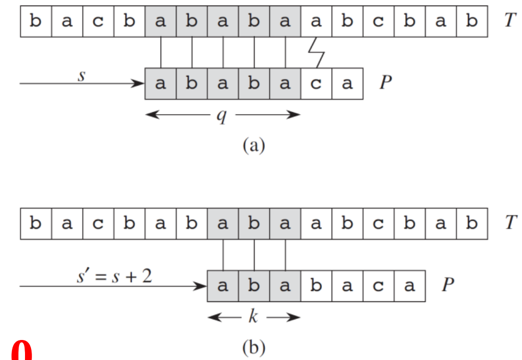
i	1	2	3	4	5	6	7
$P[i]$	a	b	a	b	a	c	a
$\pi[i]$	0	0	1	2	3	0	1

(a)



(b)

*32.4 KMP algorithm

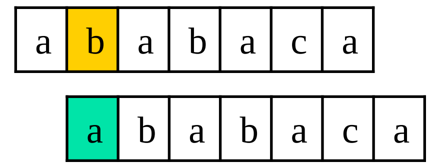



```
COMPUTE-PREFIX-FUNCTION(P)
m = P.length
let π[1..m] be a new array
π[1] = 0
k = 0
for q = 2 to m
    while k > 0 and P[k+1] ≠ P[q]
        k = π[k]
    if P[k+1] == P[q]
        k = k + 1
    π[q] = k
return π
```

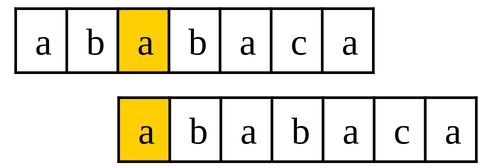
we have $P_k \sqsupset P[q\text{-}1]$,
check if $P[k+1] \neq P[q]$

q = 1: $k \leftarrow 0$, $\pi[1] \leftarrow 0$

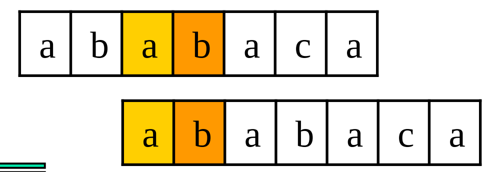
q = 2: $\pi[2] \leftarrow 0$



q = 3: $P[1] == P[3]$
$k \leftarrow 1$, $\pi[3] \leftarrow 1$



q = 4: $P[2] == P[4]$
$k \leftarrow 2$, $\pi[4] \leftarrow 2$



prefix function for the pattern P is the function..

π : {1, 2, …, m} → {0, 1, …, m-1} such that

$\pi[q] = \max\{k : k < q \text{ and } P_k \sqsupset P_q\}$.

i	1	2	3	4	5	6	7
$P[i]$	a	b	a	b	a	c	a
$\pi[i]$	0	0	1	2	3	0	1

*32.4 KMP algorithm

Running time?　　预习chapter17

Amortized analysis (accounting): **Θ(*n*), Θ(*m*)**

```
KMP-MATCHER(T, P)
n = T.length
m = P.length
π = COMPUTE-PREFIX-FUNCTION(P)
q = 0
for i = 1 to n
    while q > 0 and P[q + 1] ≠ T[i]
        q = π[q]
    if P[q + 1] == T[i]
        q = q + 1
    if q == m
        print "Pattern occurs with shift" i − m
        q = π[q]
```

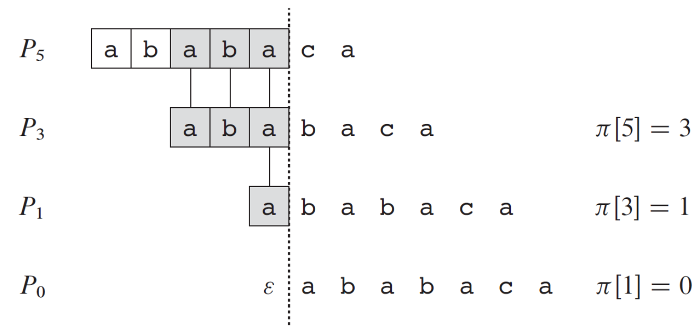
```
COMPUTE-PREFIX-FUNCTION(P)
m = P.length
let π[1 . . m] be a new array
π[1] = 0
k = 0
for q = 2 to m
    while k > 0 and P[k + 1] ≠ P[q]
        k = π[k]
    if P[k + 1] == P[q]
        k = k + 1
    π[q] = k
return π
```

prefix function:

$\pi[q] = \max\{k : k < q \text{ and } P_k \sqsupset P_q\}$

i	1	2	3	4	5	6	7
$P[i]$	a	b	a	b	a	c	a
$\pi[i]$	0	0	1	2	3	0	1

(a)



(b)

*32.4 The Knuth-Morris-Pratt algorithm

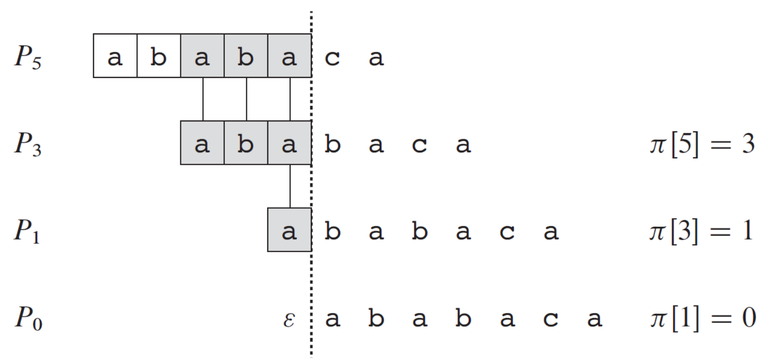
KMP algorithm avoids computing the transition function δ, and its matching time is $\boldsymbol{\Theta(n)}$ using just an auxiliary function $\boldsymbol{\pi}$, which we precompute from the pattern in time $\boldsymbol{\Theta(m)}$ and store in an array $\boldsymbol{\pi[1 \,..\, m]}$.

prefix function:

$\boldsymbol{\pi[q] = \max\{k : k < q \text{ and } P_k \sqsupset P_q\}}$

i	1	2	3	4	5	6	7
$P[i]$	a	b	a	b	a	c	a
$\pi[i]$	0	0	1	2	3	0	1

(a)

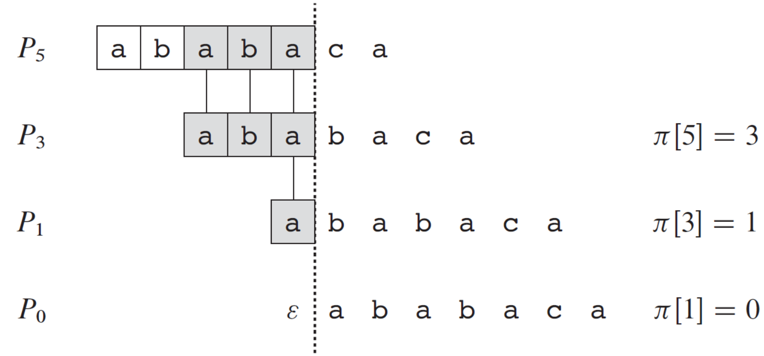


(b)

Exercises

i	1	2	3	4	5	6	7
$P[i]$	a	b	a	b	a	c	a
$\pi[i]$	0	0	1	2	3	0	1

(a)



(b)

32.1-2

32.3-1

32.3-2

32.4-1

Compute the prefix function π for the pattern:

(1) **aabaaaabab**

(2) **ababbabbabbababbabb**

32.4-7