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Engineering Notes

Robust Backstepping Magnetic Attitude Control of Satellite Subject to Unsymmetrical Mass Properties

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I. Introduction

THE attitude control problem is one of the major subdivisions in satellite development technology. Attitude control of Earth-pointing satellites using magnetic torquers has been used since the inception of satellite programs, either to manipulate the primary actuation system or as a stand-alone actuating system [1,2]. In fact, magnetic torquers were used successfully for momentum dumping and reorientation of spin-stabilized satellites and control augmentation of gravity-gradient stabilized satellites. Previous research has also shown successful application of a magnetic attitude control system for performing functions like precession control, momentum bias regulation, nutation damping, and initial acquisition [3]. Magnetic torque is generated as a result of the interaction of the geomagnetic field and onboard electromagnets. The performance of such a system depends on the size of the satellite and the torque resulting from the interaction of magnetic torquers and the magnetic field of the concerned celestial body. The resultant magnetic torque always lies in a plane perpendicular to the local magnetic field, making the system underactuated at any instant. However, the natural variability of the magnetic field along the orbit makes it possible to provide three-axes control.

A complete set of solutions for the tracking problem of a satellite with three independent actuators exists [4–6]. An approximate solution to the time-varying magnetic actuators can be found in [7]. The approximate solution method was extended to a periodic solution, making the problem more accessible [8]. It is well known that two reaction wheels or thrusters cannot even locally asymptotically stabilize using time-invariant static or dynamic feedback [9]. However, an exponential convergence can be achieved for a continuous time-varying feedback [10], provided all the axes are dynamically coupled together. The controllability of a general time-varying system was extensively discussed in [11–13]. It was shown in [13] that the attitude dynamics of a satellite actuated by magnetic torquers subject to a time-varying magnetic field is strongly accessible if the magnetic field and its time derivative are linearly independent of each other along the orbit at every instant of time. Moreover, it was also shown that for a periodic variation of the magnetic field the attitude dynamics of a satellite is fully accessible.

The conventional controllers do not give much adaptivity, and therefore to tackle this situation, feedback linearization [14], sliding mode control [15], H_∞ control [16], and backstepping control approaches [17] have been proposed. Among these, the nonlinear backstepping control is a popular design approach [18,19] that gives adaptivity and exponential stabilization in the neighborhood of the origin in a real sense. A novel approach for nonlinear and adaptive control design using backstepping was introduced in [20]. A somewhat similar approach based on quaternions for the attitude control of a rigid spacecraft equipped with four reaction thrusters and one reaction wheel was proposed in [21]. The three-axes controllability of the averaged dynamics of the magnetic satellite and its control law design with the uncertainties in moments of inertia matrix have received little attention. A robust backstepping control law using double-gimbal variable-speed control moment gyroscope was discussed in [22]. Also, a similar contribution using quaternion-based backstepping control for tracking of a microsatellite for European Student Earth Orbiter was developed in [23]. An attitude tracking problem using adaptive backstepping control based on modified Rodrigues parameters was studied in [24]. Here, an uncertainty in the inertia matrix was considered, and adaptive laws were designed accordingly. The pitch attitude maneuver of a spacecraft using the nonlinear backstepping approach was presented in [25]. In the previously mentioned literature, unsymmetrical variable-mass properties of the satellite in the formulation of its dynamics and proof of three-axes controllability for very high initial angular velocity has received little attention.

In this Note, the robust backstepping control for a magnetically actuated satellite rotating at an arbitrary high angular velocity is addressed. The control matrix in a magnetically actuated satellite dynamics is defined in terms of a unit vector along the geomagnetic field and is rank deficient, implying impossible three-axes control at any instant of time. This problem can be overcome in terms of average control by introducing the averaged control matrix in the attitude dynamics equation as shown later. The three-axes controllability and global stability of a magneto-Coulombic satellite system was addressed in [26]. However, the three-axes controllability and stability issues for a magnetically actuated satellite at high angular velocity have received little attention in the literature, and previous studies have focused largely on angular velocities that are small and comparable to the orbital angular velocity [27]. In this Note, a new real-time backstepping control is proposed and designed based on the averaged dynamics that includes external disturbance and uncertainties in the moments of inertia. To stabilize the satellite, an ideal control torque is formulated in terms of the errors in the quaternions and angular velocity components with respect to the orbital reference frame. The proposed real-time backstepping control is used to find the ideal/required torque, which in turn is used to find the required dipole moments ($\tilde{m} \in R^3$). The interaction of these dipoles with the Earth's magnetic field generates the available control torque ($\tilde{T}_m \in R^3$), which is used to actuate and stabilize the satellite. Stability analysis is also carried out based on the averaged dynamics. It is proved that the magnetic satellite system is globally stable for the nonperiodic variation of the magnetic field in the considered circular orbit. It is shown that the magnetic attitude control system can be exponentially stabilized in the neighborhood of the origin and can be stabilized for any arbitrary initial angular velocity using the proposed backstepping controller. Detailed mathematical analysis is carried out to show the stability of the system.

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II. Preliminaries

The satellite is considered to be a rigid body, and the only control actuator is the magnetic torquer of which the mass effect is included in the inertia matrix of the satellite. It is assumed that the satellite is orbiting in a circular orbit. Three different frames are used for describing the dynamics of the system: an inertial reference frame (XYZ) centered at the Earth, an orbital reference frame ($x_0y_0z_0$) of which the x axis points toward the Earth's center while the y axis is along the velocity vector in the orbit, and a body reference frame (xyz) fixed at the c.m. of the satellite. The axes of the magnetic torquers are taken along the body axes.

A. Satellite Dynamics and Kinematics

The attitude dynamics of a spacecraft subject to a gravity-gradient torque $\tilde{T}_{gg}$, magnetic control torque $\tilde{T}_c$, and disturbance torque $\tilde{T}_d$ can be expressed as [4]

$$J\dot{\tilde{\omega}} = S(\tilde{\omega})J\tilde{\omega} + 3\omega_0^2 S(J\tilde{e}_x^b)\tilde{e}_x^b + \tilde{T}_c + \tilde{T}_d \tag{1}$$

where $\tilde{T}_g$ stands for the gravity-gradient torque and is given by $\tilde{T}_g = 3\omega_0^2 S(J\tilde{e}_x^b)\tilde{e}_x^b$, $\tilde{\omega} = [\omega_x \quad \omega_y \quad \omega_z]^T$ is the inertial angular velocity of the body reference frame, $J = [J_{11} \quad J_{12} \quad J_{13};\ J_{21} \quad J_{22} \quad J_{23};\ J_{31} \quad J_{32} \quad J_{33}] \in \mathbb{R}^{3\times3}$ is the inertia matrix, ω_0 is the orbital angular velocity of the satellite, and $S(\tilde{\omega})$ is the skew symmetric matrix as defined in the following:

$$S(\tilde{\omega}) = (-\tilde{\omega}\times) = \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix} \tag{2}$$

The matrix $S(J\tilde{e}_x^b)$ has the same definition as for, $S(\tilde{\omega})$ and $\tilde{e}_x^b$ is the first column of the direction cosine matrix $A(\tilde{q})$ as given in the following, which transforms from the orbital frame to the body frame:

$$A(\tilde{q}) = \begin{bmatrix} q_4^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2+q_4q_3) & 2(q_1q_3-q_4q_2) \\ 2(q_1q_2-q_4q_3) & q_4^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3+q_4q_1) \\ 2(q_1q_3+q_4q_2) & 2(q_2q_3-q_4q_1) & q_4^2-q_1^2-q_2^2+q_3^2 \end{bmatrix} \tag{3}$$

Here, $\tilde{q} = [q_1 \quad q_2 \quad q_3 \quad q_4]^T = [\tilde{q}_r^T \quad q_4]^T$ is the attitude quaternion that describes orientation of the satellite with respect to the orbital reference frame; $\tilde{q}_r^T = [q_1 \quad q_2 \quad q_3]$. For the Earth-pointing satellite, the attitude kinematics must be described with respect to the orbital reference frame. The rotational kinematics can be expressed as

$$\dot{\tilde{q}}_r = \frac{1}{2}\Big[q_4 I_3 - S(\tilde{q}_r)\Big]\tilde{\omega}_r \tag{4}$$

$$\dot{q}_4 = -\frac{1}{2}\tilde{q}_r^T\tilde{\omega}_r \tag{5}$$

where I_3 is an indentity matrix. In Eqs. (4) and (5), $\tilde{\omega}_r$ denotes the relative angular velocity, and its expression is given in Eq. (6),

$$\tilde{\omega}_r = \tilde{\omega} - \tilde{\omega}_0 \tag{6}$$

wherein $\tilde{\omega}_0$ denotes the orbital angular velocity with respect to the inertial reference frame. The expression of $\tilde{\omega}_0$ is given in Eq. (7):

$$\tilde{\omega}_0 = -\omega_0\tilde{e}_z^b \tag{7}$$

In Eq. (7), $\tilde{e}_z^b$ denotes the third column of the direction cosine matrix defined in Eq. (3). The quaternion vector has the property of unity norm. i.e., $\tilde{q}^T\tilde{q} = 1$. Equations (1), (4), and (5) are used to propagate the angular velocity and the orientation of the satellite.

B. Relative Attitude Error Dynamics and Kinematics

Let $\tilde{q}_d = [q_{d1} \quad q_{d2} \quad q_{d3} \quad q_{d4}]^T = [\tilde{q}_{dr}^T \quad q_{d4}]^T$ be a unit quaternion vector ($\tilde{q}_d^T\tilde{q}_d = 1$) that represents the desired attitude of a satellite with respect to the orbital reference frame. Let the attitude error be defined as $\tilde{q}_e = [\tilde{q}_{er}^T \quad q_{e4}]^T$ with $\tilde{q}_{er} = [q_{e1} \quad q_{e2} \quad q_{e3}]^T$. The attitude error $\tilde{q}_e$ can be expressed in terms of $\tilde{q}_d$ and $\tilde{q}$. For this purpose, the dyadic product form of $\tilde{q}_e$ in terms of $\tilde{q}_d$ and $\tilde{q}$ can be written as in Eq. (8),

$$\tilde{q}_e = \tilde{q}^{-1} \otimes \tilde{q}_d \tag{8}$$

where $\tilde{q}^{-1}$ is the conjugate attitude quaternion of $\tilde{q}$ and is given by $\tilde{q}^{-1} = [-\tilde{q}_r^T \quad q_4]^T$. Equation (8) can be written in a simplified form in Eq. (9):

$$\tilde{q}_e = \begin{bmatrix} \tilde{q}_{er} \\ q_{e4} \end{bmatrix} = \begin{bmatrix} q_4\tilde{q}_{dr} - q_{d4}\tilde{q}_r - \tilde{q}_{dr}\times\tilde{q}_r \\ q_4q_{d4} + \tilde{q}_r^T\tilde{q}_{dr} \end{bmatrix} \tag{9}$$

As a result, the relative attitude error kinematics can be obtained as

$$\dot{\tilde{q}}_{er} = \frac{1}{2}(q_{e4}I_3 + \tilde{q}_{er}\times)\tilde{\omega}_r, \quad \dot{q}_{e4} = -\frac{1}{2}\tilde{q}_{er}^T\tilde{\omega}_r, \quad \text{for } \tilde{\omega}_{dr} = 0 \tag{10}$$

where $\tilde{\omega}_{dr}$ is the relative desired angular velocity with respect to the orbital reference frame with components along the body frame. The relative angular velocity $\tilde{\omega}_e$ in the body reference frame can be expressed as $\tilde{\omega}_e = \tilde{\omega}_r$ for $\tilde{\omega}_{dr} = 0$, and hence error dynamics becomes $\dot{\tilde{\omega}}_e = \dot{\tilde{\omega}}_r$. It is to be noted that $A(\tilde{q})^TA(\tilde{q}) = I_3$, and therefore $|A(\tilde{q})| = 1$ and $\dot{A}(\tilde{q}) = -\tilde{\omega}\times A(\tilde{q})$. As a result, the error dynamics gets reduced to

$$\begin{aligned} J\dot{\tilde{\omega}}_e &= J\dot{\tilde{\omega}} - J\tilde{\omega}\times A(\tilde{q})\tilde{\omega}_0 \\ &= S(\tilde{\omega})J\tilde{\omega} + 3\omega_0^2S(J\tilde{e}_x^b)\tilde{e}_x^b \\ &\quad + \tilde{T}_c + \tilde{T}_d - \omega_0JS(\tilde{e}_z^b)\tilde{\omega} \end{aligned} \tag{11}$$

C. Magnetic Actuators

The magnetic control torques along three body axes are generated by a set of three magnetic coils, which are aligned with the satellite body axes and can be written as

$$\tilde{T}_c = \tilde{m}_{\text{coil}} \times \tilde{B}(t) = S(\tilde{B}(t))\tilde{m}_{\text{coil}} \tag{12}$$

where $\tilde{m}_{\text{coil}} \in \mathbb{R}^3$ is the magnetic dipole moment vector, $\tilde{B}(t)$ is the Earth's magnetic field in the body reference frame, and the matrix $S(\tilde{B}(t))$ is defined in the same way as $S(\tilde{\omega})$ in Eq. (2). Let $\tilde{B}_0(t)$ be the magnetic field in the orbital reference frame. The unit vectors corresponding to $\tilde{B}(t)$ and $\tilde{B}_0(t)$ can be written as $\tilde{b}(t) = \tilde{B}(t)/\|\tilde{B}(t)\|$ and $\tilde{b}_0(t) = \tilde{B}_0(t)/\|\tilde{B}_0(t)\|$, respectively. Let $\tilde{u}(t) \in \mathbb{R}^3$ be the control torque required to stabilize the satellite. Then, in the ideal case, to obtain the expression of required dipole moment $\tilde{m}_{\text{coil}}$, we need to invert the matrix $S(\tilde{B}(t))$. However, the matrix $S(\tilde{B}(t))$ in Eq. (12) is singular in nature, and therefore its pseudo inverse is taken. Let the expression of magnetic dipole moment ($\tilde{m}_{\text{coil}}$) be given by

$$\tilde{m}_{\text{coil}} = \tilde{P}\,\tilde{u}(t) \tag{13}$$

where $\tilde{P}$ is a 3×3 matrix. The idea here is to formulate the matrix $\tilde{P}$ in such a way that the desired control torque becomes the best possible torque from the three magnetic coils. The simple formulation of $\tilde{P}$ can be done using the pseudoinverse of the matrix $S(\tilde{B}(t))$ and can be written as $S(\tilde{B}(t))^T/\|\tilde{B}(t)\|^2$, which yields $\tilde{m}_{\text{coil}}$ and $\tilde{T}_c$ in Eqs. (14) and (15), respectively,

$$\tilde{m}_{\text{coil}} = \frac{1}{\|\tilde{B}(t)\|^2}S\Big(\tilde{B}(t)\Big)^T\tilde{u}(t) \tag{14}$$

$$\tilde{T}_c = S\left(\tilde{b}(t)\right)S\left(\tilde{b}(t)\right)^T \tilde{u}(t) = \tilde{\Gamma}(t)\tilde{u}(t) \tag{15}$$

where $\tilde{\Gamma}(t) = S\left(\tilde{b}(t)\right)S\left(\tilde{b}(t)\right)^T$ is a singular control matrix.

III. Controllability Issue and Robust Backstepping Control Design

The dynamics of a magnetically actuated system in terms of control matrix $\tilde{\Gamma}(t)$ and ideal control torque can be written as

$$J\dot{\tilde{\omega}} = S(\tilde{\omega})J\tilde{\omega} + 3\omega_0^2 S(J\tilde{e}_x^b)\tilde{e}_x^b + \tilde{\Gamma}(t)\tilde{u}(t) + \tilde{T}_d \tag{16}$$

where $\tilde{\Gamma}(t)$ is defined in the body frame. It is necessary to mention that at any given time the control matrix $\tilde{\Gamma}(t) = S(\tilde{b}(t))S(\tilde{b}(t))^T \geq 0$ is singular, which implies the system defined in Eq. (16) is instantaneously uncontrollable. Let $\tilde{\Gamma}_0(t) = S(\tilde{b}_0(t))S(\tilde{b}_0(t))^T$ be the control matrix in the orbital reference frame, in which $\tilde{b}_0(t)$ is a unit vector in the orbital reference frame. The system defined by Eq. (16) will be controllable on average if the average of the control matrix $\tilde{\Gamma}(t)$ is positive definite as defined in Eq. (22), in other words $\bar{\Gamma} > 0$. This requires the condition $\bar{\Gamma}_0 > 0$ in the orbital reference frame. Thus, the positive definiteness of the average magnetic control matrix $\bar{\Gamma}$ can be ensured once the average value of $\bar{\Gamma}_0$ is positive definite as defined in Eq. (19). Proposition 1 defines the condition under which the control matrix $\bar{\Gamma}$ will be positive definite. It is to be noted that averaging over three to four data points suffices positive definiteness of $\bar{\Gamma}$, provided the satellite is in the near-polar orbit, thus making the system also temporally controllable, which plays a major role in the initial behavior of the system. It is assumed that the disturbance $\tilde{T}_d$ entering the satellite dynamics equation is unknown but bounded, i.e.,

$$|\tilde{T}_d| < \tilde{\xi} \tag{17}$$

Proposition 1: Assuming that in the considered orbit the vector $\tilde{b}_0$ satisfies the condition

$$\tilde{b}_0(t) \times \dot{\tilde{b}}_0(t) \neq 0, \qquad t \in R \tag{18}$$

then

$$\bar{\Gamma}_0 = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \left(\tilde{b}_0(t)\times\right)\left(\tilde{b}_0(t)\times\right)^T \mathrm{d}t > 0 \tag{19}$$

$$= \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau S\left(\tilde{b}_0\right)S\left(\tilde{b}_0\right)^T \mathrm{d}t > 0 \tag{20}$$

$$= \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \tilde{\Gamma}_0\left(\tilde{b}_0(t)\right) \mathrm{d}t > 0 \tag{21}$$

where $\tau > t_0$ and $0 < t_0 < \infty$. If $\|\tilde{\omega}_r\| < \infty \forall \tau > t_0$, where $0 < t_0 < \infty$, then

$$\bar{\Gamma} = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \tilde{\Gamma}\left(\tilde{b}(t)\right) \mathrm{d}t > 0 \tag{22}$$

along the trajectories of the system defined by the Eq. (16). Moreover,

$$\hat{\Gamma} = \frac{1}{\tau}\int_0^\tau \tilde{\Gamma}\left(\tilde{b}(t)\right) \mathrm{d}t > 0$$

Proof: The condition given by Eq. (19) is a weak condition and can be equivalently written in terms of persistent random excitation along the three orbital axes, which ensures that the three columns of the averaged control matrix remain linearly independent and thereby guarantees positive definiteness of the control matrix $\bar{\Gamma}_0$ ([28]).

Now, the system dynamics of the magnetic attitude control system in terms of the averaged control matrix can be written as in Eq. (23), provided $\tilde{u}$ is state dependent only,

$$J\dot{\tilde{\omega}} = S(\tilde{\omega})J\tilde{\omega} + 3\omega_0^2 S(J\tilde{e}_x^b)\tilde{e}_x^b + \bar{\Gamma}\,\tilde{u} + \bar{T}_d \tag{23}$$

If the control input $\tilde{u}$ is time dependent, then Eq. (23) can be written as

$$J\dot{\tilde{\omega}} = S(\tilde{\omega})J\tilde{\omega} + 3\omega_0^2 S(J\tilde{e}_x^b)\tilde{e}_x^b + \bar{U} + \bar{T}_d \tag{24}$$

where $\bar{U} = \lim_{\tau\to\infty}(1/\tau)\int_0^\tau \tilde{\Gamma}(\tilde{b}(t))u\,\mathrm{d}t$ and $\bar{T}_d$ is the average of the disturbance input. Equation (23) is used for designing the backstepping control algorithm as shown in the following subsection.

A. Backstepping Control Design

In this subsection, a nonlinear integrator backstepping control algorithm is proposed for the attitude control of a magnetically actuated satellite governed by Eqs. (4), (5), and (16). Integrator backstepping can be effectively used for cascaded systems of which satellite attitude control is a good example. The kinematics of the system is coupled with the dynamics of the system, and therefore the kinematics can be controlled implicitly through the control of $\tilde{\omega}_r$ using the integrator backstepping control formulated using error in the relative angular velocity ($\tilde{\omega}_e = \tilde{\omega}_r$) in Eq. (10) as a virtual control input [21]. This is followed by the design of the actual control $\tilde{u}(t)$ input, which is used in the system dynamics [Eq. (11)] to steer the system toward the desired state without destabilizing the kinematics [Eq. (10)]. The control input design procedure is presented in the following paragraphs.

The transformations are defined for the systems described by Eqs. (10) and (11),

$$\tilde{z}_1 = \tilde{q}_{er} \tag{25}$$

$$\tilde{z}_2 = \tilde{\omega}_e - \alpha(\tilde{q}_{er}) \tag{26}$$

$$= \tilde{\omega}_r - \alpha(\tilde{q}_{er}) \tag{27}$$

where $\tilde{\omega}_e = \tilde{\omega}_r$ is defined as a virtual control input and $\alpha(\tilde{q}_e)$ is a stabilizing function. The $\tilde{z}_1$ dynamics can be written as

$$\dot{\tilde{z}}_1 = \frac{1}{2}\left(\tilde{q}_{e4}I_3 + \tilde{z}_1\times\right)\tilde{z}_2 + \frac{1}{2}\left(q_{e4}I_3 + \tilde{z}_1\times\right)\alpha\left(\tilde{q}_{er}\right) \tag{28}$$

Now, the task is to stabilize the $\tilde{z}_1$ dynamics. For this, the Lyapunov function $V_1 = (1/2)\hat{z}_1^T\hat{z}_1$, where $\hat{z}_1 = [\tilde{z}_1^T, q_{e4} - 1]^T$ and further we get $\dot{\hat{z}}_1 = \dot{\hat{q}}_e$, is used. The time derivative of V_1 can be written as

$$\begin{aligned}
\dot{V}_1 &= \hat{z}_1^T\dot{\hat{z}}_1 = \hat{z}_1^T\dot{\hat{q}}_e \\
&= \hat{z}_1^T\begin{bmatrix} \frac{1}{2}(q_{e4}I_3 + \tilde{q}_{er}\times) \\ -\frac{1}{2}\tilde{q}_{er}^T \end{bmatrix}\tilde{\omega}_e \\
&= \hat{z}_1^T\begin{bmatrix} \frac{1}{2}(q_{e4}I_3 + \tilde{q}_{er}\times) \\ -\frac{1}{2}\tilde{q}_{er}^T \end{bmatrix}\tilde{z}_2 + \hat{z}_1^T\begin{bmatrix} \frac{1}{2}(q_{e4}I_3 + \tilde{q}_{er}\times) \\ -\frac{1}{2}\tilde{q}_{er}^T \end{bmatrix}\alpha(\tilde{q}_{er}) \\
&= [z_1^T, q_{e4} - 1]\begin{bmatrix} \frac{1}{2}(q_{e4}I_3 + \tilde{q}_{er}\times) \\ -\frac{1}{2}\tilde{q}_{er}^T \end{bmatrix}\tilde{z}_2 \\
&\quad + [\tilde{z}_1^T, \boldsymbol{q}_{e4} - 1]\begin{bmatrix} \frac{1}{2}(q_{e4}I_3 + \tilde{q}_{er}\times) \\ -\frac{1}{2}\tilde{q}_{er}^T \end{bmatrix}\alpha(\tilde{q}_{er}) \\
&= \frac{1}{2}\tilde{z}_1^T\tilde{z}_2 + \frac{1}{2}\tilde{z}_1^T\alpha(\tilde{q}_{er})
\end{aligned} \tag{29}$$

To prove $\dot{V}_1 \leq 0$, a nonlinear tracking function [17] for the virtual control $\alpha(\tilde{q}_{er})$ is chosen as

$$\alpha(\tilde{q}_{er}) = -a_i \arctan(b_i \tilde{q}_{ei}), \qquad i = 1, 2, 3 \tag{30}$$

where a_i and b_i are positive constant. Inserting Eq. (30) into Eq. (29) yields

$$\dot{V}_1 = \frac{1}{2}\tilde{z}_1^T \tilde{z}_2 - \frac{1}{2}\sum_{i=1}^{3} \tilde{z}_1^T a_i \arctan(b_i \tilde{z}_1) \tag{31}$$

Clearly, if $\tilde{z}_2 = 0$, then

$$\dot{V}_1 = -\frac{\mathbf{1}}{\mathbf{2}}\sum_{i=1}^{3} \tilde{z}_1^T a_i \arctan(b_i \tilde{z}_1) < 0 \tag{32}$$

and $\tilde{z}_1$ is guaranteed to asymptotically converge to zero. We also need to prove that the $\tilde{z}_2$ dynamics is stable. The time derivative of $\tilde{z}_2$ in Eq. (26) is multiplied by the inertia matrix and can be written using the averaged dynamics of Eq. (24) as

$$\begin{aligned} J\dot{\tilde{z}}_2 &= J\dot{\omega}_e - J\dot{\alpha}(\tilde{q}_{er}) \\ &= S(\tilde{\omega})J\tilde{\omega} + 3\omega_0^2 S(J\tilde{e}_x^b)\tilde{e}_x^b + \bar{U} \\ &\quad + \bar{T}_d - \omega_0 J S(\tilde{e}_z^b)\omega - J\dot{\alpha}(\tilde{q}_{er}) \end{aligned} \tag{33}$$

where $\tilde{u}$ is assumed to be formulated in terms of state variables. In Eq. (33), the term $\dot{\alpha}(\tilde{q}_e)$ can be formulated as

$$\dot{\alpha}(\tilde{q}_e) = -\begin{bmatrix} \frac{a_1 b_1}{1+b_1^2 q_{e1}^2} & 0 & 0 \\ 0 & \frac{a_2 b_2}{1+b_2^2 q_{e2}^2} & 0 \\ 0 & 0 & \frac{a_3 b_3}{1+b_3^2 q_{e3}^2} \end{bmatrix} \dot{\tilde{q}}_{er} \tag{34}$$

Until now, we have considered the inertia matrix $J \in \mathbb{R}^{3\times3}$ to be constant, but in practice, it is imprecisely known or even may have large uncertainty. In this case, it can be replaced by an estimate of the inertia and update the estimate by a robust adaptive scheme. For this purpose, an uncertain parameter is obtained from the inertia matrix terms. To obtain the uncertain parameters in vector form, a linear operator $L: \mathbb{R}^3 \to \mathbb{R}^{3\times6}$ acting on the vector $\tilde{c} = [c_1, c_2, c_3]^T$ is defined by

$$L(\tilde{c}) = \begin{bmatrix} c_1 & 0 & 0 & 0 & c_3 & c_2 \\ 0 & c_2 & 0 & c_3 & 0 & c_1 \\ 0 & 0 & c_3 & c_2 & c_1 & 0 \end{bmatrix} \tag{35}$$

Let an uncertain parameter denoted as $\tilde{\theta} \in \mathbb{R}^6$ defined by $\tilde{\theta} = [J_{11} \quad J_{22} \quad J_{33} \quad J_{23} \quad J_{13} \quad J_{12}]^T$, then it can be written as [29]

$$J\tilde{c} = L(\tilde{c})\tilde{\theta} \tag{36}$$

Applying the linear parameter matrix L and the parameter vector θ to Eq. (33) yields

$$\begin{aligned} J\dot{\tilde{z}}_2 &= S(\tilde{\omega})L(\tilde{\omega})\tilde{\theta} + 3\omega_0^2 S(L(\tilde{e}_x^b)\tilde{\theta})\tilde{e}_x^b \\ &\quad + \bar{U} + \bar{T}_d - \omega_0 \boldsymbol{L}(\boldsymbol{S}(\tilde{\boldsymbol{e}}_z^b)\tilde{\omega})\tilde{\theta} - L(\dot{\alpha}(\tilde{q}_{er}))\tilde{\theta} \\ &= S(\tilde{\omega})L(\tilde{\omega})\tilde{\theta} - 3\omega_0^2 (S(\tilde{e}_x^b)L(\tilde{e}_x^b)\tilde{\theta}) \\ &\quad + \bar{U} + \bar{T}_d - \omega_0 \boldsymbol{L}(\boldsymbol{S}(\tilde{\boldsymbol{e}}_z^b)\tilde{\omega})\tilde{\theta} - L(\dot{\alpha}(\tilde{q}_{er}))\tilde{\theta} \\ &= [S(\tilde{\omega})L(\tilde{\omega}) - 3\omega_0^2 S(\tilde{e}_x^b)L(\tilde{e}_x^b)]\tilde{\theta} \\ &\quad [-\omega_0 \boldsymbol{L}(\boldsymbol{S}(\tilde{\boldsymbol{e}}_z^b)\tilde{\omega}) - L(\dot{\alpha}(\tilde{q}_{er}))]\tilde{\theta} + \bar{T}_d + \bar{U} \\ &= \tilde{M}\tilde{\theta} + \bar{T}_d + \bar{U} \end{aligned} \tag{37}$$

where $\tilde{M}$ is

$$\tilde{M} = S(\tilde{\omega})L(\tilde{\omega}) - 3\omega_0^2 S(\tilde{e}_x^b)L(\tilde{e}_x^b) - \omega_0 L(S(\tilde{e}_z^b)\tilde{\omega}) - L(\dot{\alpha}(\tilde{q}_{er})) \tag{38}$$

Let $\bar{\theta}$ be the estimated value of the inertia matrix terms; then, the deviation between the real inertia parameters $\tilde{\theta}$ and the estimated one $\bar{\theta}$ can be written as

$$\hat{\theta} = \tilde{\theta} - \bar{\theta} \tag{39}$$

Consider the Lyapunov function for the overall system,

$$V_2 = V_1 + \frac{1}{2}\tilde{z}_2^T J \tilde{z}_2 + \frac{1}{2}\hat{\theta}^T \psi \hat{\theta} \tag{40}$$

where ψ is a positive definite diagonal matrix. The time derivative of V_2 is written as

$$\dot{V}_2 = \dot{V}_1 + \tilde{z}_2^T J \dot{\tilde{z}}_2 + \hat{\theta}^T \psi \dot{\hat{\theta}} \tag{41}$$

Substituting Eqs. (25), (29), (37), and (39) into Eq. (41) yields

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2}\tilde{z}_1^T \tilde{z}_2 + \frac{1}{2}\tilde{z}_1^T \alpha(\tilde{q}_e) + \tilde{z}_2^T J \dot{\tilde{z}}_2 + \hat{\theta}^T \psi \dot{\hat{\theta}} \\ &= \frac{1}{2}\tilde{z}_1^T \alpha(\tilde{q}_{er}) + \frac{1}{2}\tilde{z}_1^T \tilde{z}_2 + \tilde{z}_2^T[\tilde{M}\tilde{\theta} + \bar{T}_d + \bar{U}] + \hat{\theta}^T \psi \dot{\hat{\theta}} \\ &= \frac{1}{2}\tilde{z}_1^T \alpha(\tilde{q}_{er}) + \hat{\theta}^T \psi \dot{\hat{\theta}} + \tilde{z}_2^T\left[\frac{1}{2}\tilde{z}_1 + \tilde{M}(\hat{\theta} + \bar{\theta}) + \bar{T}_d + \bar{U}\right] \\ &= \frac{1}{2}\tilde{z}_1^T \alpha(\tilde{q}_{er}) + \tilde{z}_2^T \tilde{M}\hat{\theta} + \hat{\theta}^T \psi \dot{\hat{\theta}} + \tilde{z}_2^T\left[\frac{1}{2}\tilde{z}_1 + \tilde{M}\bar{\theta} + \bar{T}_d + \bar{U}\right] \\ &= \frac{1}{2}\tilde{z}_1^T \alpha(\tilde{q}_{er}) + \hat{\theta}^T \psi[\psi^{-1}\tilde{M}^T \tilde{z}_2 - \dot{\bar{\theta}}] \\ &\quad + \tilde{z}_2^T\left[\frac{1}{2}\tilde{z}_1 + \tilde{M}\bar{\theta} + \bar{T}_d + \bar{U}\right] \end{aligned} \tag{42}$$

According to Eq. (32), $0.5\tilde{z}_1^T \alpha(\tilde{q}_e)$ satisfies the condition $0.5\tilde{z}_1^T \alpha(\tilde{q}_e) < 0$. To render $\dot{V}_2 \leq 0$, one natural choice of $\tilde{\boldsymbol{u}}$ to ensure the asymptotic stability of the system is

$$\tilde{\boldsymbol{u}}(\boldsymbol{t}) = \hat{\boldsymbol{\Gamma}}(\boldsymbol{t})^{-1}\left(-\frac{\mathbf{1}}{\mathbf{2}}\tilde{\boldsymbol{z}}_1 - \boldsymbol{K}_1 \tilde{\boldsymbol{z}}_2 - \tilde{\boldsymbol{M}}\bar{\boldsymbol{\theta}} - \bar{\boldsymbol{\xi}}\right) \tag{43}$$

$$\dot{\bar{\boldsymbol{\theta}}} = \boldsymbol{\psi}^{-1}\tilde{\boldsymbol{M}}^T \tilde{\boldsymbol{z}}_2 \tag{44}$$

which cancels all the nonlinear terms in Eq. (42), when inserted into it after premultiplying with $\tilde{\Gamma}(t)$ and averaging as $\tau \to \infty$. Here, $\hat{\Gamma}^{-1}$ is the inverse of the temporally averaged control matrix as defined in Proposition 1, and $\boldsymbol{K}_1 \in \mathbb{R}^{3\times3}$ is a positive-definite symmetric matrix. This can be verified by using the fact that $\lim_{\tau\to\infty}(1/\tau)\int_0^\tau \tilde{\Gamma}(t)\hat{\Gamma}(t)^{-1}\,\mathrm{d}t$ is an identity matrix. Therefore, using Eqs. (43), (44), and (42), it can be observed that $\dot{V}_2 \leq 0$. Thus, it completes the proof of the stability of the closed-loop system. The closed-loop block diagram of backstepping control system for the attitude control of a magnetically actuated satellite subjected to unsymmetrical mass properties is shown in Fig. 1.

IV. Results and Discussion

Simulations are presented to illustrate the effectiveness of the proposed control for the attitude control of an Earth-pointing satellite in a circular orbit. The orbit with radius 7000 km and inclination 97.8 deg w.r.t. EME 2000 is considered. The geomagnetic field shown in Fig. 2 belongs to that of International Geomagnetic

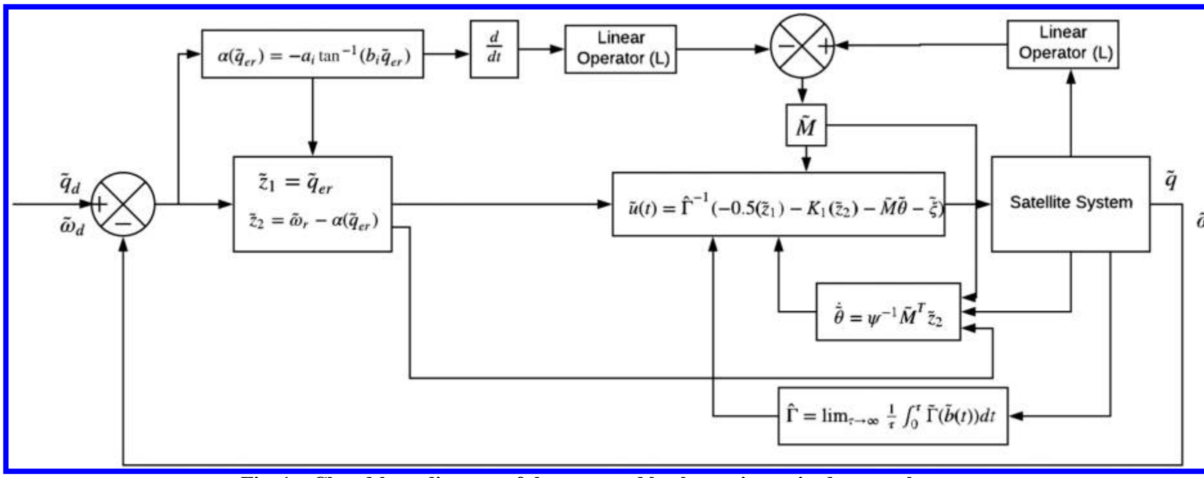


Fig. 1 Closed-loop diagram of the proposed backstepping attitude control system.

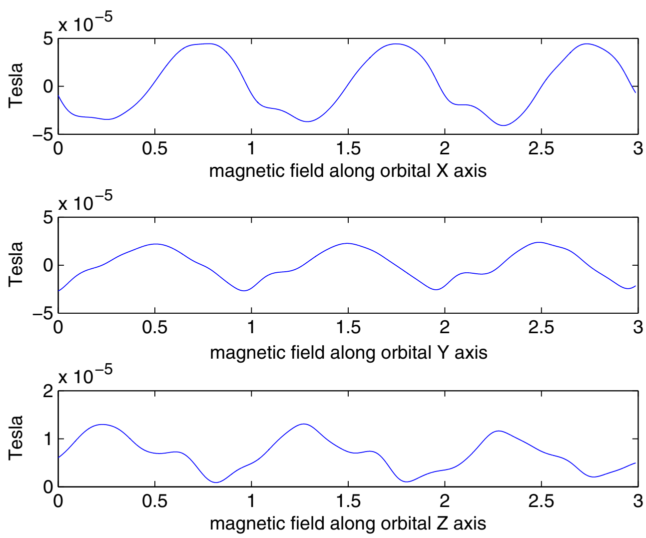


Fig. 2 Magnetic field along the orbital axes plotted against the orbit number, which is used for simulation purposes.

Reference Field commencing at 9:30 a.m. on 5 May 2005 Indian Standard Time. The disturbance torque is made up of a constant and a periodic term, as given in the following:

$$\tilde{T}_d = 0.1 \times 10^{-5} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 0.2 \times 10^{-5} \begin{bmatrix} \sin(\omega_0 t) \\ \sin(\omega_0 t + \pi/4) \\ \sin(\omega_0 t - \pi/4) \end{bmatrix} \mathrm{N \cdot m} \tag{45}$$

From the disturbance torque $\tilde{T}_d$ model considered in Eq. (45), the components of $\tilde{\xi}$ are taken as 3×10^{-6} N · m Let us assume that the off-diagonal terms in the inertia matrix are nonzero. The gravity-gradient torque $\tilde{T}_g$ in Eq. (1) will be stabilizing or destabilizing depending on the moments of inertia along the body axes. However, if the products of inertia terms J_{xy}, J_{xz}, and J_{yz} are present in the inertia matrix, then the stability may get affected adversely because of coupling. In this Note, the instability condition is considered based on the slow or fast growing response of the attitude quaternions. In other words, a solution with small bounded oscillation in the neighborhood of the desired orientation and angular velocity is considered stable attitude convergence to the desired attitude. Therefore, in the present Note, destabilizing gravity-gradient torque in addition to the nonzero product of inertia is considered for all the simulations. The actual inertia matrix and the initial estimated value $\tilde{\theta}$ for all the simulations are chosen as

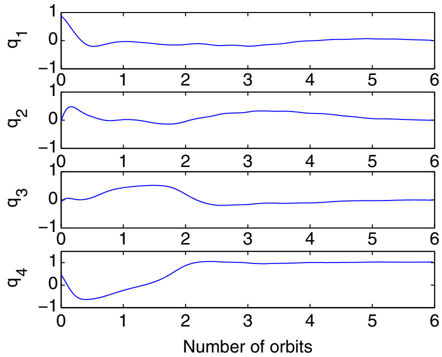
$$J = \begin{bmatrix} 140 & 1 & -2 \\ 1 & 120 & 3 \\ -2 & 3 & 130 \end{bmatrix} \mathrm{kg \cdot m^2}$$

$$\tilde{\theta}(0) = [\,139 \quad 121 \quad 129 \quad 2.8 \quad -1.5 \quad .9\,]^T \ \mathrm{kg \cdot m^2}$$

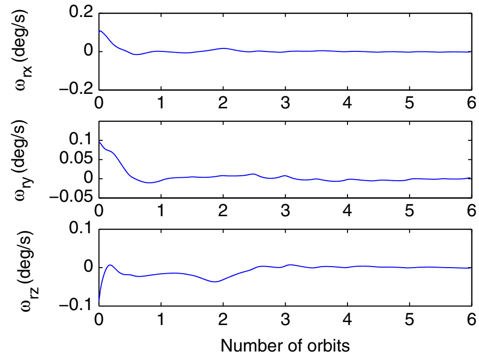
respectively; the latter is used for attitude propagation and updated according to the scheme proposed in the previous section. In the first example, the angular velocity of the satellite is kept to a low value. The initial conditions are chosen as $\tilde{\omega}(0) = [\,0.001 \quad 0.001 \quad -0.001\,]^T$ rad/s and $\tilde{q}(0) = [\,0.860 \quad 0.080 \quad 0.402 \quad 0.303\,]^T$, where $\tilde{q}(0)$ corresponds to yaw, pitch, and roll angles of 30, –40, and 130 deg, respectively. The desired orientation for all the examples is represented by $\tilde{q}_d = [\,0 \quad 0 \quad 0 \quad \pm 1\,]^T$, which corresponds to yaw, pitch, and roll angles of 0 deg. For the numerical simulation, the gains of the robust control law in Eq. (43) are selected as $a = 1.1 \times 10^{-5} I_3$, $b = 3 \times 10^2 I_3$, $K_1 = 8.5 \times 10^2$, and $\psi = 0.01 I_6$. The cutoff limit for the magnetic dipole moment is set to $m_{\mathrm{lim}} = \pm 18\ \mathrm{Am}^2$. The results are shown in Fig. 3. It can be observed from Figs. 3a and 3b that the convergence of the attitude quaternion and relative angular velocity components takes place within six orbits. Figure 3b corresponds to errors in the Euler angle rates less than 1 deg for the level of disturbance considered. In Fig. 3c, the required magnetic moments along the body axes are plotted.

In the second example, the initial angular velocity is kept the same as in the previous example, and the quaternion vector is chosen as $\tilde{q}(0) = [\,0.518 \quad 0.511 \quad 0.560 \quad -0.395\,]^T$. For the numerical simulation, the gains of the control law in Eq. (43) are kept the same as that in the first example. The results of this example are shown in Fig. 4. It can be observed that the convergence of attitude quaternions and angular velocity components is achieved within six orbits. It can be concluded from the first and second examples that the convergence of the attitude quaternion to the desired attitude is weakly dependent on the initial orientation of the satellite.

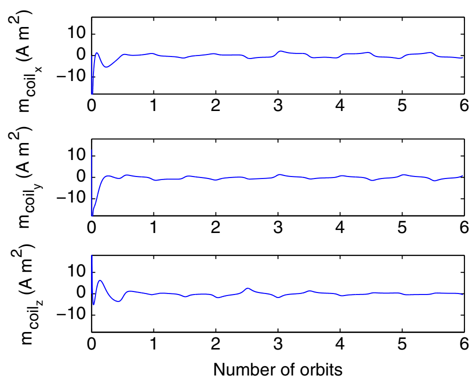
The third simulation is carried out to show the effect of the high angular velocity of the satellite on the attitude and angular velocity convergence. The initial conditions are $\tilde{q}(0) = [\,0.860 \quad 0.080 \quad 0.402 \quad 0.303\,]^T$ and $\tilde{\omega}(0) = [\,0.1 \quad 0.1 \quad -0.1\,]^T$ rad/s. The gains of the control law in Eq. (5) are chosen to be $a = 1.1 \times 10^{-5} I_3$, $b = 4 \times 10^2 I_3$, $K_1 = 5 \times 10^2$, and $\psi = 0.01 I_6$. To provide the necessary control torque to control the high angular velocity, the cutoff limit of the magnetic dipole moment is set to a high value ($m_{\mathrm{lim}} = \pm 120\ \mathrm{Am}^2$). The simulation results for this example are shown in Figs. 5a and 5c. It can be observed from the Figs. 5a and 5b that the convergence of the



a) Relative angular velocity

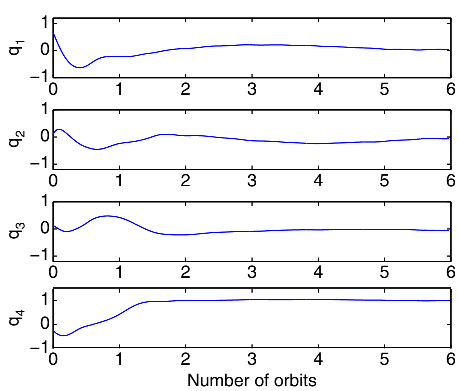


b) And dipole moments

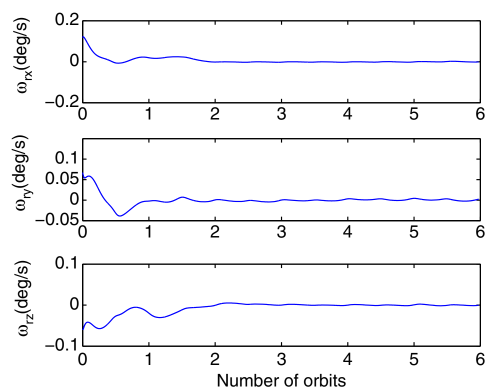


c) Along the body axes

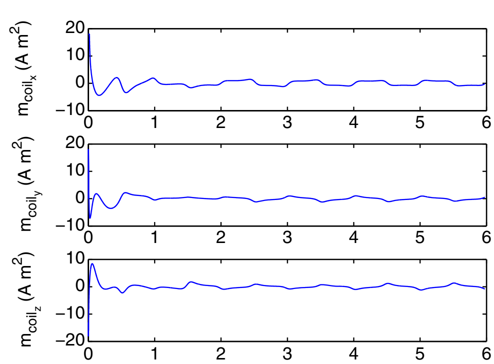
Fig. 3 Variation of the quaternions, relative angular velocity, and dipole moments along the body axes.



a) Relative angular velocity



b) And dipole moments

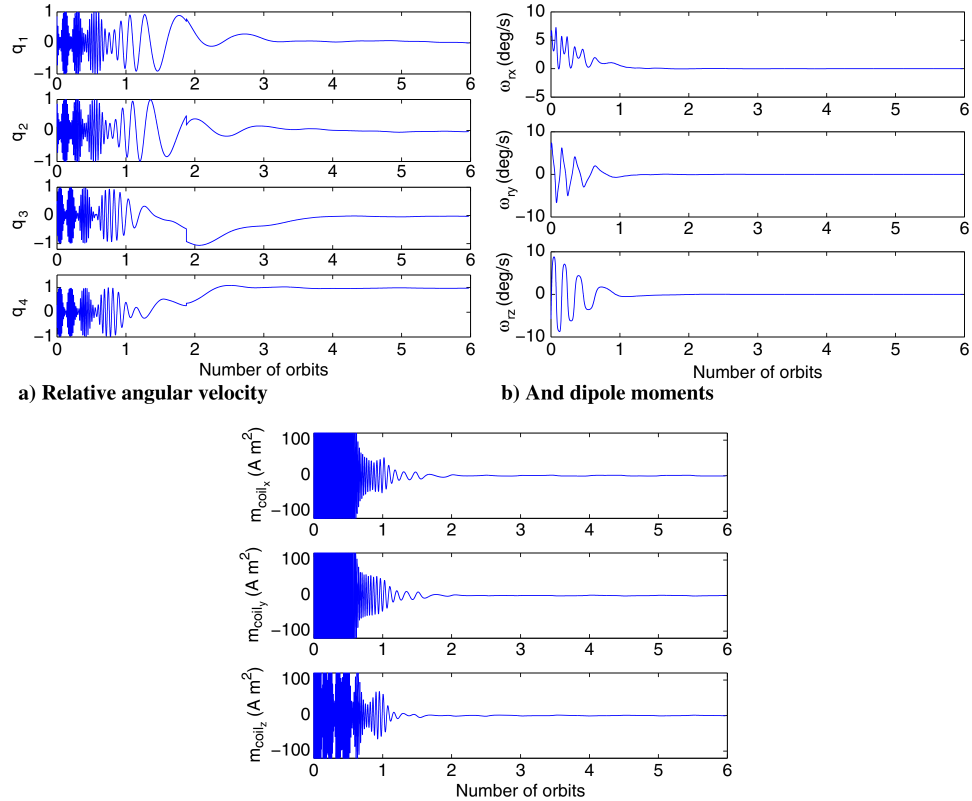


c) Along the body axes

Fig. 4 Variation of the quaternions, relative angular velocity, and dipole moments along the body axes.

quaternions and angular velocity components with respect to the orbital reference frame is achieved within four orbits in spite of the high initial angular velocity and destabilizing gravity-gradient torque. Thus, it can be concluded that, using the robust control law in Eq. (43), the adaptive attitude tracking and rapid convergence are achieved when the inertia matrix is uncertain.

The results for the robust backstepping control law for a magnetically actuated satellite with unsymmetrical mass properties



a) Relative angular velocity **b) And dipole moments**

c) Along the body axes

Fig. 5 Variation of the quaternions, relative angular velocity, and dipole moments along the body axes.

and uncertainties shows that the convergence takes place within six orbits. The delay in the quaternion convergence (six orbits) is primarily due to the uncertainty in the inertia terms along with unsymmetrical mass distribution.

V. Conclusions

The attitude control of a magnetically actuated satellite with uncertainty in an inertia matrix using backstepping control was presented. The equation for the ideal control torque was derived using robust backstepping control and was used for finding the required dipole moments. This was followed by the determination of the available magnetic torque using the required dipole moments. It was validated that the proposed controller effectively overcomes the influences of the external disturbances and uncertainties in the moments of the inertia matrix. It was also shown that it is possible to orient and control the satellite from any arbitrary attitude to the Earth-pointing attitude. It was observed that even a very high initial angular velocity can be controlled, provided a high cutoff limit for the magnetic dipole moment is taken. The performance of the proposed control was satisfactory when there was a uncertainities in the inertia matrix.

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