Abstract algebra Quiz $\sharp 1$
(Mar. 28, 2022)

Provide a suitable explanation for all your answers. It must be **accurate** at every step. (10 points per each problem)

1. Suppose that a ring R has a unique element e such that $xe = x$ for all $x \in R$. Show that $ey = y$ for all $y \in R$.

2. Let R be a ring with $1 \neq 0$ and $a, b \in R$. Assume that neither a nor b is a zero divisor. If ab is a unit, show that a and b are units.

3. Describe **all** ring homomorphisms from $\mathbb{Z} \times \mathbb{Z}$ to $\mathbb{Z}$.

4. Prove or disprove that $2\mathbb{Z}$ is isomorphic to $3\mathbb{Z}$ as rings.