20181288 권성훈 추대 2차 과제.

1. Lemma: If $|G|$ is even, G contains an element of order 2.

proof) Suppose G is a finite group with no element of order 2. Then every element $a \neq e$ has $a \neq a^{-1}$ so the non-identity elements come in pairs. Therefore $|G| = 1 + 2k$ is odd, where k is the number of those pairs.

By Lemma, G contains an element of order 2, since $|G| = 10$ which is even. ▫

2. (a) 4, 8, 12, 16

(b) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}

{0, 2, 4, 6, 8, 10, 12, 14, 16, 18}

{0, 4, 8, 12, 16}

{0, 5, 10, 15}

{0, 10}

{0}

3. (a) Let $G' = \{g \in G \mid g^{15} = e\}$

(i) Since $e^{15} = e$, thus $e \in G'$ which means G' is not empty.

(ii) Let $g_1, g_2 \in G'$, then $(g_1 g_2)^{15} = g_1^{15} g_2^{15} = e \cdot e = e$. Thus $g_1 g_2 \in G'$.

(iii) Let $g_1 \in G'$ then $g_1^{15} = e$. Since $g_1 \in G$, g_1^{-1} exists. Then we can multiply g_1^{-1} on the equation. Then we have $e = g_1^{-15} = (g_1^{-1})^{15}$. Thus $g_1^{-1} \in G$.

By (i), (ii), (iii), it is a subgroup of G ∎

3. (b) There is only 1 element, which is the identity of G.

Thus, the order is 1 $(\because 15 \nmid 100)$.