a) $$\int \sqrt{5+2x}\,dx = \left| \begin{array}{l} 5+2x=y \\ 2dx=dy \\ dx=\frac{dy}{2} \end{array} \right| = \frac{1}{2}\int \sqrt{y}\,dy = \frac{1}{2}\cdot\frac{y^{3/2}}{\frac{3}{2}} + C = \frac{y^{3/2}}{3} + C = \frac{(5+2x)\sqrt{5+2x}}{3} + C$$

f) $$\int \frac{x}{x^2+1}\,dx = \left| \begin{array}{l} x^2+1=y \\ 2x\,dx=dy \\ x\,dx=\frac{1}{2}dy \end{array} \right| = \frac{1}{2}\int \frac{dy}{y} = \frac{1}{2}\ln|y| + C = \frac{1}{2}\ln(x^2+1)+C$$

c) $$\int \frac{\ln^2 x}{x}\,dx = \left| \begin{array}{l} \ln x=y \\ \frac{1}{x}dx=dy \end{array} \right| = \int y^2\,dy = \frac{y^3}{3} + C = \frac{\ln^3 x}{3} + C$$

d) $$\int e^{\cos^2 x}\sin 2x\,dx = \left| \begin{array}{l} \cos^2 x=y \\ -\sin 2x\,dx=dy \\ \sin 2x\,dx=-dy \end{array} \right| = -\int e^{y}\,dy = -e^{y}+C = -e^{\cos^2 x}+C$$

n) $$\int \frac{e^{1/x}}{x^2}\,dx = \left| \begin{array}{l} \frac{1}{x}=y \\ -\frac{1}{x^2}dx=dy \\ \frac{1}{x^2}dx=-dy \end{array} \right| = -\int e^{y}\,dy = -e^{y}+C = -e^{1/x}+C$$

z) $$\int \frac{x}{(x^2-4)^3}\,dx = \left| \begin{array}{l} x^2-4=y \\ 2x\,dx=dy \\ x\,dx=\frac{1}{2}dy \end{array} \right| = \frac{1}{2}\int \frac{dy}{y} = \frac{1}{2}\ln(y)+C = \frac{1}{2}\ln(x^2-4)+C$$

Z) $$\int \frac{1}{x^3}\sin\frac{1}{x^2}\,dx = \left| \begin{array}{l} \frac{1}{x^2}=y \\ -\frac{2}{x^3}dx=dy \\ \frac{1}{x^3}dx=-\frac{1}{2}dy \end{array} \right| = -\frac{1}{2}\int \sin y\,dy = +\frac{1}{2}\cos y + C = \frac{1}{2}\cos\frac{1}{x^2} + C$$

1. Integrals

1) $\int(5-6x)\,dx = 5x - \frac{6x^2}{2} + C$

2) $\int\left(1-x^2-3x^5\right)dx = x - \frac{x^3}{3} - \frac{3x^6}{6} + C$

3) $\int(-5\sin t)\,dt = 5\cos t + C$

4) $\int(4\sec x\cdot\tan x - 2\sec^2 x)\,dx = 4\sec x - 2\tan x + C$

5) $\int \frac{1+\cos 4t}{2}\,dt = \frac{1}{2}t + \frac{\sin 4t}{8} + C$

6) $\frac{dy}{dx} = 2x - 7 \quad , y(2)=0$

$y = \int \frac{dy}{dx}\,dx = \int(2x-7)\,dx = x^2 - 7x + C$

$y(2) = 4 - 14 + C = 0$

$C = 10$

$y = x^2 - 7x + 10$

7) $\frac{dr}{d\theta} = -\pi\sin\pi\theta \quad , r(\theta)=0$

$r = \int -\pi\sin\pi\theta\,d\theta = \cos\pi\theta + C$

$r(0) = 1 + C = 0$

$C = -1$

$\cancel{y}\ r = \cos\pi\theta - 1$