

Distributions

The Basics

Definition

The way in which quantities are distributed among a sample space.

Random Variables

- Definition:
 - A variable whose value is subject to variations due to chance
- Types:
 - Discrete Random Variables
 - Continuous Random Variables

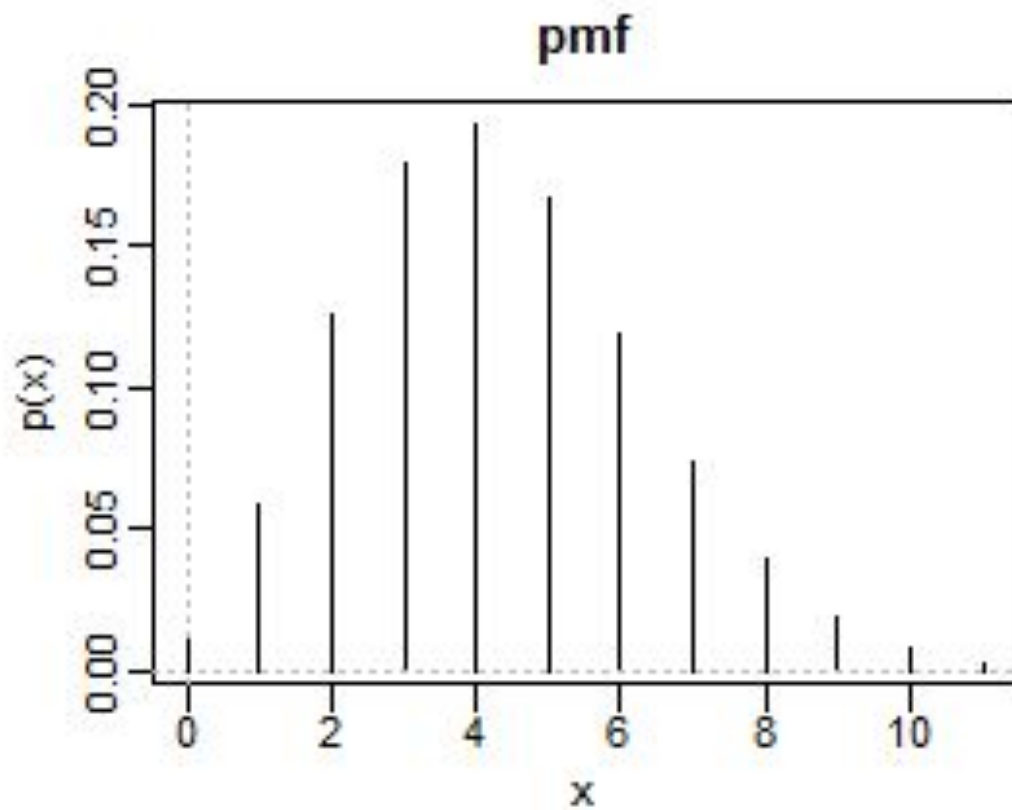
Probability Distribution Functions

PMF vs PDF

CMF vs CDF

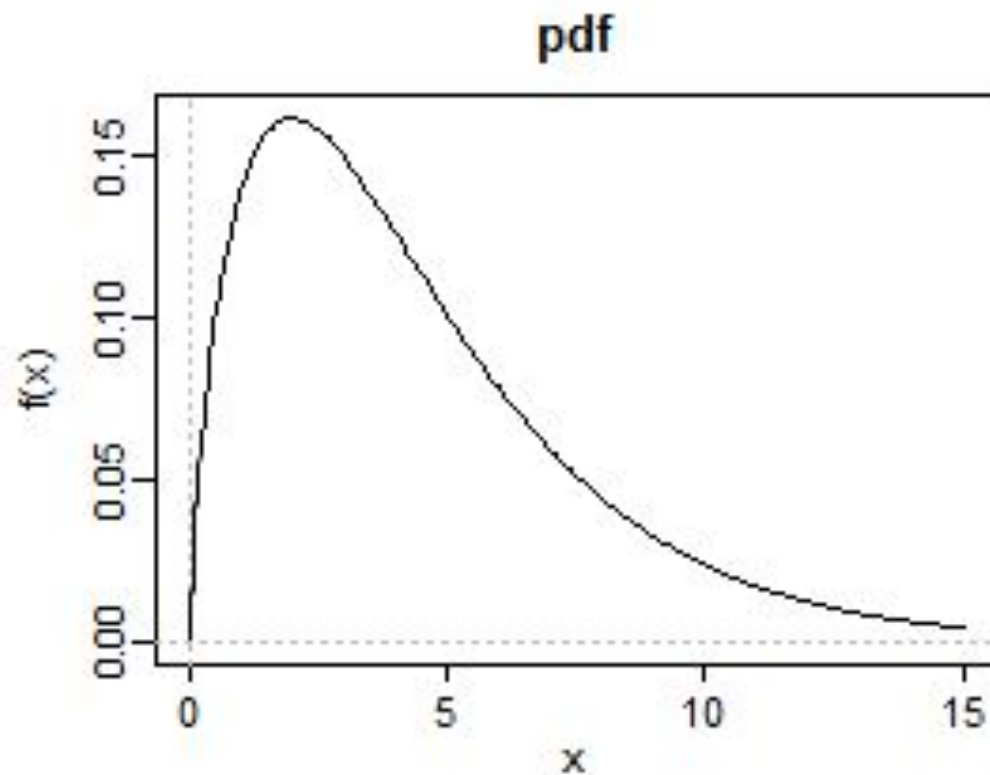
PMF vs. PDF

- PMF:
 - The Probability Mass Function (PMF) is a function that gives the probability that a DISCRETE random variable is exactly equal to some value.
 - Ex: $p(x = 3) = 78\%$



PMF vs. PDF

- PDF:
 - The Probability Density Function (PDF) is a function that gives the probability that a CONTINUOUS variable lies within a particular range. We cannot say anything about the probability of a variable equaling a single value in the space.
 - In other words: We must refer to probabilities in terms of intervals.
 - Ex: $p(1 < x < 3) = 43\%$



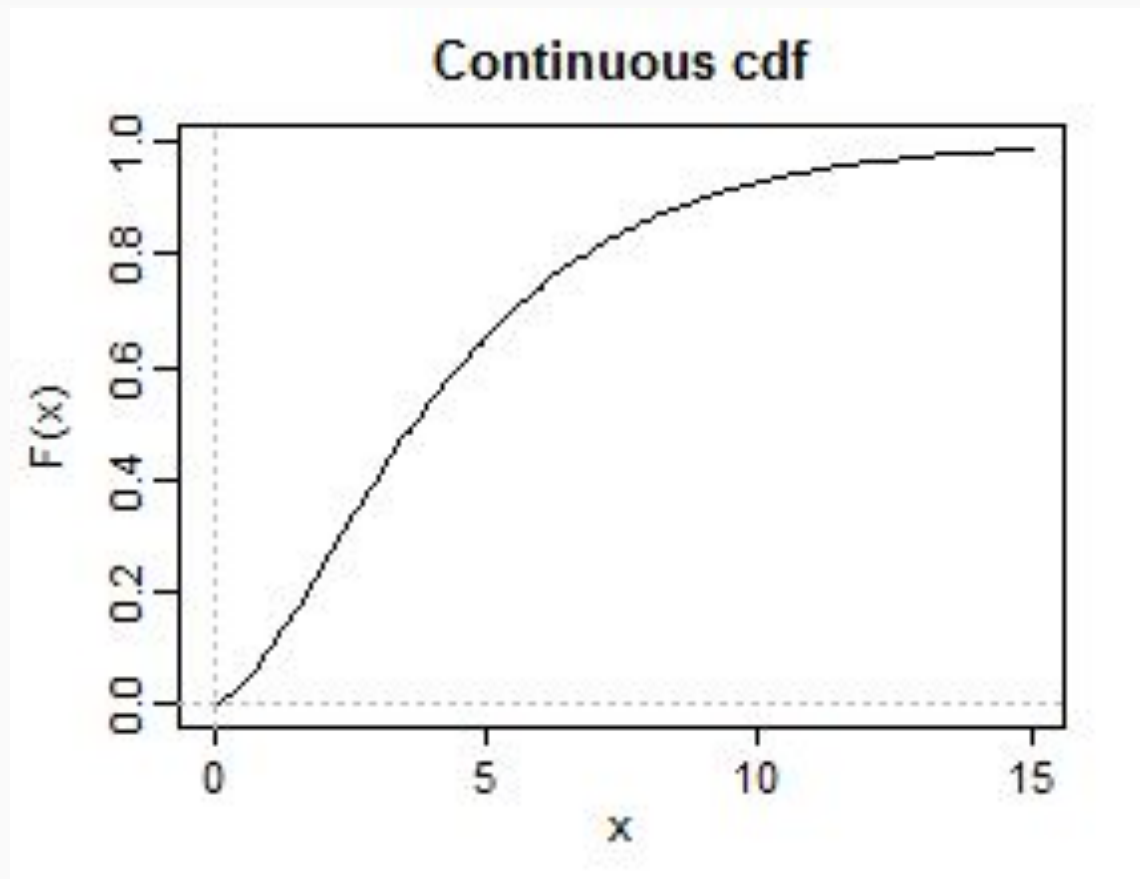
CMF vs. CDF

- CMF:
 - The Cumulative Mass Function (CMF) Is often represented as a series of probabilities on a table, that add up to 1.
 - Ex: $\text{Sum}(p(X)) = 1$; where X represents $\{x_1, x_2, x_3, \dots, x_n\}$

CMF table					
x	1	2	3	4	
f(X = x)	0.5	0.1	0.2	0.2	
Total (sum)	1				

CMF vs. CDF

- CDF:
 - The Cumulative Density Function (CDF) is a probability distribution that is used to find the probability that the variable takes a value less than or equal to x . The probability that the variable is EXACTLY x will ALWAYS be 0.
 - Ex: $p(x \leq 3) = 72\%$
 - Ex: $p(x = 3) = 0$

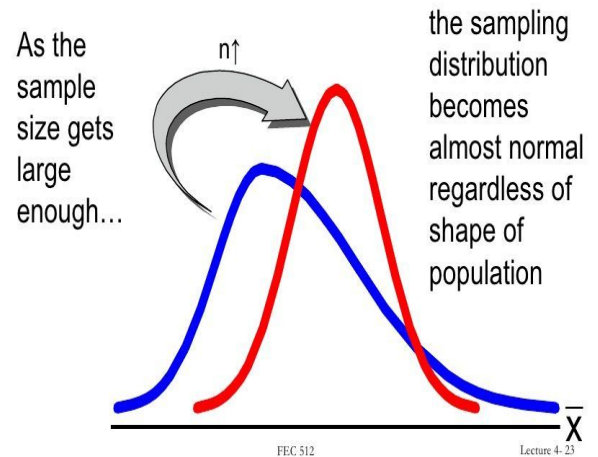


Properties of Distributions

Normal Distributions and more...

Central Limit Theorem

- Given certain conditions, the arithmetic mean of a sufficiently large enough set will be approximately normally distributed, regardless of the underlying distribution.
- This implies that methods that normally work for NORMAL distributions (bell curve), can be applied to many problems involving OTHER types of distributions.



Binomial Distribution and Bernoulli

Bernoulli Trials are a series of binary experiments with each trial either being a success/ failure.

A Binomial distribution is the distribution of the number of successes in a series of n independent bernoulli trials.

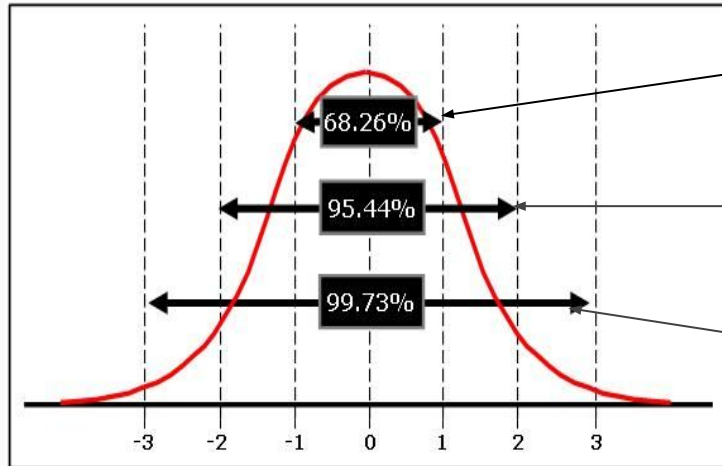
Ex: 100 coin tosses in a series are a Bernoulli Trial.

Ex: The resulting sample space = $\{H,H,T,T,T,H,T,T,H, \dots, H\}$ is a Binomial Distribution.

6-Sigma: 68-95-99 Rule

The 6 Sigma rule:

Essentially, if you are able to represent your set as a normal distribution, normalized by standard deviation (Z-score) as such:



68% of all values lie within 1 std dev from the mean.

95% of all values lie within 2 std devs from the mean.

99% of all values lie within 3 std devs from the mean.

Example Graphs

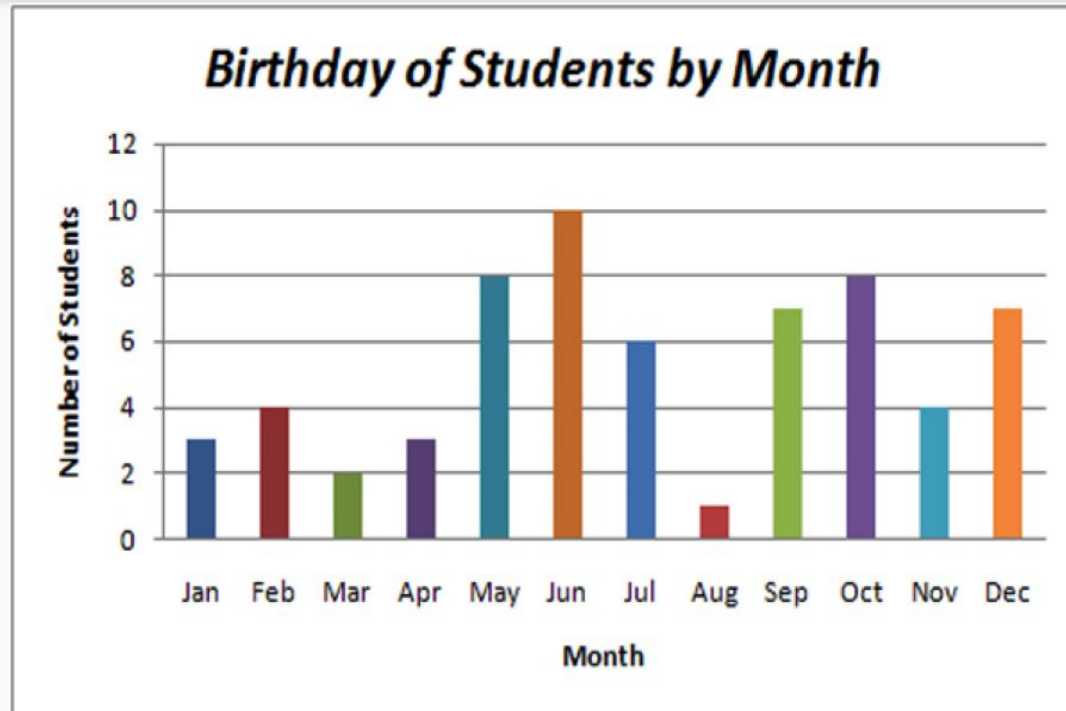
Histograms and Scatter Plots

Histogram

A Histogram is a diagram consisting of rectangles.

Each rectangle's area is proportional to the frequency of a variable

Each rectangle's width is equal to the class interval.



Scatter Plot

A Scatter Plot is a diagram consisting of 2 axes, and several points in an outcome space.

Each point represents a point in the outcome space

Great for finding trends in the data

