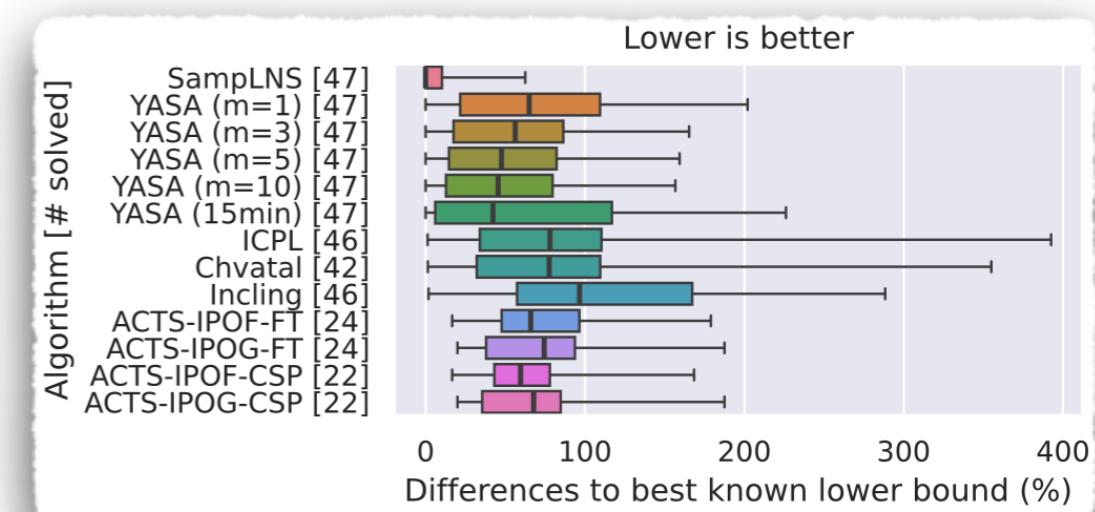
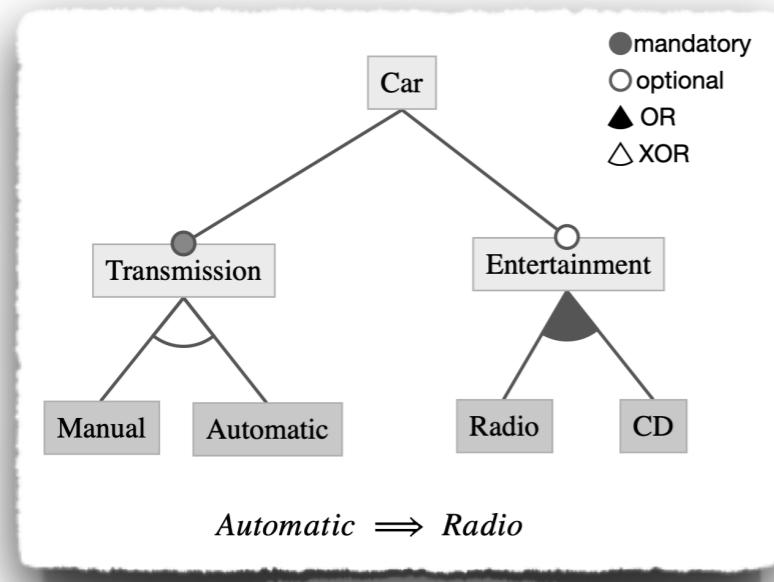




How Low Can We Go? Minimizing Interaction Samples for Configurable Systems



All errors and outrageous lies
are mine, and only mine

Co-conspirators:



Dominik Krupke, Ahmad Moradi, Michael Perk,
Phillip Keldenich, Gabriel Gehrke, Sebastian Krieter,
Thomas Thüm, Sándor P. Fekete



Testing configurable systems is critical



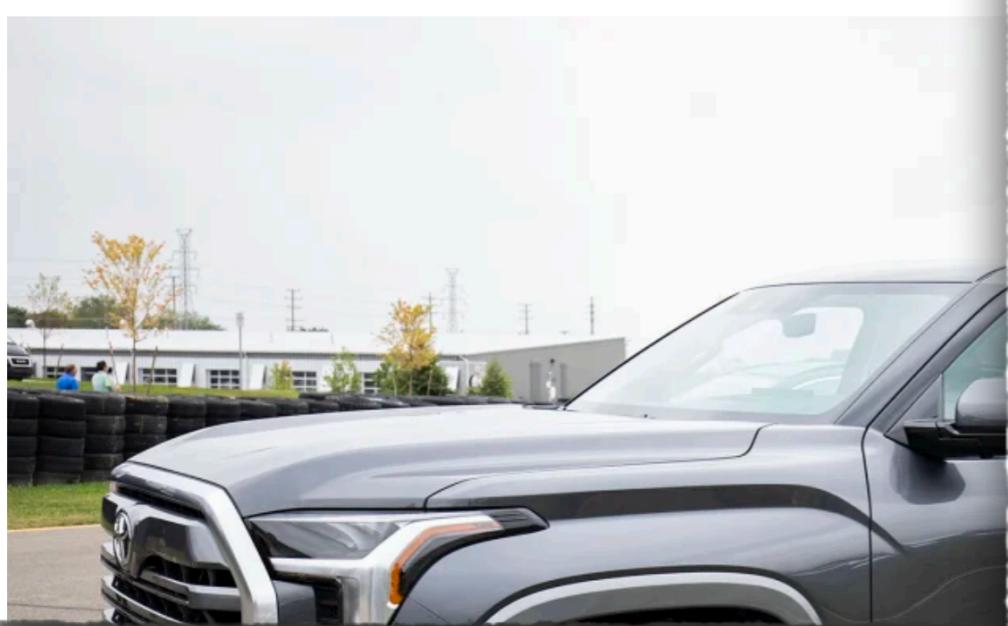
US World Politics Business Opinion Health Entertainment Style Travel Sports Underscored

Toyota recalls 280,000 vehicles because they may ‘creep forward’ in neutral

By Diksha Madhok, CNN

2 minute read · Updated 9:48 AM EST Thu February 22, 2024

f X e



The company will inform the owners of recalled vehicles by late April and update the software for the transmission, Toyota said.

The recall is one of three in the United States that the company announced Wednesday.

Toyota said it was recalling another 19,000 vehicles over a software problem that means “the rearview image may not display within the period of time required by certain US safety regulations after the driver shifts the vehicle into reverse, increasing the risk of a crash while backing the vehicle.”

That safety recall involves certain Mirai and Lexus LS, LC, ES models in North America, made between 2023 and 2024, it added.

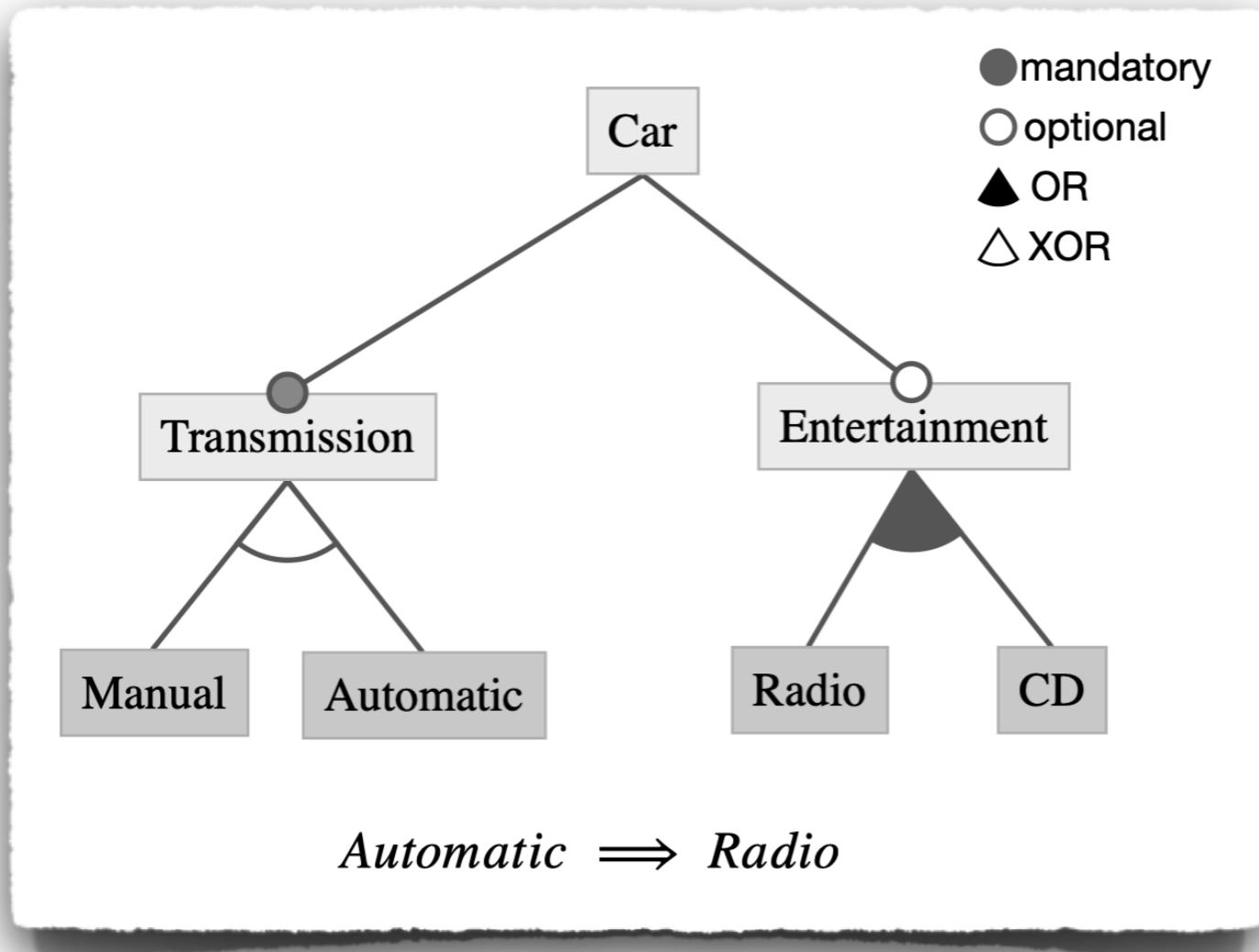
Additionally, some 4,000 Toyota Camry and Camry Hybrid vehicles are also being recalled over safety issues with the head restraints on rear fold-down seats that “increase the risk of injury during certain collisions.”

Toyota is the world's biggest automaker by sales, but it risks becoming bogged down in safety scandals.



Technische
Universität
Braunschweig

The Feature Model (on binary features)



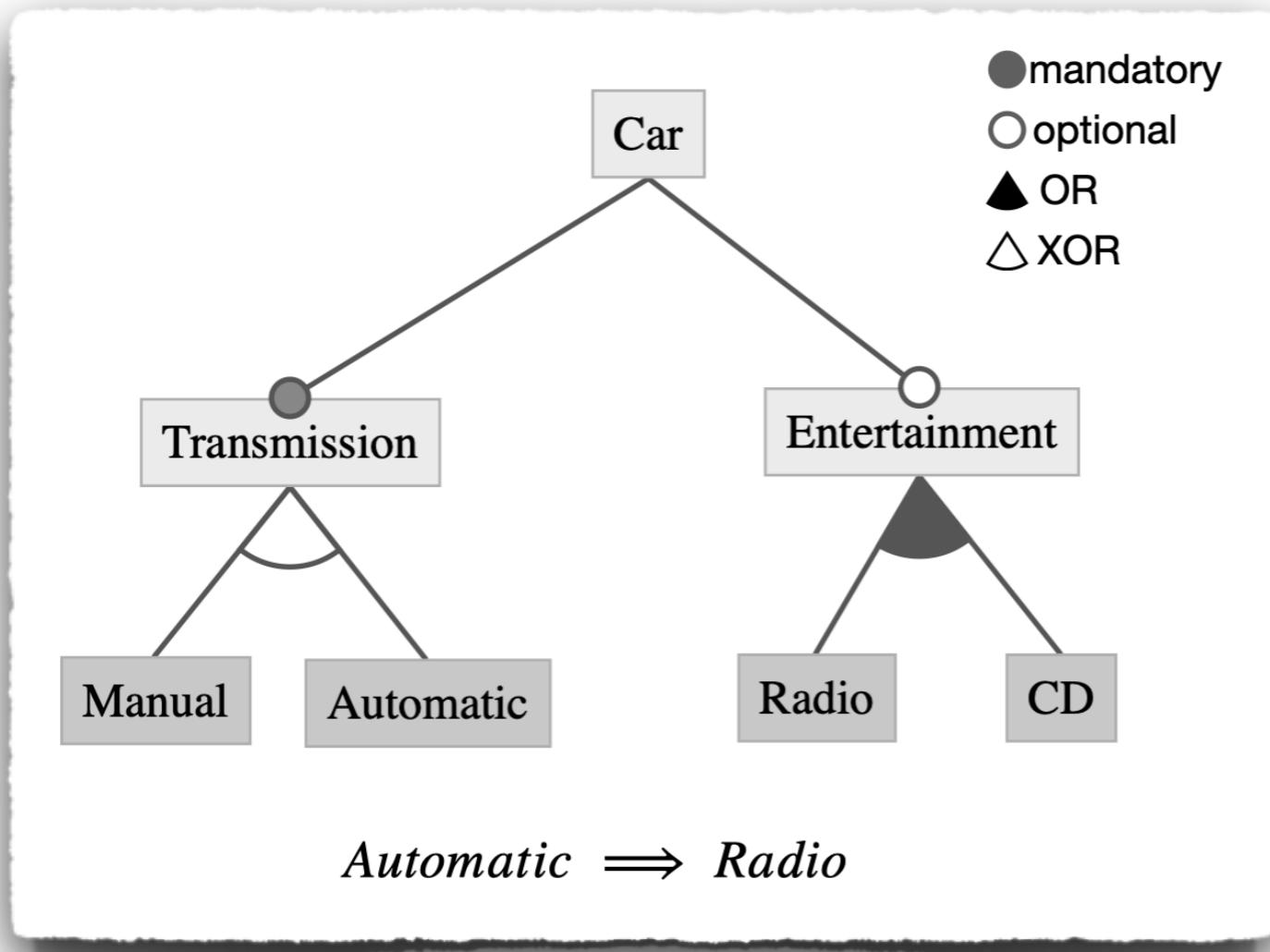
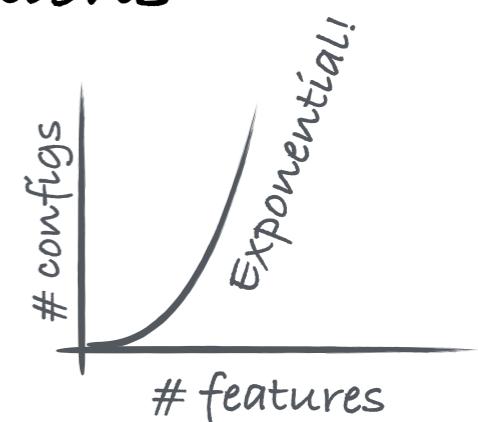
$SAT\ S$

Any solution = valid



t-wise Interactions

Testing all configurations
is not feasible.



just cover all t-wise interactions

Interactions = tuples of literals of size t

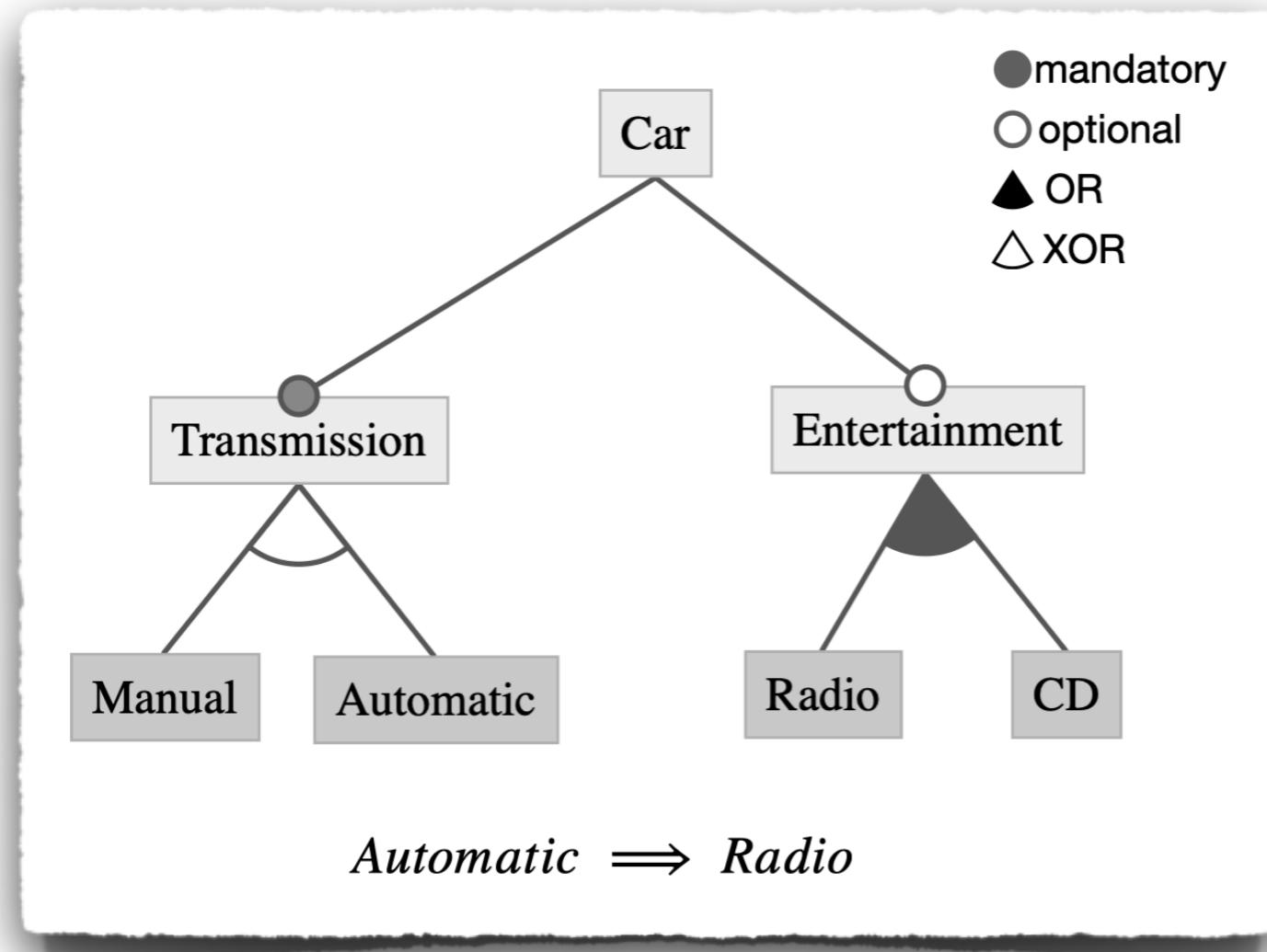
e.g. for t=2

- (CD, Radio)
- (-CD, Radio)
- (-CD, -Radio)



(Complete) Pairwise-Interaction Sampling Problem

$t=2$



Features $F = \{1, \dots, n\}$

Literals

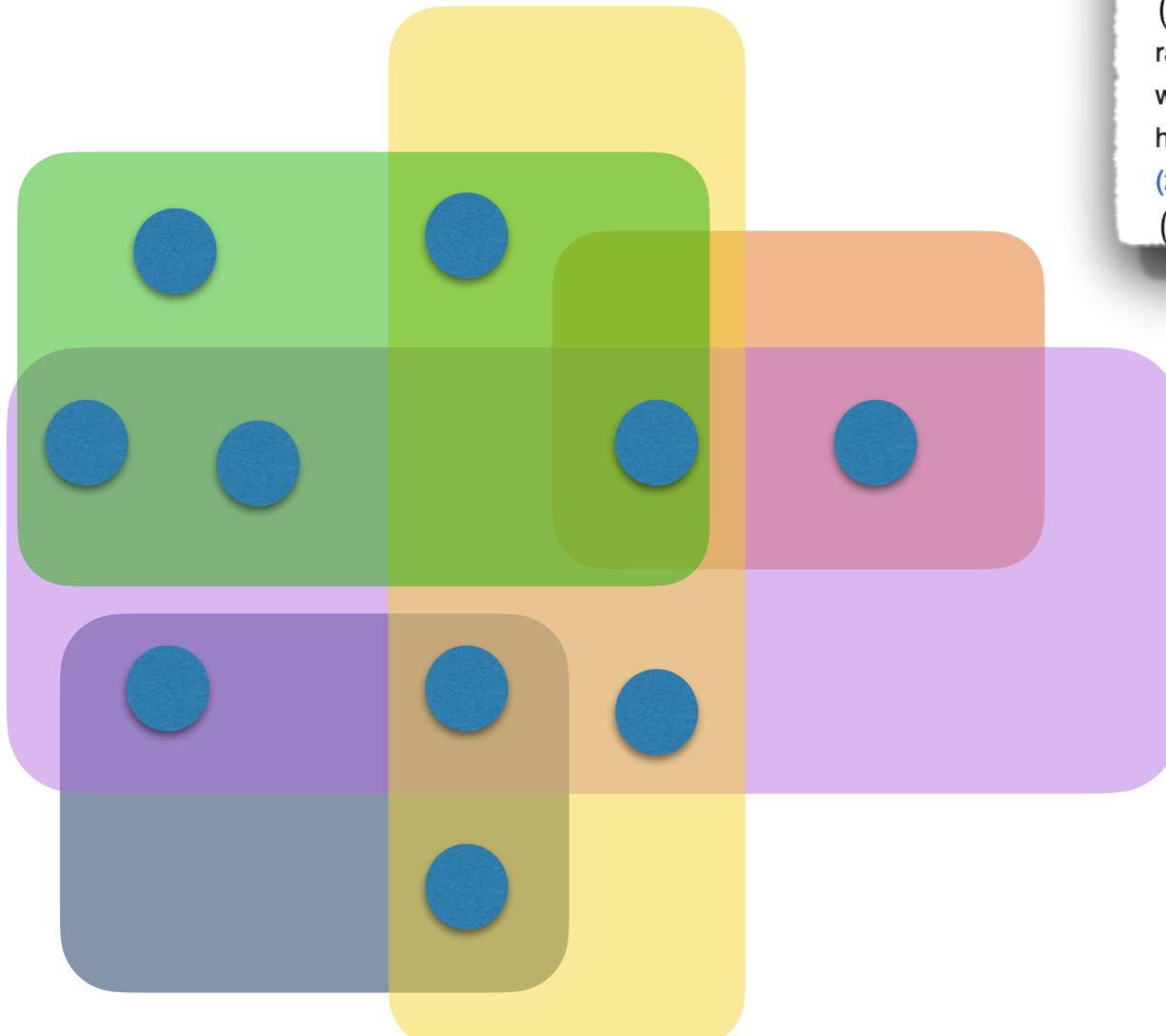
$L = \{-n, \dots, -1, 1, \dots, n\}$

valid interactions $I \subseteq L \times L$

Goal: Find minimum cardinality set of configurations that covers I

↗ we call this a sample

The Set Cover Problem



Inapproximability results [edit]

When n refers to the size of the universe, [Lund & Yannakakis \(1994\)](#) showed that set covering cannot be approximated in polynomial time to within a factor of $\frac{1}{2} \log_2 n \approx 0.72 \ln n$, unless NP has [quasi-polynomial time](#) algorithms. [Feige \(1998\)](#) improved this lower bound to $(1 - o(1)) \cdot \ln n$ under the same assumptions, which essentially matches the approximation ratio achieved by the greedy algorithm. [Raz & Safra \(1997\)](#) established a lower bound of $c \cdot \ln n$, where c is a certain constant, under the weaker assumption that $\text{P} \neq \text{NP}$. A similar result with a higher value of c was recently proved by [Alon, Moshkovitz & Safra \(2006\)](#). [Dinur & Steurer \(2013\)](#) showed optimal inapproximability by proving that it cannot be approximated to $(1 - o(1)) \cdot \ln n$ unless $\text{P} = \text{NP}$.

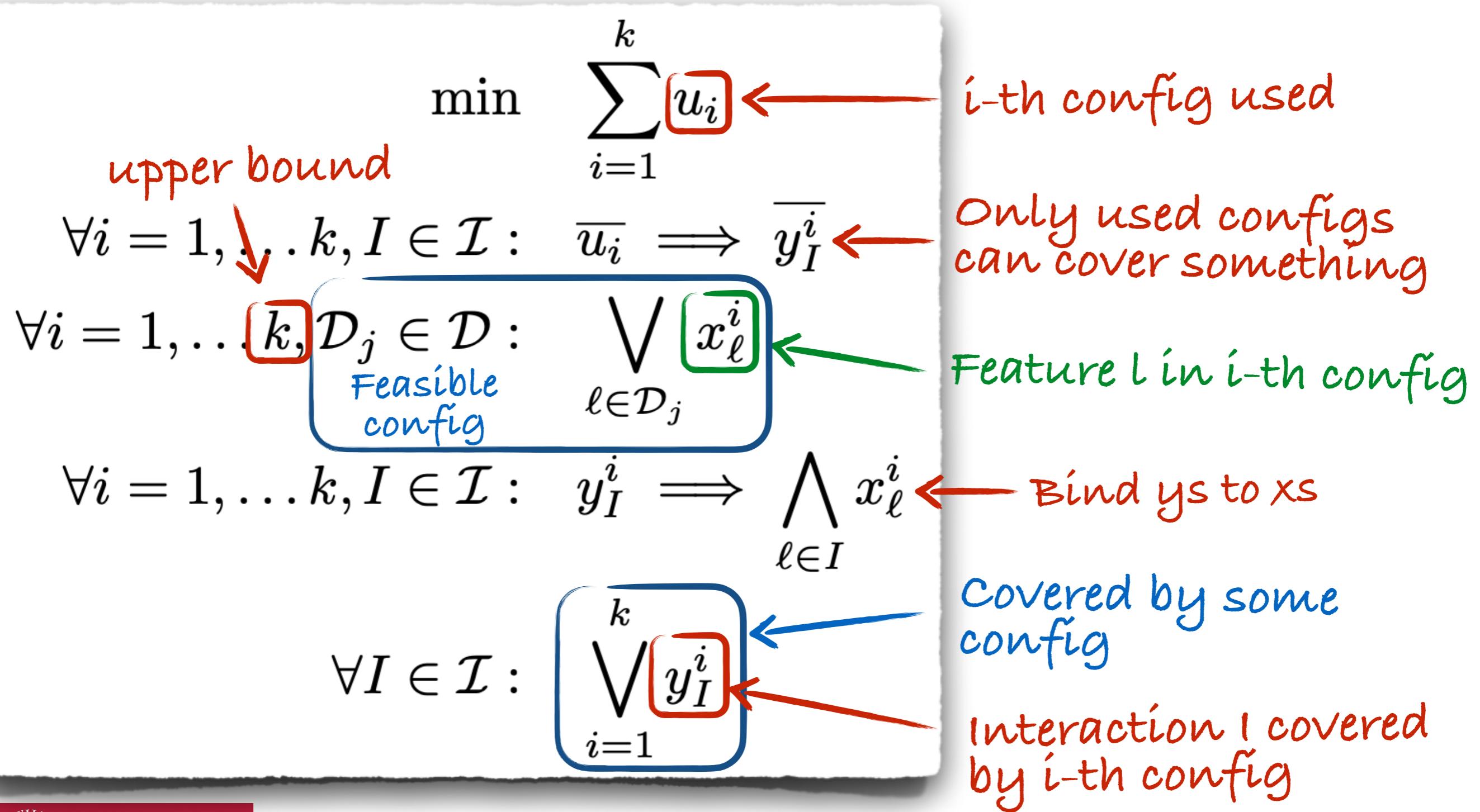
https://en.wikipedia.org/wiki/Set_cover_problem

+ Challenge 1:
Elements NP-hard to
identify

+ Challenge 2:
Exponential number of
potential covering sets



We can express the problem as a SAT-formula
(plus objective) to solve for optimality!



We have k -x the same
model!
Symmetries!!!

$$\min \sum_{i=1}^k u_i$$

$$\forall i = 1, \dots, k, I \in \mathcal{I}: \overline{u_i} \implies \overline{y_I^i}$$

$$\forall i = 1, \dots, k, \mathcal{D}_j \in \mathcal{D}: \bigvee_{\ell \in \mathcal{D}_j} x_\ell^i$$

$$\forall i = 1, \dots, k, I \in \mathcal{I}: y_I^i \implies \bigwedge_{\ell \in I} x_\ell^i$$

$$\forall I \in \mathcal{I}: \bigvee_{i=1}^k y_I^i$$



why are symmetries so bad?

$$\begin{array}{llllll} x_1^1 := 1 & x_2^1 := 1 & x_3^1 := 0 & x_4^1 := 1 & \dots & \\ x_1^2 := 0 & x_2^2 := 0 & x_3^2 := 0 & x_4^2 := 1 & \dots & \\ x_1^3 := 1 & x_2^3 := 0 & x_3^3 := 1 & x_4^3 := 0 & \dots & \\ x_1^4 := 0 & x_2^4 := 1 & x_3^4 := 0 & x_4^4 := 1 & \dots & \\ \vdots & \vdots & \vdots & \vdots & & \\ x_1^n := 1 & x_2^n := 1 & x_3^n := 0 & x_4^n := 0 & \dots & \end{array}$$

*k! equivalent
representations that
look different to the
solver!*

$$10! = 3628800, 15! = 1.3076744e+12, 20! = 2.432902e+18$$

Mutually Exclusive Interactions

Symmetry Breaking

cannot appear in same sample

$(x_1 := 1, x_2 := 1)$

cover in first configuration

$(x_1 := 0, x_2 := 1)$

cover in second configuration

$(x_1 := 1, x_2 := 0)$

cover in third configuration

$(x_1 := 0, x_2 := 0)$

cover in fourth configuration

can we do more?

$$(\neg x_1 \vee \neg x_2 \vee \neg x_3 \vee \neg x_4) \\ (x_1 := 1, x_2 := 1) \quad \text{⚡} \quad (x_3 := 1, x_4 := 1)$$

Find Largest set of
mutually excluding
interactions!

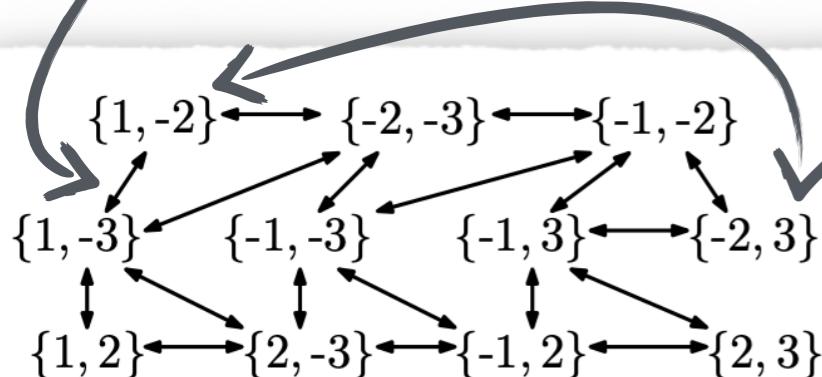
ohoh... isn't that the max clique problem!?



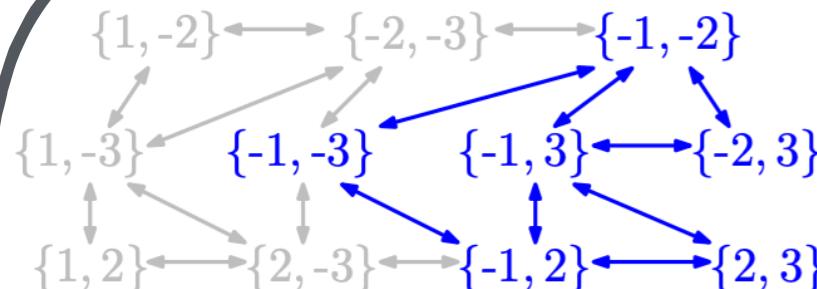
use a Large Neighborhood Search

could interact

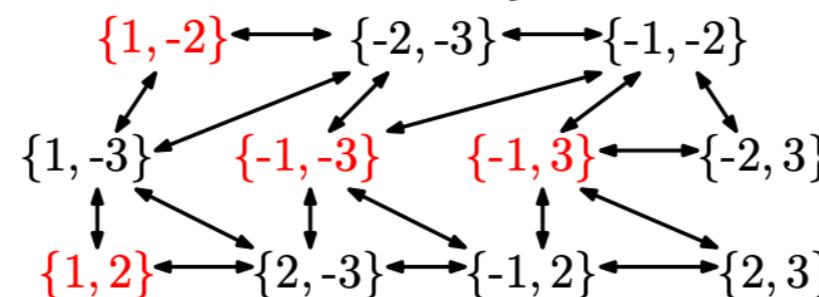
Proven incompatibility



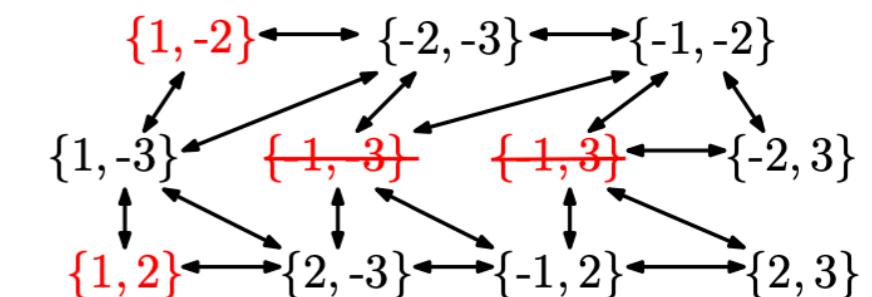
(a) Compatibility Graph



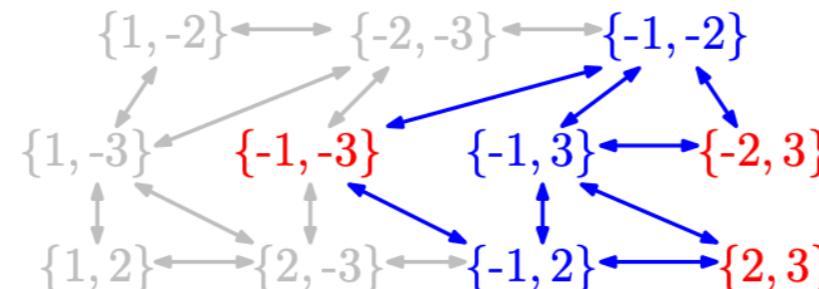
(d) Subgraph of potential extension



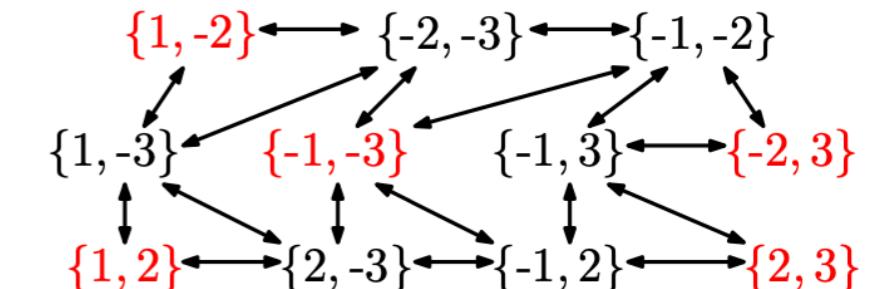
(b) Set of mutually excluding interactions



(c) Remove part of set



(e) Solution of subgraph



(f) Combination to larger set

Inverse of incompatibility graph

Result is a lower bound!



Next problem...
the size...

\min

Medium

$$\sum_{i=1}^k u_i$$

Small

$$\forall i = 1, \dots, k, \forall I \in \mathcal{I} : \cancel{\exists} \quad \overline{u_i} \Rightarrow y_I^i$$

$$\forall i = 1, \dots, k, \forall \mathcal{D}_j \in \mathcal{D} : \quad \bigvee_{\ell \in \mathcal{D}_j} x_\ell^i$$

$$\forall i = 1, \dots, k, \forall I \in \mathcal{I} : \cancel{\exists} \quad y_I^i \Rightarrow \bigwedge_{\ell \in I} x_\ell^i$$

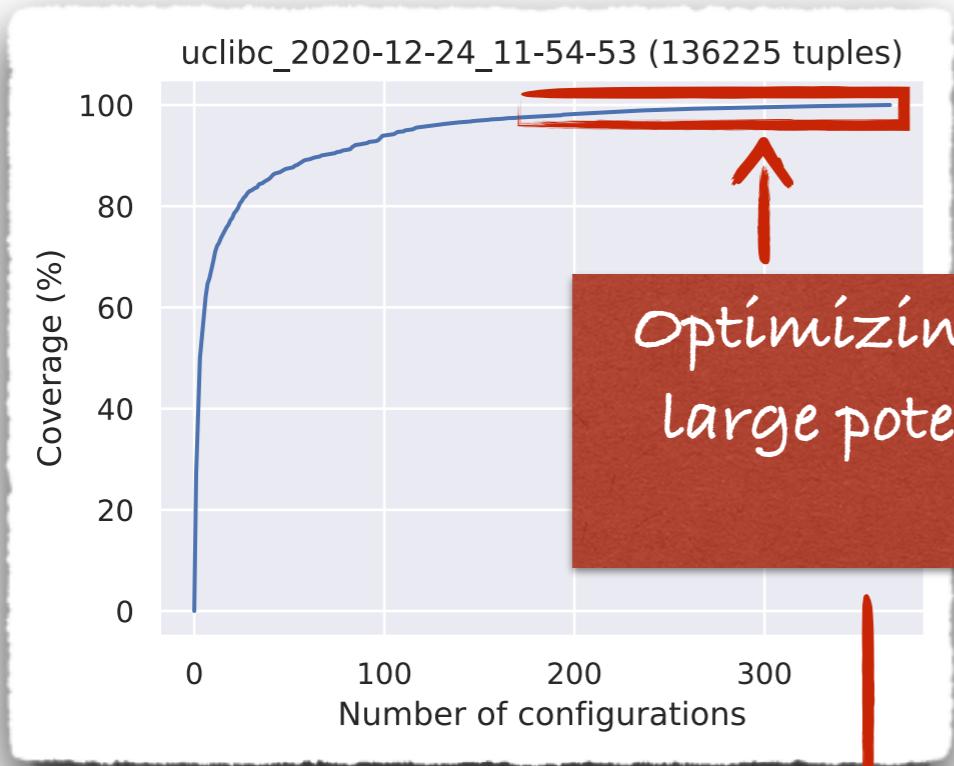
LARGE

$$\forall I \in \mathcal{I} : \cancel{\exists}$$

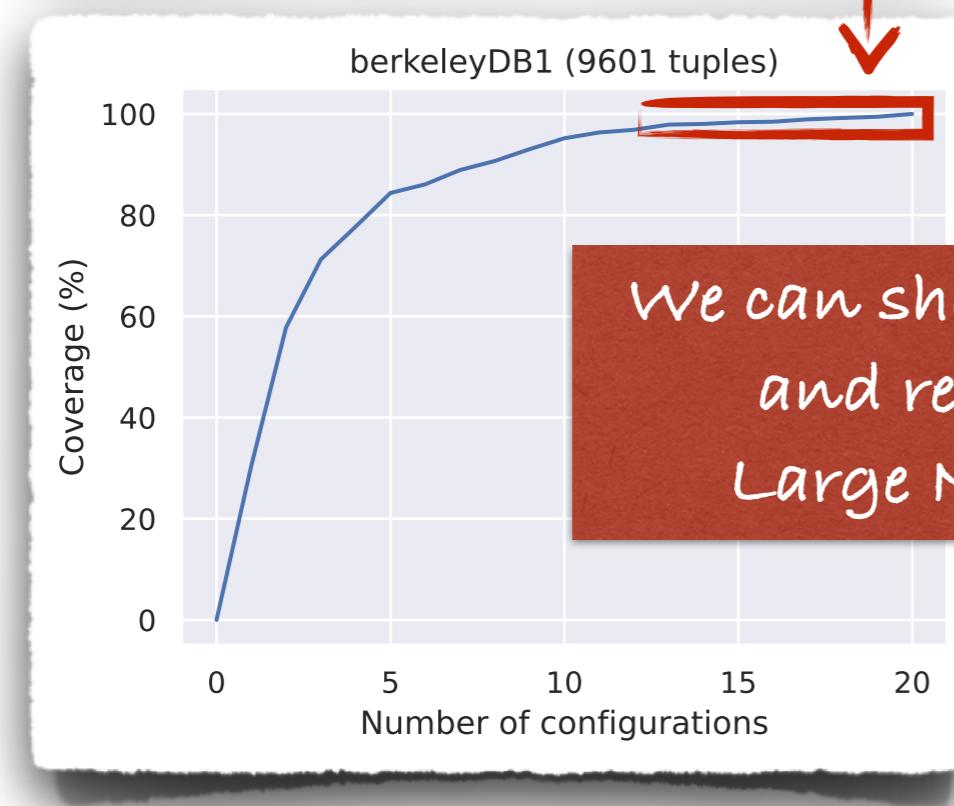
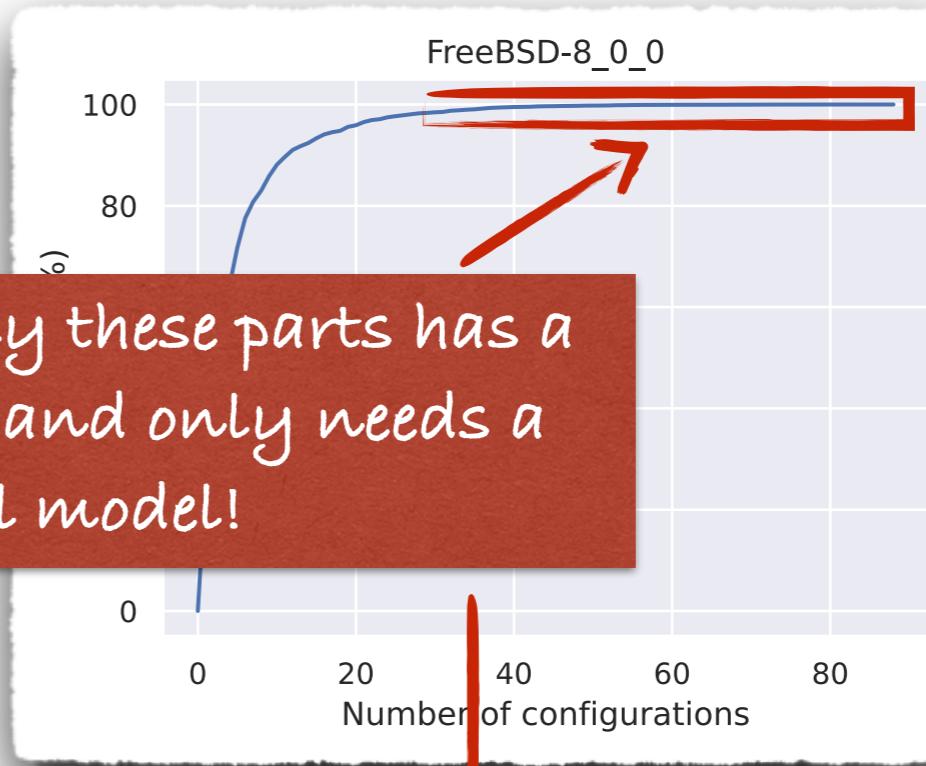
Medium

$$\bigvee_{i=1}^k y_I^i$$

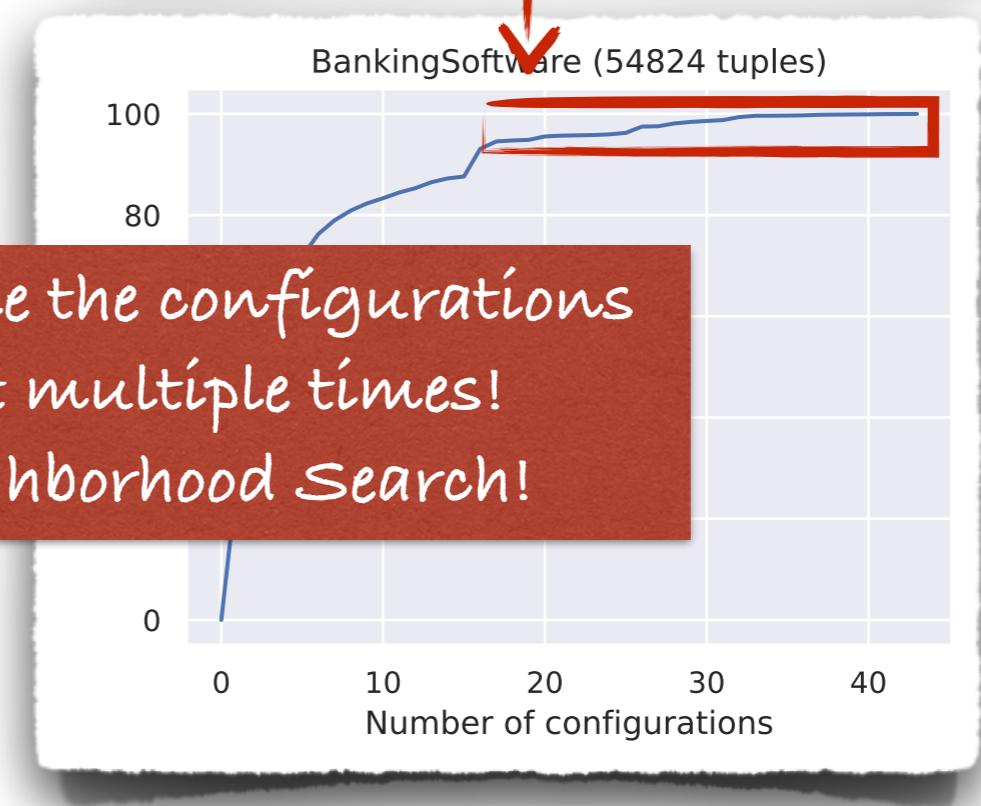




Optimizing only these parts has a large potential and only needs a small model!



We can shuffle the configurations and repeat multiple times!
Large Neighborhood Search!



Features:

$$\mathcal{F} = \{1, 2, 3, 4\}$$

clauses:

$$\mathcal{D} = \{\{1, 2\}, \{3, 4\}\}$$

Example!

Interactions:

$$\mathcal{I} = \{\{3, 4\}, \{1, -3\}, \{2, -4\}, \{1, 3\}, \{-2, 4\}, \{-1, 4\}, \{2, 4\}, \{1, 2\}, \{1, -4\}, \{-2, -3\}, \{-1, -3\}, \{-2, 3\}, \{-1, 3\}, \{3, -4\}, \{-3, 4\}, \{2, -3\}, \{1, -2\}, \{1, 4\}, \{2, 3\}, \{-1, -4\}, \{-2, -4\}, \{-1, 2\}\}$$

Initial sample:

(locally optimal)

$$S = \{\{1, 2, 3, 4\}, \{1, -2, 3, -4\}, \{1, -2, -3, 4\}, \{-1, 2, 3, 4\}, \{-1, 2, 3, -4\}, \{-1, 2, -3, 4\}\}$$

DESTROY: Select random subset...

$$S' = \{\{1, 2, -3, 4\}, \{-1, 2, 3, 4\}, \{-1, 2, 3, -4\}\}$$

Removal leaves uncovered:

(much smaller!)

$$\mathcal{I}' = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{2, -4\}, \{-1, 3\}, \{-1, -4\}\}$$

REPAIR: Compute optimal sample for it...

$$S'' = \{\{1, 2, 3, 4\}, \{-1, 2, -3, 4\}\}$$

Build better sample:

5 instead of 6!

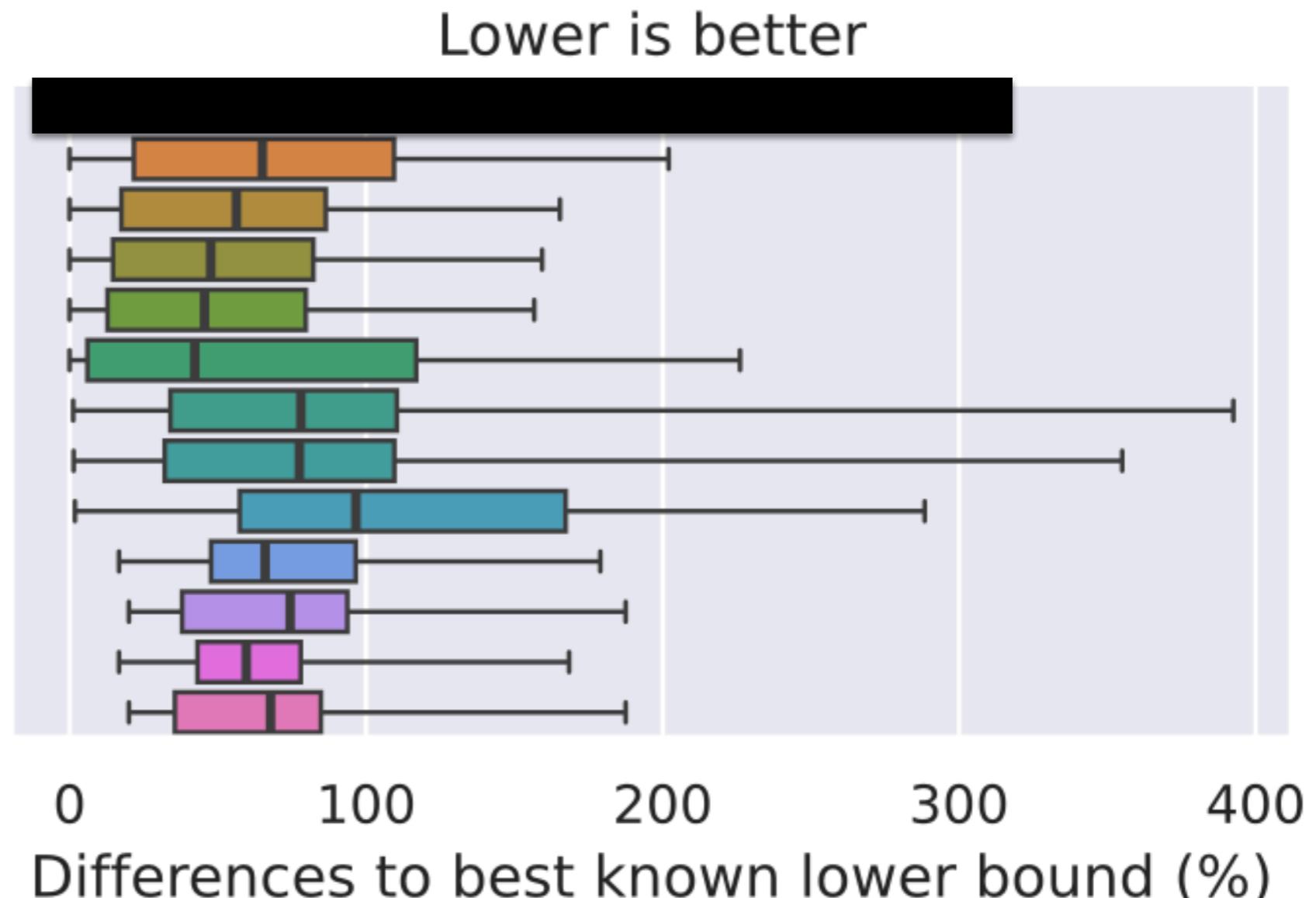
$$S = (S \setminus S') \cup S'' = \{\{1, -2, 3, -4\}, \{1, -2, -3, 4\}, \{-1, 2, -3, 4\}, \{1, 2, 3, 4\}, \{-1, 2, -3, 4\}\}$$

Repeat!

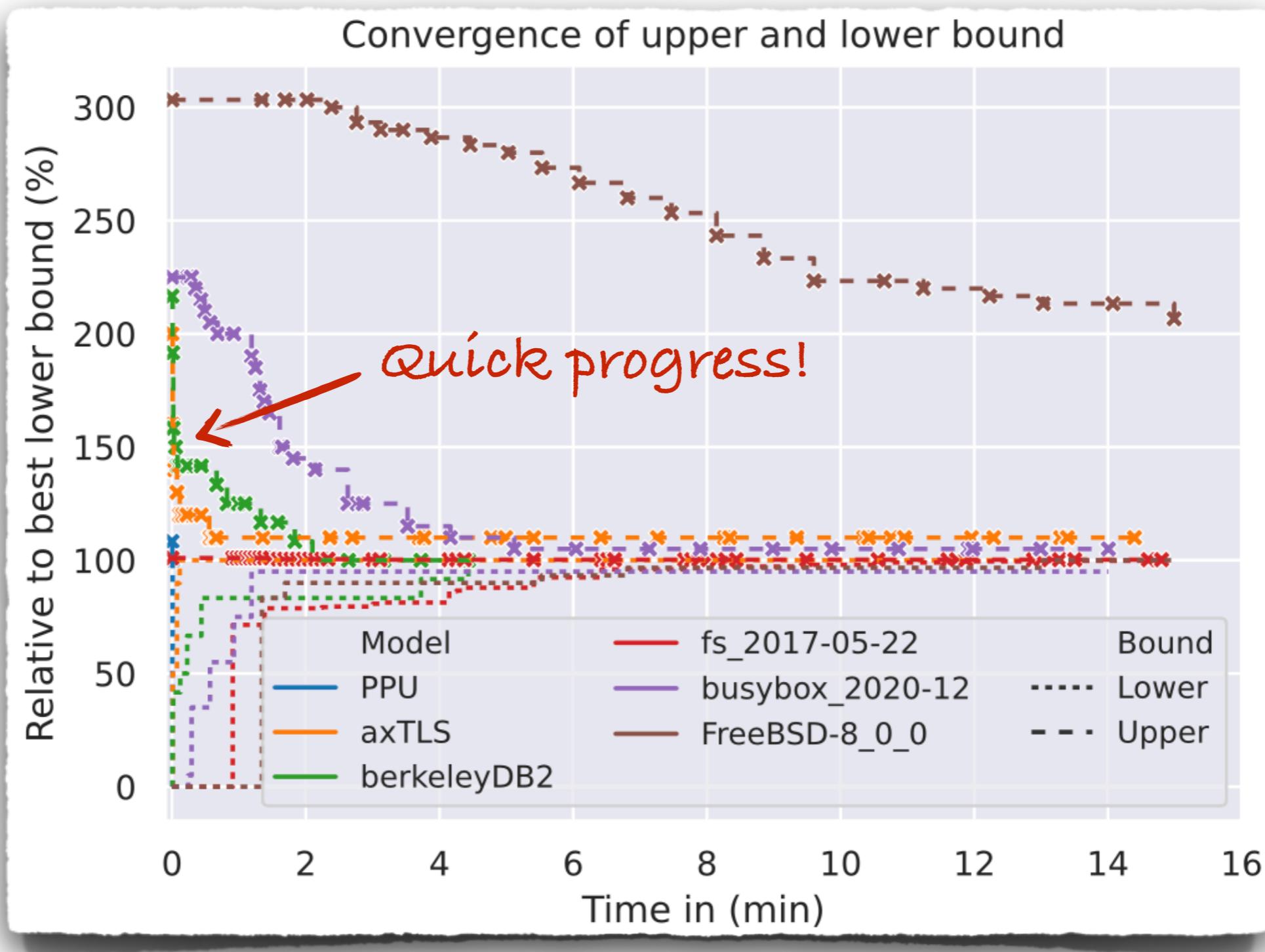


How good are the lower bounds and solutions?

Algorithm [# solved]	
SampLNS [47]	
YASA ($m=1$) [47]	
YASA ($m=3$) [47]	
YASA ($m=5$) [47]	
YASA ($m=10$) [47]	
YASA (15min) [47]	
ICPL [46]	
Chvatal [42]	
Inclning [46]	
ACTS-IPOF-FT [24]	
ACTS-IPOG-FT [24]	
ACTS-IPOF-CSP [22]	
ACTS-IPOG-CSP [22]	

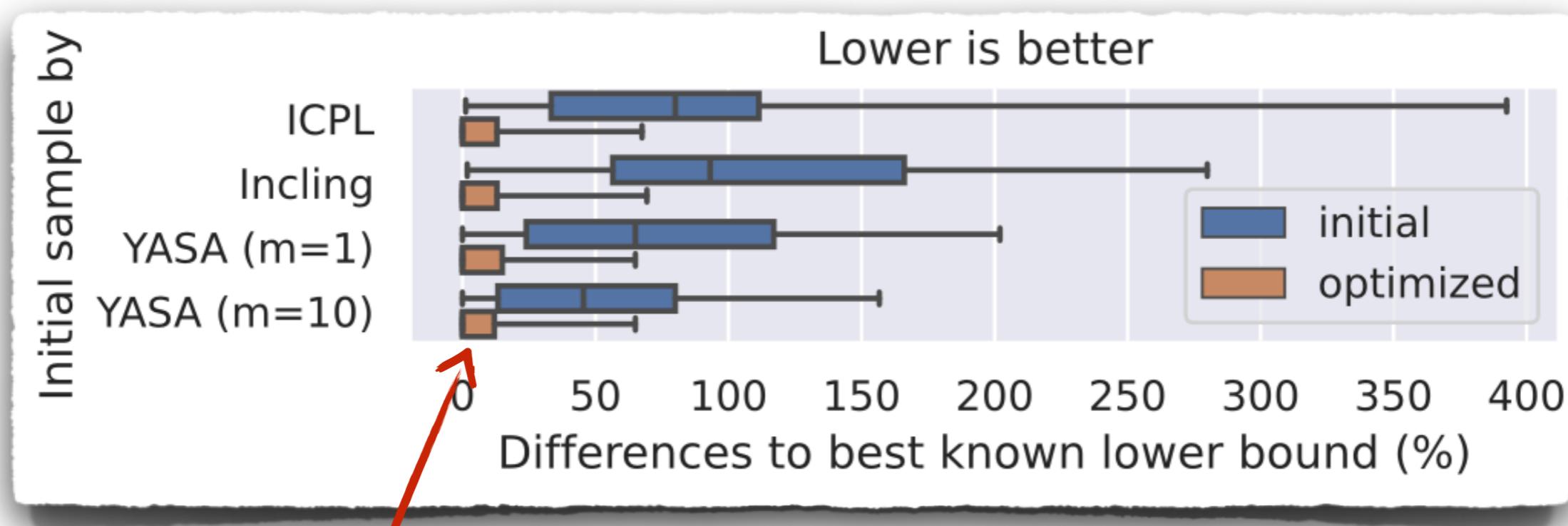


How efficient is the algorithm?



Our algorithm needs an initial solution...

... how important is its quality?



No serious differences

... just use the fastest initial solution!

Feature Model	$ \mathcal{F} $	$ \mathcal{D} $	Baseline min	SampLNS UB mean (min)	SampLNS LB mean (max)	Savings	SampLNS UB/LB	Time to Bounds
calculate	9	15	9	5 (5)	5 (5)	44 % (44 %)	1.00 (1.00)	< 1 s (1 s)
lcm	9	16	8	6 (6)	6 (6)	25 % (25 %)	1.00 (1.00)	< 1 s (< 1 s)
email	10	17	6	6 (6)	6 (6)	0 % (0 %)	1.00 (1.00)	< 1 s (< 1 s)
ChatClient	14	20	7	7 (7)	7 (7)	0 % (0 %)	1.00 (1.00)	1 s (2 s)
toybox_2006-10-31...	16	13	9	8 (8)	8 (8)	11 % (11 %)	1.00 (1.00)	1 s (1 s)
car	16	33	6	5 (5)	5 (5)	17 % (17 %)	1.00 (1.00)	< 1 s (< 1 s)
FeatureIDE	19	27	9	8 (8)	8 (8)	11 % (11 %)	1.00 (1.00)	271 s (128 s)
FameDB	22	40	8	8 (8)	8 (8)	0 % (0 %)	1.00 (1.00)	1 s (1 s)
APL	23	35	9	7 (7)	7 (7)	22 % (22 %)	1.00 (1.00)	1 s (1 s)
SafeBali	24	45	11	11 (11)	11 (11)	0 % (0 %)	1.00 (1.00)	< 1 s (< 1 s)
TightVNC	28	39	11	8 (8)	8 (8)	27 % (27 %)	1.00 (1.00)	16 s (21 s)
APL-Model	28	40	10	8 (8)	8 (8)	20 % (20 %)	1.00 (1.00)	14 s (15 s)
gpl	38	99	17	16 (16)	16 (16)	5.9 % (5.9 %)	1.00 (1.00)	3 s (3 s)
SortingLine	39	77	12	9 (9)	9 (9)	25 % (25 %)	1.00 (1.00)	8 s (9 s)
dell	46	244	32	31 (31)	31 (31)	3.1 % (3.1 %)	1.00 (1.00)	29 s (45 s)
PPU	52	109	12	12 (12)	12 (12)	0 % (0 %)	1.00 (1.00)	2 s (2 s)
berkeleyDB1	76	147	19	15 (15)	15 (15)	21 % (21 %)	1.00 (1.00)	77 s (137 s)
axTLS	96	183	16	11 (11)	10 (10)	31 % (31 %)	1.10 (1.10)	20 s (20 s)
Violet	101	203	23	17 (17)	16 (16)	26 % (26 %)	1.06 (1.06)	476 s (656 s)
berkeleyDB2	119	346	20	12 (12)	12 (12)	40 % (40 %)	1.00 (1.00)	162 s (282 s)
soletta_2015-06-2...	129	192	30	24 (24)	24 (24)	20 % (20 %)	1.00 (1.00)	21 s (60 s)
BattleofTanks	144	769	451	320 (295)	256 (256)	29 % (35 %)	1.25 (1.15)	887 s (160 min)
BankingSoftware	176	280	40	29 (29)	29 (29)	28 % (28 %)	1.00 (1.00)	306 s (429 s)
fiasco_2017-09-26...	230	1,181	234	225 (225)	225 (225)	3.8 % (3.9 %)	1.00 (1.00)	382 s (579 s)
fiasco_2020-12-01...	258	1,542	209	196 (196)	196 (196)	6.1 % (6.2 %)	1.00 (1.00)	438 s (478 s)
uclibc_2008-06-05...	263	1,699	505	505 (505)	505 (505)	0 % (0 %)	1.00 (1.00)	104 s (67 s)
uclibc_2020-12-24...	272	1,670	365	365 (365)	365 (365)	0 % (0 %)	1.00 (1.00)	108 s (112 s)
E-Shop	326	499	19	12 (12)	9 (10)	37 % (37 %)	1.30 (1.20)	268 s (64 min)
toybox_2020-12-06...	334	92	18	13 (13)	7 (8)	28 % (28 %)	1.71 (1.62)	532 s (35 min)
DMIE	366	627	26	16 (16)	16 (16)	38 % (38 %)	1.00 (1.00)	104 s (135 s)
soletta_2017-03-0...	458	1,862	56	37 (37)	31 (37)	34 % (34 %)	1.16 (1.00)	387 s (24 min)
busybox_2007-01-2...	540	429	34	21 (21)	21 (21)	38 % (38 %)	1.00 (1.00)	164 s (237 s)
fs_2017-05-22	557	4,992	398	396 (396)	396 (396)	0.5 % (0.5 %)	1.00 (1.00)	478 s (575 s)
WaterlooGenerated	580	879	144	82 (82)	82 (82)	43 % (43 %)	1.00 (1.00)	223 s (310 s)
financial_services	771	7,238	4,384	4,368 (4,340)	4,274 (4,336)	0.36 % (1 %)	1.02 (1.00)	862 s (102 min)
busybox-1_18_0	854	1,164	26	16 (16)	11 (13)	35 % (38 %)	1.53 (1.23)	233 s (59 min)
busybox-1_29_2	1,018	997	36	22 (22)	17 (21)	38 % (39 %)	1.26 (1.05)	465 s (60 min)
busybox_2020-12-1...	1,050	996	33	21 (20)	17 (19)	36 % (39 %)	1.19 (1.05)	407 s (17 min)
am31_sim	1,178	2,747	60	36 (33)	26 (29)	39 % (45 %)	1.36 (1.14)	699 s (77 min)
EMBToolkit	1,179	5,414	1,881	1,879 (1,872)	1,821 (1,872)	0.1 % (0.48 %)	1.03 (1.00)	863 s (47 min)
atlas_mips32_4kc	1,229	2,875	66	38 (36)	31 (33)	41 % (45 %)	1.22 (1.09)	548 s (50 min)
eCos-3-0_i386pc	1,245	3,723	64	43 (39)	31 (36)	32 % (39 %)	1.38 (1.08)	621 s (146 min)
integrator_arm7	1,272	2,980	66	38 (36)	30 (33)	41 % (45 %)	1.28 (1.09)	681 s (82 min)
XSEngine	1,273	2,942	63	38 (36)	31 (32)	39 % (43 %)	1.23 (1.12)	572 s (52 min)
aaed2000	1,298	3,036	87	55 (52)	51 (51)	36 % (40 %)	1.09 (1.02)	707 s (75 min)
FreeBSD-8_0_0	1,397	15,692	76	47 (41)	27 (30)	38 % (46 %)	1.72 (1.37)	831 s (120 min)
ea2468	1,408	3,319	65	38 (36)	31 (32)	41 % (45 %)	1.24 (1.12)	721 s (67 min)

optimality

7

[15 %]

 ≥ 26

[55 %]

improvements

40 [85 %]



Summary

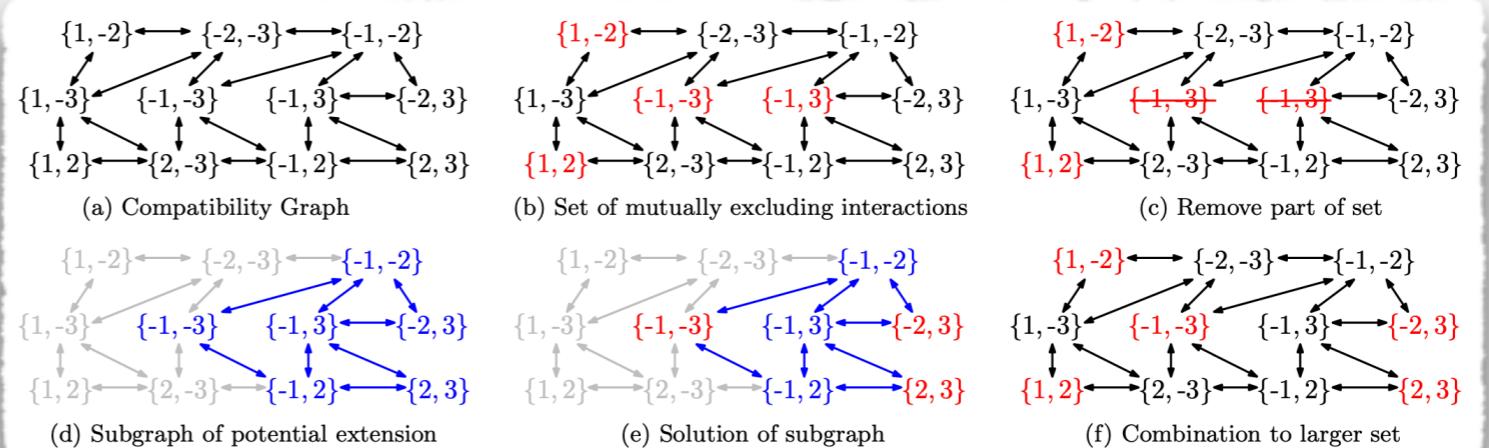
$$\min \sum_{i=1}^k u_i$$

$$\forall i = 1, \dots, k, I \in \mathcal{I} : \bar{u}_i \implies \bar{y}_I^i$$

$$\forall i = 1, \dots, k, \mathcal{D}_j \in \mathcal{D} : \bigvee_{\ell \in \mathcal{D}_j} x_\ell^i$$

$$\forall i = 1, \dots, k, I \in \mathcal{I} : y_I^i \implies \bigwedge_{\ell \in I} x_\ell^i$$

$$\forall I \in \mathcal{I} : \bigvee_{i=1}^k y_I^i$$



Medium

$$\min \sum_{i=1}^k u_i$$

$$\forall i = 1, \dots, k, I \in \mathcal{I} : \bar{u}_i \implies \bar{y}_I^i$$

$$\forall i = 1, \dots, k, \mathcal{D}_j \in \mathcal{D} : \bigvee_{\ell \in \mathcal{D}_j} x_\ell^i$$

$$\forall i = 1, \dots, k, I \in \mathcal{I} : y_I^i \implies \bigwedge_{\ell \in I} x_\ell^i$$

$$\forall I \in \mathcal{I} : \bigvee_{i=1}^k y_I^i$$

