



[Elbe, Magdeburg]

Tseitin or not Tseitin?

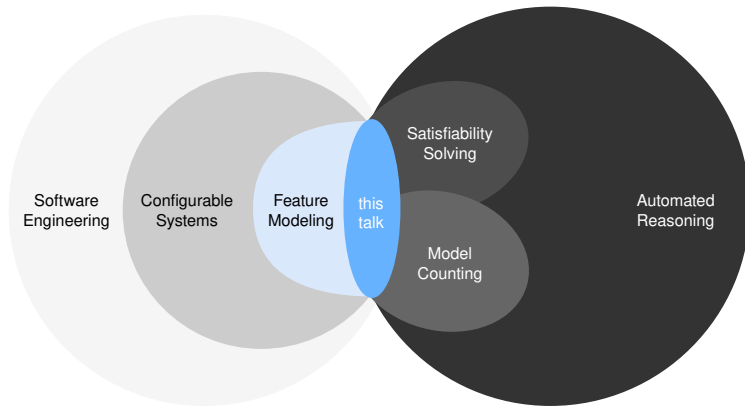
The Impact of CNF Transformations on Feature-Model Analyses

MCW@SAT 2023 (ASE 2022) — July 4 — Alghero, Italy

Elias Kuter¹, Sebastian Krieter², Chico Sundermann², Thomas Thüm², Gunter Saake¹

University of Magdeburg¹, Ulm², Germany

Software Engineering Meets Automated Reasoning



Implementing Configurable Software Systems

A Configurable Graph

```
class Node {  
    #ifdef LABELED  
        std::string label;  
    #endif  
    #ifdef COLORED  
        std::string color;  
    #endif  
};  
  
class Edge {  
    #ifdef DIRECTED  
        Node from, to;  
    #elif UNDIRECTED && HYPER  
        std::set<Node> nodes;  
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```

Product Line Implementation

(here: C++ with C preprocessor)

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Product Line Implementation
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A Labeled Directed Graph

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Configuration

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A Colored Undirected Hypergraph

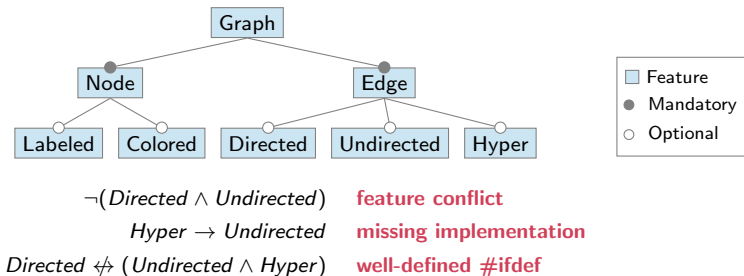
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Product Implementation

Modeling Features and their Dependencies

Feature Models

- tree models **features**
- cross-tree **constraints** model dependencies
- solver-based **analyses** can be used to understand the configuration space better



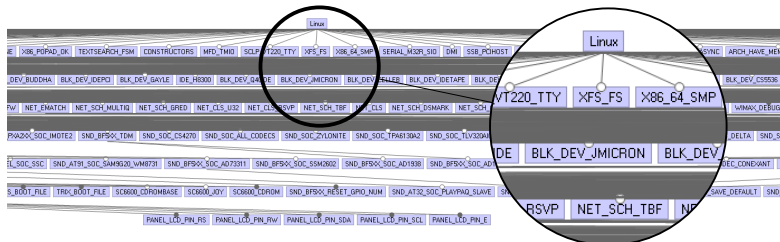
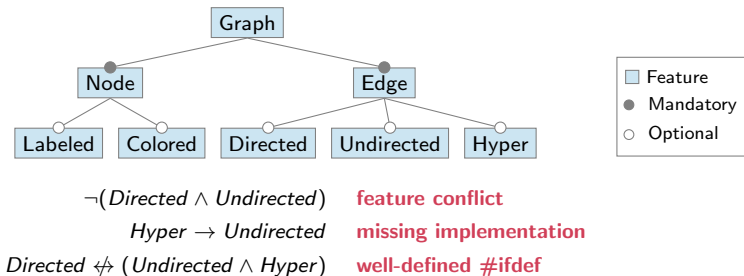
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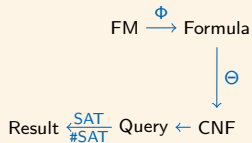
The Linux Kernel

- > 13000 features [2018]
- > 10^{700} products [2007]
- 114 dead features [2013]
- 151 reverse dependency bugs [2019]



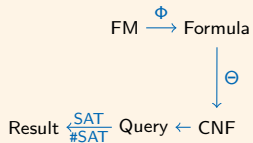
Analyzing Feature Models with SAT and #SAT Solvers

Feature-Model Analysis

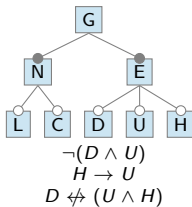


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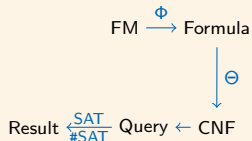


A Feature Model FM

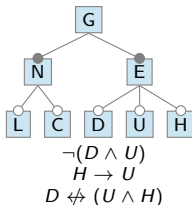


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$\Phi \rightarrow$

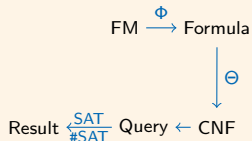
As a Formula $\Phi(FM)$

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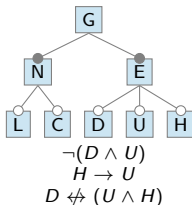
$$\begin{aligned} &G \\ &\wedge (N \leftrightarrow G) \wedge (E \leftrightarrow G) \\ &\wedge ((L \vee C) \rightarrow N) \\ &\wedge ((D \vee U \vee H) \rightarrow E) \\ &\wedge \neg(D \wedge U) \wedge (H \rightarrow U) \\ &\wedge (D \nleftrightarrow (U \wedge H)) \end{aligned}$$

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Φ

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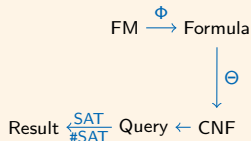
$\downarrow \Theta$

As a CNF $\Theta(\Phi(FM))$

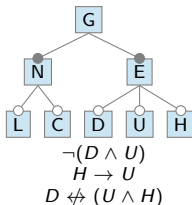
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Core Features

$\{G, N, E\}$

Core Feature F ?

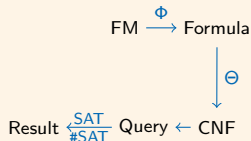
$\text{SAT}(\Theta(\Phi(FM)) \wedge \neg F)$

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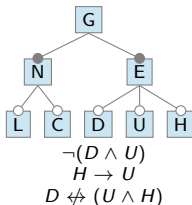
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Feature Model Cardinality

8

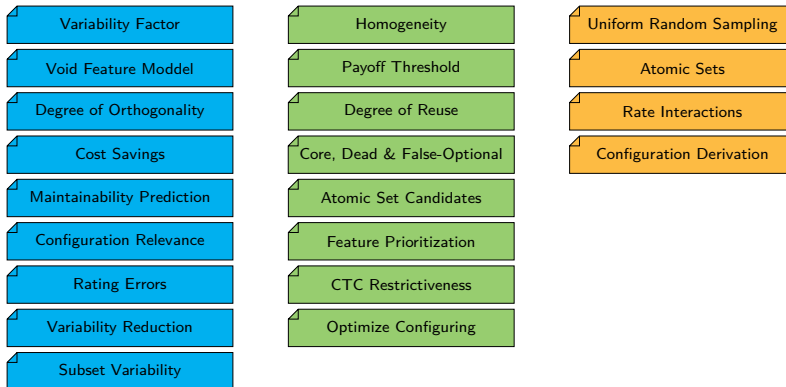
Products in FM ?

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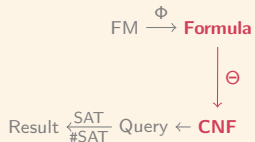
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Feature Modeling Meets Model Counting



Often Overlooked: Conjunctive Normal Form (CNF)

Feature-Model Analysis



From Formula ...

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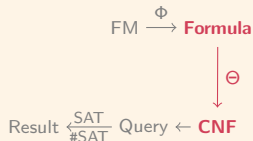
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Often Overlooked: Conjunctive Normal Form (CNF)

Feature-Model Analysis



Conjunctive Normal Form

- **conjunction** \wedge of **disjunctions** \vee of **literals** $X, \neg X$
- here: a set of **clauses**, which are sets of literals
- used by almost all solvers

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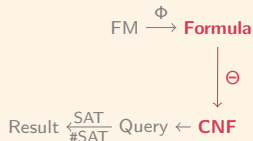
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Our Goal: Raise Awareness for CNF Transformations

[ASE'22]

- how to **transform** feature-model formulas **into CNF**?
 \Rightarrow describe and classify CNF transformations
- does this **impact** the work of **practitioners and researchers**?
 \Rightarrow evaluate efficiency and correctness on feature models

... to CNF

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CNF Transformations

Distributive $\Theta = D$

apply laws of logic (**De Morgan's laws** and **distributivity**)

$$D \not\leftrightarrow (U \wedge H)$$

$$\xrightarrow{D} (D \vee (U \wedge H)) \wedge (\neg D \vee \neg(U \wedge H))$$

$$\xrightarrow{D} \{\{D, U\}, \{D, H\}, \{\neg D, \neg U, \neg H\}\}$$

✓ **equivalence**

SAT ✓, #SAT = 4

✓ easy to implement

✗ **exponential** complexity

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Tseitin $\Theta = T$

[83]

abbreviate a subformula ϕ with an auxiliary variable $x_\phi \leftrightarrow \phi$

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Plaisted-Greenbaum $\Theta = PG$ [86]

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$$\xrightarrow{PG} (D \not\leftrightarrow x) \wedge x \rightarrow (U \wedge H)$$

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- ✓ **equi-assignability** SAT ✓
- ✓ **linear** complexity < T
- ✗ **equi-countability** #SAT = 5

Evaluation

Research Questions

- RQ 1 **efficiency** of CNF transformations?
- RQ 2 CNF transformation → **efficiency** of analyses?
- RQ 3 CNF transformation → **correctness** of analyses?

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- 22 configurable software systems
- 3 CNF transformation tools
- 23 SAT and #SAT solvers

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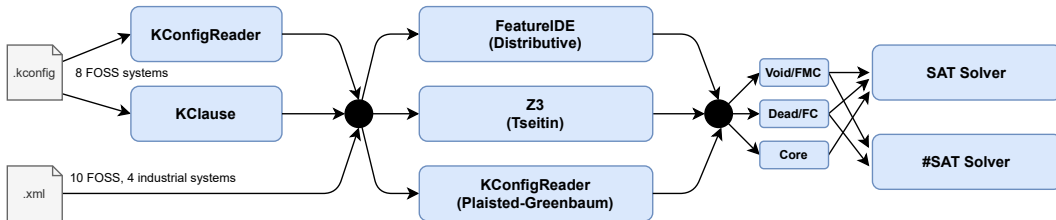
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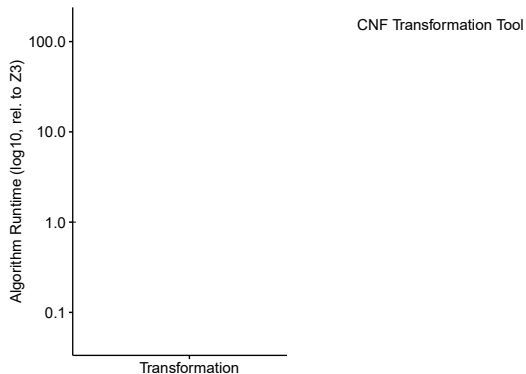
Stage 1: Formula Extraction

Stage 2: CNF Transformation

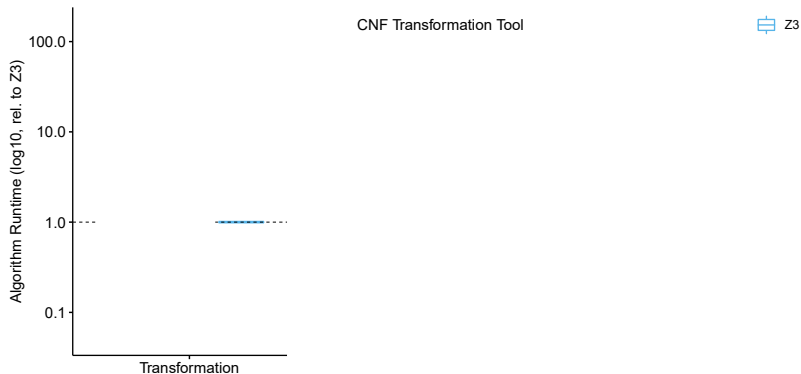
Stage 3: Automated Analysis



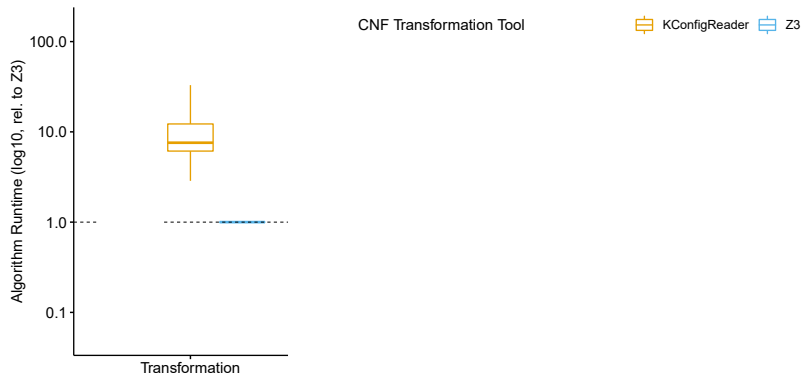
Efficiency of CNF Transformations (RQ 1) and Analyses (RQ 2)



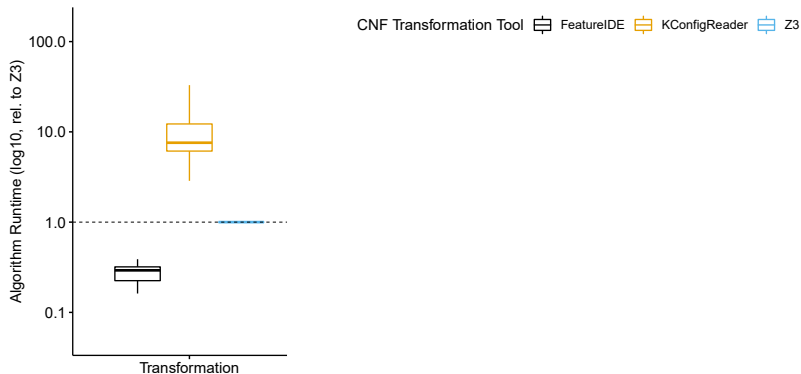
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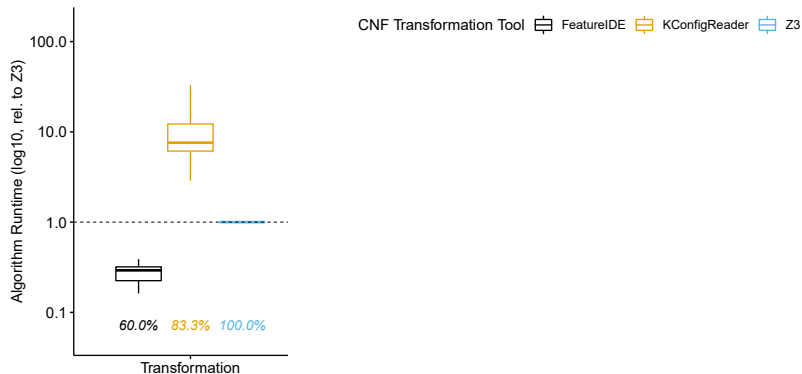
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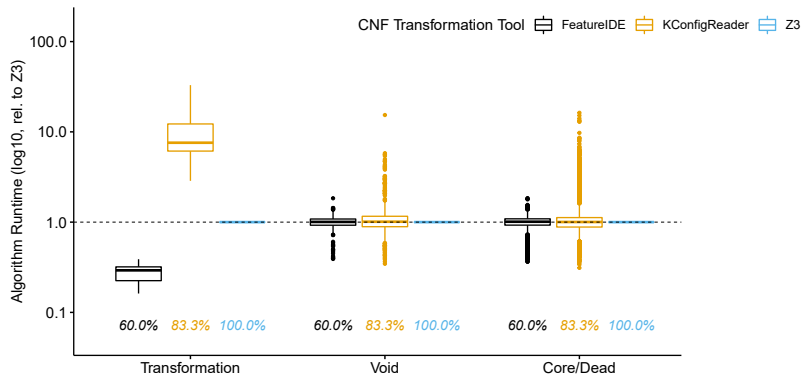


Efficiency of CNF Transformations (RQ 1) and Analyses (RQ 2)



RQ 1: *D* often fails,
tools differ significantly

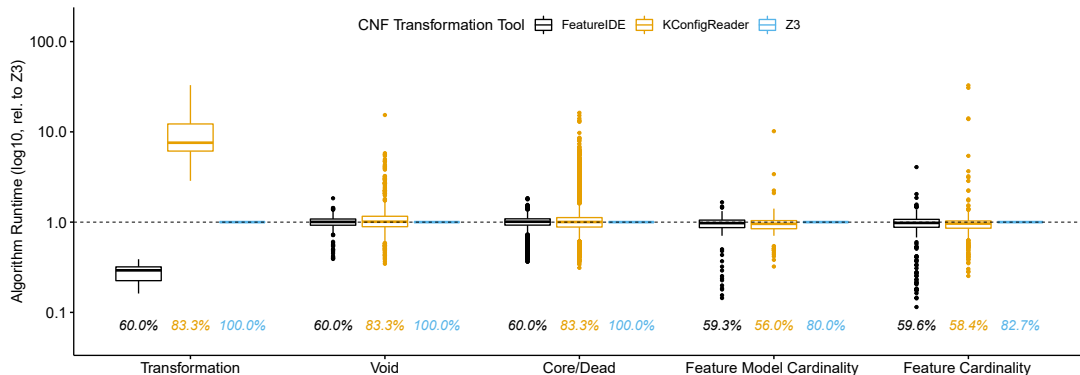
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solve time varies by factor 0.31–16.27

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RQ 1: *D* often fails,
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RQ 2 (SAT): almost all calls succeed,
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RQ 2 (#SAT): 81.6% of calls succeed,
solve time varies by factor 0.11–32.7

Correctness of #SAT-Based Analyses (RQ 3)

How Many Valid Configurations in BusyBox 1.35.0?

FeatureIDE (Distributive) says:

```
47842046044873008384 13517649496919484532 17980737275928522342 35800557238486733859
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- with PG , $\approx 70\%$ of #SAT calls return **incorrect results**
- incorrect by factor ≈ 3 (median)
- incorrect by factor $\approx 10^{77}$ (worst)

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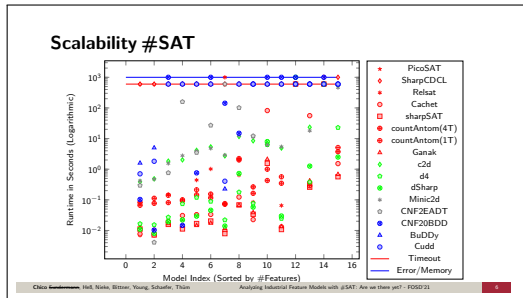
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Our Recommendations

- RQ 1** D for small, T for large models
- RQ 2** largely depends on the model
 \Rightarrow future work
- RQ 3** do not use PG for #SAT

Perspective: Model Counting Meets Feature Modeling?

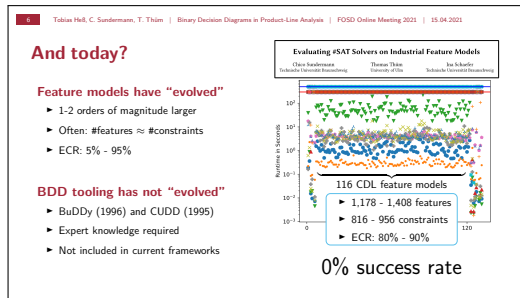
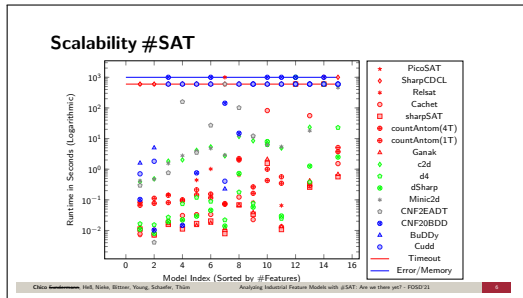
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Compiling to d-DNNFs [Chico Sundermann]

works on most models (but not Linux/Automotive)
15 software systems, 130 models (some industrial)

Perspective: Model Counting Meets Feature Modeling?



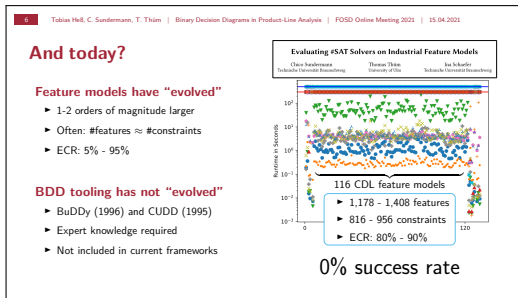
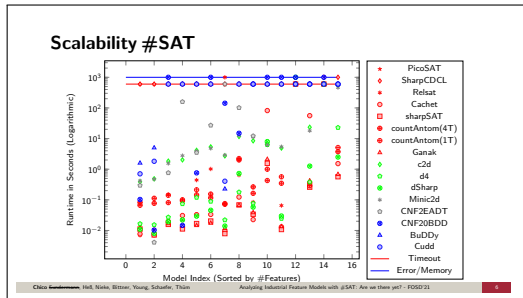
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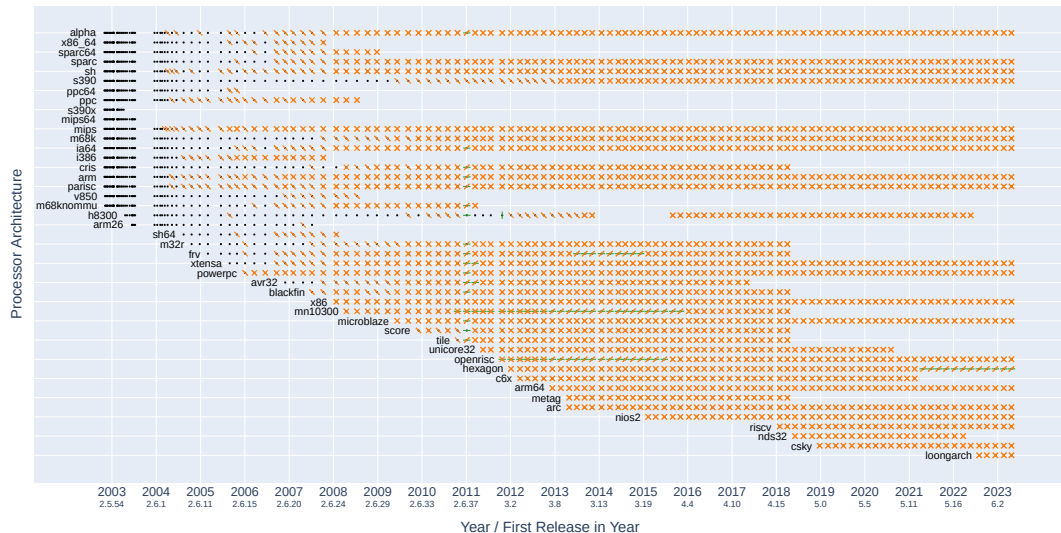
our opinion: feature models are interesting, diverse, and potentially hard problem instances for benchmarking (#)SAT and knowledge compilation. maybe they could contribute to the SAT and MC competition as well?

Perspective: How Hard can Linux be, Really?

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[illegible]

Perspective: How Hard can Linux be, Really? [SharpSAT+td+Arjun, d4]



Conclusion

The Impact of CNF Transformations on Feature-Model Analyses

Distributive

apply laws of logic

- ✓ **equivalence**
- ✓ easy to implement
- ✗ **exponential** complexity

FeatureIDE

often fails on large models

Tseitin

abbreviate ϕ with $x_\phi \leftrightarrow \phi$

- ✓ **quasi-equivalence**
- ✓ **linear** complexity
- ✗ take care of new variables

Z3

succeeds correctly on all models

Plaisted-Greenbaum

abbreviate ϕ with $x_\phi \rightarrow \phi$

- ✓ **equi-assignability**
- ✓ **linear** complexity
- ✗ **equi-countability**

KConfigReader

often incorrect for #SAT calls

Compiling to **d-DNNFs**

Compiling to **BDDs**

Counting **Linux** is hard

find out more:



github.com/ekuiter/tseitin-or-not-tseitin

