4. Feature Modeling – Handout

Software Product Lines | Elias Kuiter, Thomas Thüm, Timo Kehrer | April 26, 2023

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4. Feature Modeling

4a. Feature Models and Configurations

4b. Transforming Feature Models

4c. Analyzing Feature Models

Recap: Software Product Lines [Lecture 1]

Software Product Line [Northrop et al. 2012, p. 5]

"A **software product line** is

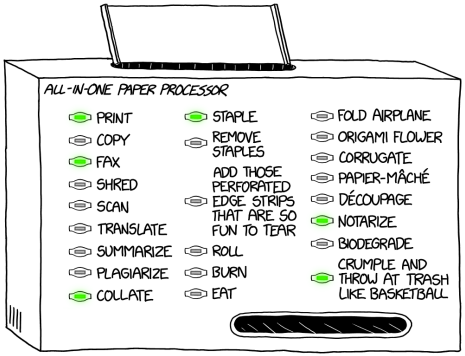
- a set of software-intensive systems
- that share a common, managed set of features
- satisfying the specific needs of a particular market segment or mission
- and that are developed from a common set of core assets in a prescribed way." [Software Engineering Institute, Carnegie Mellon University]

Product [Apel et al. 2013, p. 19]

"A **product of a product line** is specified by a valid feature selection (a subset of the features of the product line). A feature selection is **valid** if and only if it fulfills all feature dependencies."

Feature [Apel et al. 2013, p. 18]

"A **feature** is a characteristic or end-user-visible behavior of a software system."



Features Have Dependencies

Ordering a Waffle . . .

. . . with Sugar

. . . with Cherries

This is Nice, But . . .

- plate and sugar seem to always be included, a fork is only included for some orders
  $\Rightarrow$ limitations seem **arbitrary**
- children get special treatment
  $\Rightarrow$ order process is **unfair**
- what exactly am I paying for?
  $\Rightarrow$ investments are **unclear**

In This Lecture

1. how to **model and configure** features and their dependencies?
2. how to **store and communicate**?
3. how to **analyze and understand**?

Specifying Valid Configurations

Configuration

- a **configuration** over a set of features F selects and deselects features in F
- formally: a pair (S, D) such that $S, D \subseteq F$ and S, D are disjoint ($S \cap D = \varnothing$)
- is **complete** if all features are covered ($S \cup D = F$) and **partial** otherwise
- a complete configuration is **valid** if it "makes sense" in the domain and **invalid** otherwise
- we often abbreviate complete configurations with $S \equiv (S, F \setminus S)$

Feature set $F = \{\mathbf{C}onfigDB, \mathbf{G}et, \mathbf{P}ut, \mathbf{D}elete, \mathbf{T}ransactions, \mathbf{W}indows, \mathbf{L}inux\}$

Examples for **complete** configurations:

- **valid** (read-only database on Windows): $(\{C, G, W\}, \{P, D, T, L\})$
- **valid** (fully functional database on Linux): $(\{C, G, P, D, T, L\}, \{W\})$
- **invalid** ($\lightning$ no operating system): $(\{C, G\}, \{P, D, T, W, L\})$
- **invalid** (transactions $\lightning$ read-only database): $(\{C, G, T, L\}, \{P, D, W\})$

Examples for **partial** configurations: $(\{C, G\}, \{P, D\})$, $(\varnothing, \varnothing)$

Specifying Valid Configurations – Natural Language

Valid Configuration

A complete configuration over F is valid if it "makes sense" in the domain. $\rightsquigarrow$ **"makes sense"?**

Natural Language

- informal description of relationships between features in F
- a complete configuration S is **valid** if it conforms to the description

\+ succinct

– sometimes ambiguous

– not machine-readable

"A **configurable database** has an API that allows for at least one of the request types **Get**, **Put**, or **Delete**. Optionally, the database can support **transactions**, provided that the API allows for Put or Delete requests. Also, the database targets a supported operating system, which is either **Windows** or **Linux**."

Specifying Valid Configurations – Configuration Map

Valid Configuration

A complete configuration over F is valid if it "makes sense" in the domain. $\rightsquigarrow$ **"makes sense"?**

Configuration Map

- a **configuration map** over F is a set of complete configurations $M \subseteq \mathcal{P}(F)$
- a complete configuration S is **valid** if it occurs in the configuration map ($S \in M$)
- also known as product map

\+ precise

– not human-readable

– redundant, explodes in size ($0 \leq |M| \leq 2^{|F|}$)

Feature set $F = \{\mathbf{C}onfigDB, \mathbf{G}et, \mathbf{P}ut, \mathbf{D}elete, \mathbf{T}ransactions, \mathbf{W}indows, \mathbf{L}inux\}$

Configuration map:

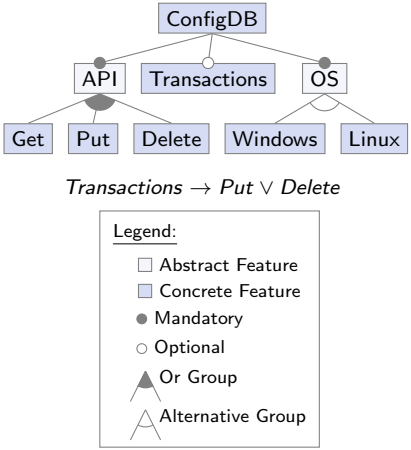
$\{C, G, W\}$	$\{C, G, L\}$
$\{C, P, W\}$	$\{C, P, L\}$
$\{C, G, P, W\}$	$\{C, G, P, L\}$
$\{C, D, W\}$	$\{C, D, L\}$
$\{C, G, D, W\}$	$\{C, G, D, L\}$
$\{C, P, D, W\}$	$\{C, P, D, L\}$
$\{C, G, P, D, W\}$	$\{C, G, P, D, L\}$
$\{C, P, T, W\}$	$\{C, P, T, L\}$
$\{C, G, P, T, W\}$	$\{C, G, P, T, L\}$
$\{C, D, T, W\}$	$\{C, D, T, L\}$
$\{C, G, D, T, W\}$	$\{C, G, D, T, L\}$
$\{C, P, D, T, W\}$	$\{C, P, D, T, L\}$
$\{C, G, P, D, T, W\}$	$\{C, G, P, D, T, L\}$

Specifying Valid Configurations – Configuration Map in Excel

	A	B	C	D	E	F	G
1	**ConfigDB**	**Get**	**Put**	**Delete**	**Transactions**	**Windows**	**Linux**
2	x	x				x	
3	x		x			x	
4	x	x	x			x	
5	x			x		x	
6	x	x		x		x	
7	x		x	x		x	
8	x	x	x	x		x	
9	x		x		x	x	
10	x	x	x		x	x	
11	x			x	x	x	
12	x	x		x	x	x	
13	x		x	x	x	x	
14	x	x	x	x	x	x	
15	x	x					x
16	x		x				x
17	x	x	x				x
18	x			x			x
19	x	x		x			x
20	x		x	x			x
21	x	x	x	x			x
22	x		x		x		x
23	x	x	x		x		x
24	x			x	x		x
25	x	x		x	x		x
26	x		x	x	x		x
27	x	x	x	x	x		x

Can we do better?

Feature Models – Syntax [Apel et al. 2013; Kang et al. 1990, pp. 63–72; Batory 2005]



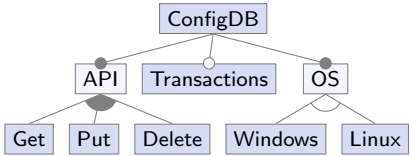
Feature Model

- hierarchy of features
- dependencies between features modeled by tree and cross-tree constraints
- **tree constraints**: defined by the hierarchy
- **cross-tree constraints**: propositional formulas over features
- **abstract features** are used to group other features
- **concrete features** have an implementation
- also known as feature diagram or feature tree
- notation for **optional/mandatory features** and **or/alternative groups**

Feature Models – Semantics [Apel et al. 2013; Batory 2005]

Tree Constraints

- the **root feature** is always required
- each feature requires its parent (aka. **parent-child-relationship**)
- an **optional feature** can be (de-)selected freely when its parent is selected
- a **mandatory feature** is required by its parent
- **or group**: at least one child feature must be selected when the parent is selected
- **alternative group**: exactly one child feature must be selected when the parent is selected

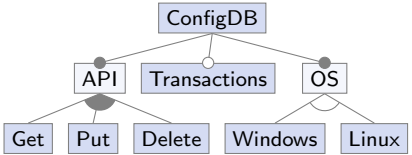


$Transactions \rightarrow Put \vee Delete$

Cross-Tree Constraints

- a list of **propositional formulas** expressing further dependencies between features
- each cross-tree constraint must be satisfied

Feature Models – Examples

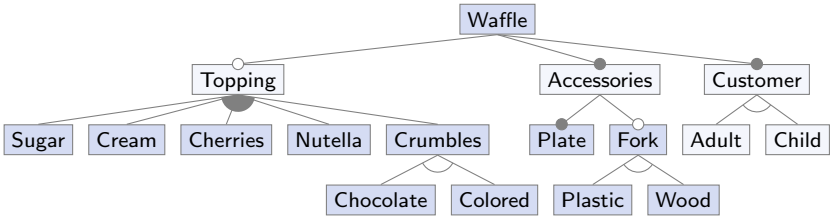


$Transactions \rightarrow Put \vee Delete$

Is This a Valid Configuration?

- **valid** (read-only database on Windows): $(\{C, A, G, O, W\}, \{P, D, T, L\})$
- **valid** (fully functional database on Linux): $(\{C, A, G, P, D, T, O, L\}, \{W\})$
- **invalid** ($\lightning$ no operating system): $(\{C, A, G\}, \{P, D, T, O, W, L\})$
- **invalid** (transactions $\not\rightarrow$ read-only database): $(\{C, A, G, T, O, L\}, \{P, D, W\})$

Feature Models – Examples



$$Sugar$$
$$Cherries \rightarrow Sugar \wedge Fork$$
$$Nutella \vee Crumbles \rightarrow Child$$
$$Fork \rightarrow Adult$$

- every feature (leaf or compound) can be abstract or concrete
- groups can be used anywhere
- directly below groups, no optional or mandatory markers are allowed

Discussion of Feature Models

Pro: Making Tacit Knowledge Explicit

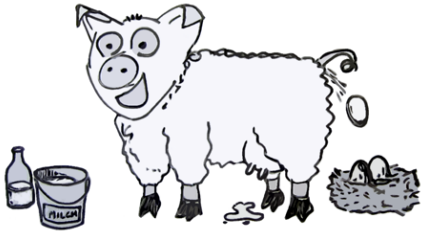
"I think the best [about feature modeling] is you can see relationships, to actually know what configurations are allowed and what are not allowed. That was also not so easy to express in the past [...] This is from the developer's point of view. But it's also [...] important, because before we noticed that **the same functionality was implemented twice** within the same project, basically they haven't realized that. They implemented the same features." – Interview with Practitioners [Berger et al. 2014]

Pro: Tool Support

, Gears, pure::variants, ...

Con: Challenges

- **domain scoping**: which features?
- **feature interactions**: which dependencies?
- requires infrastructure, consulting, and training



Feature Models and Configurations – Summary

Lessons Learned

- features, dependencies between features, and configurations
- feature models: abstract and concrete features, tree and cross-tree constraints
- tree constraints: optional, mandatory, or group, alternative group

Further Reading

- Apel et al. 2013, Section 2.3, pp. 26–39 — introduction to feature modeling
- Thorsten Berger et al. (2013): A Survey of Variability Modeling in Industrial Practice
- Damir Nešić et al. (2019): Principles of Feature Modeling

Practice

1. sketch a feature model with features A, B, C, D, E, F that has exactly those 5 valid configurations (pen and paper preferred):

   $\{A, B\}$ $\{A, C, E\}$ $\{A, C, E, F\}$
   $\{A, B, D\}$ $\{A, C, F\}$

2. discuss in groups whether your feature models are syntactically correct and specify exactly the above configurations

4. Feature Modeling

4a. Feature Models and Configurations

4b. Transforming Feature Models

- Representations and Transformations
- UVL, the Universal Variability Language
- Propositional Formulas
- CNF as a Universal Formula Language
- Summary

4c. Analyzing Feature Models

Representations and Transformations

Natural Language

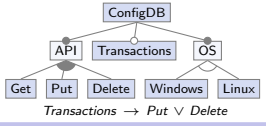
"A **configurable database** has an API that allows for at least one of the request types **Get**, **Put**, or **Delete**. Optionally, the database can support **transactions**, provided that the API allows for Put or Delete requests. Also, the database targets a supported operating system, which is either **Windows** or **Linux**."

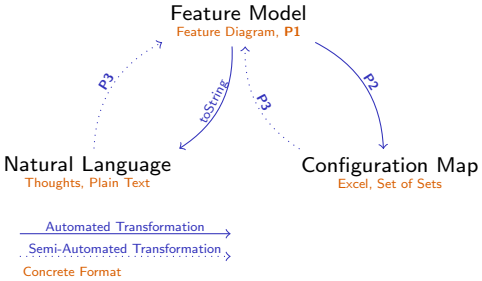
Configuration Map

$\{C, G, W\}$ $\quad$ $\{C, G, L\}$

$\{C, G, P, D, T, W\}$ $\quad$ $\{C, G, P, D, T, L\}$

Feature Diagram (Graphical Feature Model)





Problems

P1 How to express feature models **textually**?

P2 How to (a) validate configurations and (b) get all valid configurations **automatically**?

P3 (How to reverse engineer feature models?)

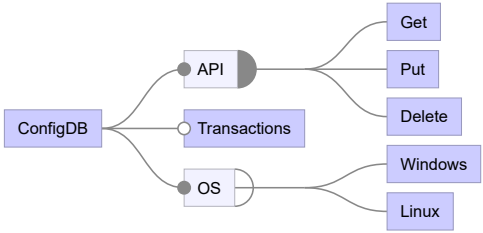
UVL, the Universal Variability Language [UVL]

```
features
  ConfigDB
    mandatory
      API {abstract}
        or
          Get
          Put
          Delete

    optional
      Transactions
    mandatory
      OS {abstract}
        alternative
          Windows
          Linux

constraints
  Transactions => Put | Delete
```

A Feature Model "Sideways"

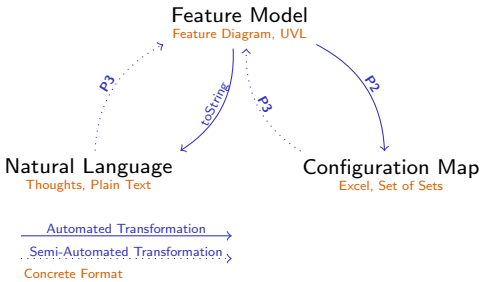


Transactions $\rightarrow$ *Put* $\vee$ *Delete*

Universal Variability Language (UVL)

- textual language for feature modeling
- adds advanced modeling constructs (e.g., attributes, cardinalities, submodels, . . .)

Representations and Transformations



Problems

P1 How to express feature models **textually**?

P2 How to (a) validate configurations and (b) get all valid configurations **automatically**?

P3 (How to reverse engineer feature models?)

Solutions

P1 Universal Variability Language ⇒ **Syntax**

P2 **Semantics**?

P3 –

Propositional Formulas – Recap

Syntax of Propositional Formulas

Inductive definition of **propositional formulas**

- the **Boolean truth values** $\top$, $\bot$
- any **Boolean variable** X
- any **negation** $\neg\phi$ of a formula ϕ
- any **conjunction** $(\phi \wedge \psi)$ of formulas ϕ and ψ
- any **disjunction** $(\phi \vee \psi)$, **implication** $(\phi \rightarrow \psi)$, or **biimplication** $(\phi \leftrightarrow \psi)$

Informal Semantics of Propositional Formulas

$\top$, $\bot$, $\neg\phi$, $\phi \wedge \psi$, $\phi \vee \psi$, $\phi \rightarrow \psi$, $\phi \leftrightarrow \psi$ means

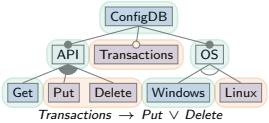
- "true" (or **tautology**)
- "false" (or **contradiction**)
- "not ϕ"
- "ϕ and ψ"
- "ϕ or ψ" (inclusive or!)
- "if ϕ, then ψ" (and else?)
- "ϕ if and only if ψ"

Operator Precedence: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

$$\begin{aligned} & \mathit{Transactions} \rightarrow (\mathit{Put} \vee \mathit{Delete}) \\ \equiv\ & \mathit{Transactions} \rightarrow \mathit{Put} \vee \mathit{Delete} \\ \not\equiv\ & (\mathit{Transactions} \rightarrow \mathit{Put}) \vee \mathit{Delete} \end{aligned}$$

Propositional Formulas – Example

A Feature Model FM ...



... as a Propositional Formula $\Phi(FM)$

$$
\begin{aligned}
\Phi(FM) = \; & ConfigDB \\
& \wedge (API \leftrightarrow ConfigDB) \\
& \wedge (Transactions \rightarrow ConfigDB) \\
& \wedge (OS \leftrightarrow ConfigDB) \\
& \wedge (Get \vee Put \vee Delete \leftrightarrow API) \\
& \wedge (Windows \vee Linux \leftrightarrow OS) \\
& \wedge \neg(Windows \wedge Linux) \\
& \wedge (Transactions \rightarrow Put \vee Delete)
\end{aligned}
$$

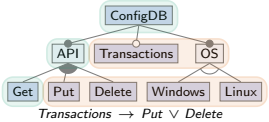
Is This a Valid Configuration?

$$
\begin{aligned}
& \Phi(FM)(\{C, A, G, O, W\}) \\
\equiv \; & \Phi(FM)((\{C, A, G, O, W\}, \{P, D, T, L\})) \\
\equiv \; & C \wedge (A \leftrightarrow C) \wedge (\top \rightarrow C) \wedge (O \leftrightarrow C) \\
& \wedge (G \vee P \vee D \leftrightarrow A) \wedge (W \vee L \leftrightarrow O) \\
& \wedge \neg(W \wedge L) \wedge (T \rightarrow P \vee D) \\
\equiv \; & \top \wedge (\top \leftrightarrow \top) \wedge (\bot \rightarrow \top) \wedge (\top \leftrightarrow \top) \\
& \wedge (\top \vee \bot \vee \bot \leftrightarrow \top) \wedge (\top \vee \bot \leftrightarrow \top) \\
& \wedge \neg(\top \wedge \bot) \wedge (\bot \rightarrow \bot \vee \bot) \\
\equiv \; & \top \wedge \top \wedge \top \wedge \top \wedge \top \wedge \top \wedge \top \wedge \top \\
\equiv \; & \top
\end{aligned}
$$

$\rightsquigarrow$ **configuration is valid**
(read-only database on Windows)

Propositional Formulas – Example

A Feature Model FM ...



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& \wedge (Windows \vee Linux \leftrightarrow OS) \\
& \wedge \neg(Windows \wedge Linux) \\
& \wedge (Transactions \rightarrow Put \vee Delete)
\end{aligned}
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& \wedge (G \vee P \vee D \leftrightarrow A) \wedge (W \vee L \leftrightarrow O) \\
& \wedge \neg(W \wedge L) \wedge (T \rightarrow P \vee D) \\
\equiv \; & \top \wedge (\top \leftrightarrow \top) \wedge (\bot \rightarrow \top) \wedge (\bot \leftrightarrow \top) \\
& \wedge (\top \vee \bot \vee \bot \leftrightarrow \top) \wedge (\bot \vee \bot \leftrightarrow \bot) \\
& \wedge \neg(\bot \wedge \bot) \wedge (\bot \rightarrow \bot \vee \bot) \\
\equiv \; & \top \wedge \top \wedge \top \wedge \bot \wedge \top \wedge \top \wedge \top \wedge \top \\
\equiv \; & \bot
\end{aligned}
$$

$\rightsquigarrow$ **configuration is invalid**
($\lightning$ no operating system)

Propositional Formulas – Algorithm

Algorithm: Translate *FM* Into $\Phi(FM)$

1. translate each tree constraint
   - **Root feature**: R is always required
   - **Optional feature**: C requires P
   - **Mandatory feature**:
     Optional + P requires C
   - **Or group**:
     Optional + P requires at least one C_i
   - **Alternative group**:
     Optional + P requires exactly one C_i
2. conjoin translated tree constraints
   $\Phi(TC) \leftarrow \bigwedge_{tc \in TC} \Phi(tc)$
3. conjoin **cross-tree constraints**
   $\Phi(CTC) \leftarrow \bigwedge_{ctc \in CTC} ctc$
4. $\Phi(FM) \leftarrow \Phi(TC) \wedge \Phi(CTC)$

$$\Phi(\boxed{\text{Root}}) = Root$$

$$\Phi\left(\begin{array}{c}\boxed{\text{P}}\\ \boxed{\text{C}}\end{array}\right) = C \rightarrow P$$

$$\Phi\left(\begin{array}{c}\boxed{\text{P}}\\ \boxed{\text{C}}\end{array}\right) = C \leftrightarrow P$$

$$\Phi\left(\begin{array}{c}\boxed{\text{P}}\\ \boxed{C_1}\ \boxed{\cdots}\ \boxed{C_n}\end{array}\right) = \bigvee_{1 \le i \le n} C_i \leftrightarrow P$$

$$\Phi\left(\begin{array}{c}\boxed{\text{P}}\\ \boxed{C_1}\ \boxed{\cdots}\ \boxed{C_n}\end{array}\right) = \bigvee_{1 \le i \le n} C_i \leftrightarrow P \wedge \bigwedge_{1 \le i < j \le n} \neg(C_i \wedge C_j)$$

CNF as a Universal Formula Language

Recap: Conjunctive Normal Form

- a **literal** L is a variable X or its negation $\neg X$
- a **clause** C is a disjunction of literals $\bigvee_j L_j$
- a **conjunctive normal form (CNF)** is a conjunction of clauses $\bigwedge_i C_i = \bigwedge_i \bigvee_j L_j$
- intuitively: a set of "rules" to be satisfied
- any formula ϕ can be transformed into a CNF ϕ' that is logically equivalent ($\phi \Leftrightarrow \phi'$)

Recap: Laws of Propositional Logic

- implication: $\phi \rightarrow \psi \Leftrightarrow \neg\phi \vee \psi$
- biimplication: $\phi \leftrightarrow \psi \Leftrightarrow (\neg\phi \vee \psi) \wedge (\neg\psi \vee \phi)$
- De Morgan's laws: $\neg(\phi \wedge \psi) \Leftrightarrow \neg\phi \vee \neg\psi$
- distributivity: $(\phi \wedge \psi) \vee \chi \Leftrightarrow (\phi \vee \chi) \wedge (\psi \vee \chi)$

Transforming Part of $\Phi(FM)$ into $CNF(\Phi(FM))$

$$
\begin{aligned}
&C \\
&\wedge\ (T \rightarrow C) \\
&\wedge\ (O \leftrightarrow C) \\
&\wedge\ (W \vee L \leftrightarrow O) \\
&\wedge\ \neg(W \wedge L)
\end{aligned}
\qquad
\begin{aligned}
&C \\
&\wedge\ (\neg T \vee C) \\
&\wedge\ (\neg O \vee C) \wedge (\neg C \vee O) \\
&\wedge\ (\neg(W \vee L) \vee O) \\
&\wedge\ (\neg O \vee W \vee L) \\
&\wedge\ \neg(W \wedge L)
\end{aligned}
$$

$$
\begin{aligned}
&C \\
&\wedge\ (\neg T \vee C) \\
&\wedge\ (\neg O \vee C) \wedge (\neg C \vee O) \\
&\wedge\ ((\neg W \wedge \neg L) \vee O) \\
&\wedge\ (\neg O \vee W \vee L) \\
&\wedge\ (\neg W \vee \neg L)
\end{aligned}
\qquad
\begin{aligned}
&C \\
&\wedge\ (\neg T \vee C) \\
&\wedge\ (\neg O \vee C) \wedge (\neg C \vee O) \\
&\wedge\ (\neg W \vee O) \wedge (\neg L \vee O) \\
&\wedge\ (\neg O \vee W \vee L) \\
&\wedge\ (\neg W \vee \neg L)
\end{aligned}
$$

CNF as a Universal Formula Language – DIMACS

$$
\begin{aligned}
& C \\
\wedge\ & (\neg T \vee C) \\
\wedge\ & (\neg O \vee C) \wedge (\neg C \vee O) \\
\wedge\ & (\neg W \vee O) \wedge (\neg L \vee O) \\
\wedge\ & (\neg O \vee W \vee L) \\
\wedge\ & (\neg W \vee \neg L)
\end{aligned}
$$

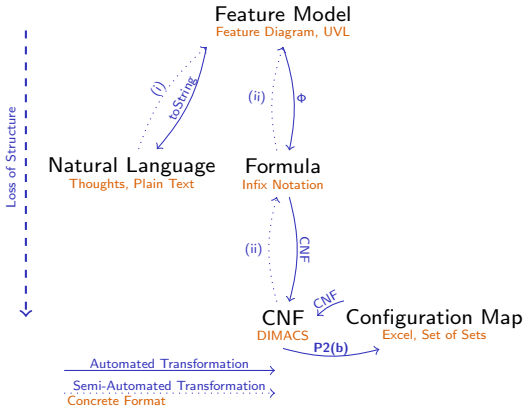
```
c 1 C
c 2 T
c 3 O
c 4 W
c 5 L
p cnf 5 8
1 0
-2 1 0
-3 1 0 -1 3 0
-4 3 0 -5 3 0
-3 4 5 0
-4 5 0
```

DIMACS Format [DIMACS 1993]

- de facto industry standard for storing CNF
- machine-readable, automated analyses, ...
- comments start with `c ...`
- problem line:
  `p cnf #variables #clauses`
- clause $\bigvee_i L_i$ translates to `L1 ... Ln 0`
- intuitively:

$$
\left.\begin{array}{l} 0 \\ \text{␣} \\ - \end{array}\right\} \text{ means } \left\{\begin{array}{l} \wedge \\ \vee \\ \neg \end{array}\right.
$$

Representations and Transformations



Problems

P1 How to express feature models **textually**?

P2 How to
- (a) validate configurations and
- (b) get all valid configurations

automatically?

P3 (How to reverse engineer feature models?)

Solutions

P1 Universal Variability Language ⇒ **Syntax**

P2 Propositional Formulas ⇒ **Semantics**
- (a) evaluate feature-model formula
- (b) Lecture 4c

P3 (i) e.g., Bakar et al. 2015
(ii) e.g., Czarnecki and Wasowski 2007

Transforming Feature Models – Summary

Lessons Learned

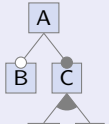
- to understand large configuration spaces, we need formal semantics and machine-readable representations
- propositional formulas satisfy many (though not all) needs for such a representation

Further Reading

- Don Batory (2005): Feature Models, Grammars, and Propositional Formulas
- UVL — official website for the Universal Variability Language with examples, grammar, literature pointers
- Alexander Knüppel et al. (2017): Is There a Mismatch Between Real-World Feature Models and Product-Line Research?

Practice

1. translate the following feature diagram into a propositional formula:



$D \rightarrow B$

2. check formulas of your colleagues

4. Feature Modeling

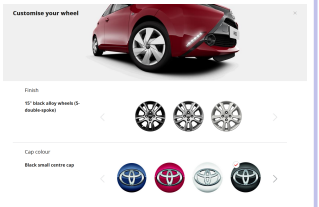
4a. Feature Models and Configurations

4b. Transforming Feature Models

4c. Analyzing Feature Models

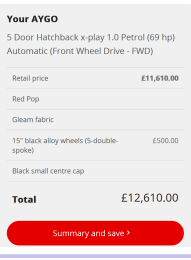
Configurators in the Wild – Cars

Configuring a Car . . .

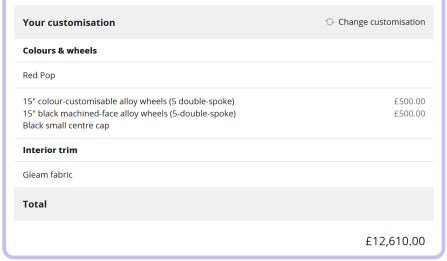


Demo Video

with a Weird Price



with 8 Wheels!



- canceling the dialog was not considered and lead to an invalid state (i.e., configuration)
- humans check these configurations, but some errors are only found during production
- many constraints: appear arbitrary, not explained

Configurators in the Wild – Cars

Configuring a German Car [example from Lecture 1]

Configuration Assistant.

Show instructions

Your most recent action requires your configuration to be adjusted.

Your choice	Price
+ Enhanced Bluetooth telephone with USB & Voice Control	+ £ 350.00
Adding	
+ BMW Navigation	£ 0.00
Removing	
- Enhanced Bluetooth with wireless charging	- £ 395.00
- Navigation system Professional	£ 0.00
- WiFi hotspot preparation	£ 0.00
- Media package - Professional	- £ 900.00
- Online Entertainment	£ 0.00
- Microsoft Office 365	- £ 150.00

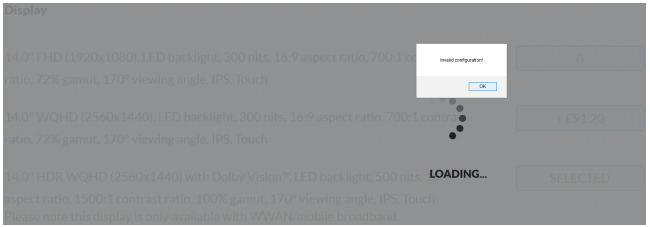
Why does the telephone conflict with Microsoft Office?

Configurators in the Wild – Notebooks

Configuring a Notebook

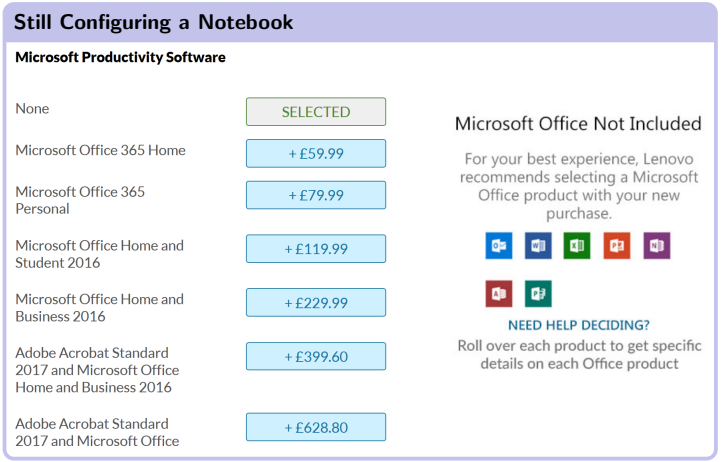
Display

14.0" FHD (1920x1080), LED backlight, 300 nits, 16:9 aspect ratio, 700:1 contrast ratio, 72% gamut, 170° viewing angle, IPS, Touch	SELECTED
14.0" WQHD (2560x1440), LED backlight, 300 nits, 16:9 aspect ratio, 700:1 contrast ratio, 72% gamut, 170° viewing angle, IPS, Touch	+ £91.20
14.0" HDR WQHD (2560x1440) with Dolby Vision™, LED backlight, 500 nits, 16:9 aspect ratio, 1500:1 contrast ratio, 100% gamut, 170° viewing angle, IPS, Touch Please note this display is only available with WWAN/mobile broadband.	+ £159.60



can detect mistakes, but provides no explanations or fixes

Configurators in the Wild – Notebooks



allows some feature combinations and not others, prices are opaque

Automated Analysis of Feature Models

Open Questions

- How do such configurators work?
- How to avoid inconsistencies?
- How to provide explanations and fixes?
- How to get all valid configurations automatically? (**P2(b)**)

Automated Analysis of Feature Models

- up until now: **creation** and **transformation** of feature models
- now: **analysis** of feature models to improve our understanding of a configuration space
- for brevity: product = valid configuration

Asking Questions About Feature Models

- Is a given configuration valid?
- Is there any product at all?
  How many/which products are there?
- Is a given feature (de-)selectable at all?
  How many/which products include it?
- Is a given partial configuration consistent?
  How many/which products include it?
- (Which features always occur together?)
- (Is a given constraint redundant?)
- (How do two feature model versions differ?)
- (Why is . . . ? How to fix . . . ?)

SAT, #SAT, and AllSAT

Recap: Boolean Satisfiability Problem (SAT)

- **decision problem**: is there any assignment A that satisfies a given formula?
- formally: $SAT(\phi) \Leftrightarrow \exists A \colon \phi(A) = \top$
- known to be **NP-complete**:
  in theory, difficult to solve if $P \neq NP$;
  in practice, solvability depends on domain
- answered by **SAT solvers**:
  highly-optimized, off-the-shelf tools;
  competitively developed over several decades;
  takes a CNF in DIMACS format as input

- $X \rightarrow Y$ is satisfiable
- $X \vee \neg X$ is satisfiable (even a tautology)
- $X \wedge \neg X$ is not satisfiable (why?)

Sharp Satisfiability Problem (#SAT)

- **counting problem**: how many assignments satisfy a given formula?
- $\#SAT(\phi) = |\{A \mid \phi(A) = \top\}|$
- known to be **#P-complete**:
  at least as hard as SAT (probably harder)
- answered by **#SAT solvers**

Solution Enumeration Problem (AllSAT)

- **enumeration problem**: which assignments satisfy a given formula?
- $AllSAT(\phi) = \{A \mid \phi(A) = \top\}$
- at least as hard as #SAT (probably harder)
- answered by **AllSAT solvers**

Automated Analysis of Feature Models

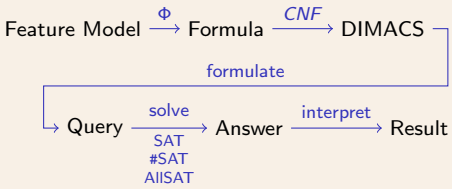
Asking Questions About Feature Models

- Is a given configuration valid? ⇒ **evaluate**
- **Is** there any valid configuration at all?
  How many/**which** valid configurations are there?
- **Is** a given feature (de-)selectable at all?
  How many/**which** valid configurations include it?
- **Is** a given partial configuration consistent?
  How many/**which** valid configurations include it?

Choosing the Right Solver

- "is?" ≈ SAT solver query
- "how many?" ≈ #SAT solver query
- "which?" ≈ AllSAT solver query

Typical Feature-Model Analysis Process



for brevity, we assume that $\phi = CNF(\Phi(FM))$ for a given feature model FM

Consistency, Cardinality, and Enumeration – Feature Model

Consistency of Feature Models (SAT)

Void/Consistent Feature Model

- are there grave modeling errors?
- is it possible to configure any product at all?

$$\phi \xrightarrow{\text{SAT}} \bot/\top \begin{array}{l} \xrightarrow{\bot} \textit{FM} \text{ is } \textbf{void} \\ \xrightarrow{\top} \textit{FM} \text{ is } \textbf{consistent} \end{array}$$

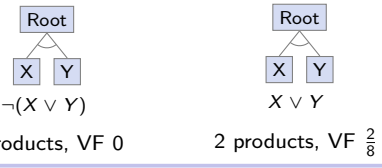
Cardinality of Feature Models (#SAT)

How Many Products Are There?

$$\phi \xrightarrow{\#\text{SAT}} |C|$$

Variability Factor: Share of Products?

$$\phi \xrightarrow{\#\text{SAT}} |C| \longrightarrow \frac{|C|}{2^{|F|}}$$



Consistency, Cardinality, and Enumeration – Feature Model

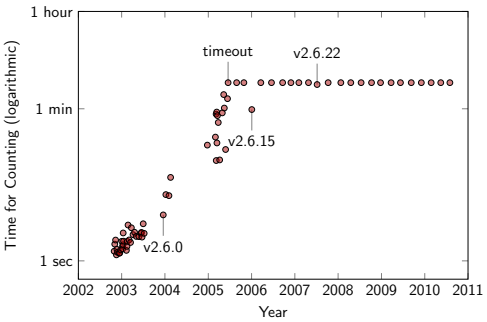
Feasibility of SAT-Based Analyses

Is SAT-Based Analysis "Easy"?

- provocative claim: "SAT-based analysis of feature models is easy" [Mendonca et al. 2009]
- easy = performs much better than expected (although NP-complete)
- easy = fast?
  - what about formulating the query? (e.g., CNF transformation)
  - what about many queries? (e.g., what we discuss next)

Feasibility of #SAT-Based Analyses

Time to Count Products of Linux



- the solver from 2023 can solve models from 2003
- can we analyze the models from 2023 in 2043?

Consistency, Cardinality, and Enumeration – Feature Model

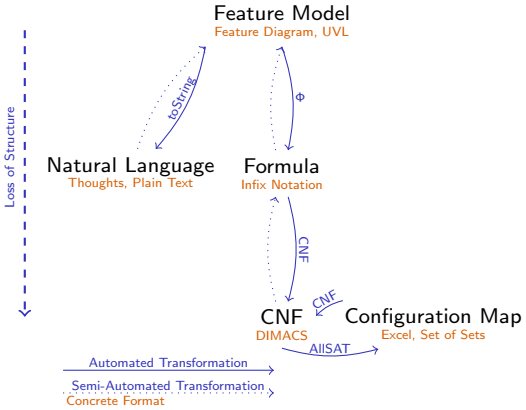
Enumeration of Feature Models (AllSAT)

Which Products Are There?

- **P2(b)**: How to get all products?

$$\phi \xrightarrow{\text{AllSAT}} C$$

AllSAT does not scale to realistic feature models!
50 features, configurations à 1 Byte ≈ 1 Petabyte



Consistency, Cardinality, and Enumeration – Features

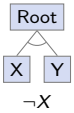
Consistency of Features (SAT)

Core/Dead Feature

- can a feature F be (de-)selected at all?

$$\phi \wedge F \xrightarrow{\text{SAT}} \bot/\top \begin{cases} \xrightarrow{\bot} F \text{ is } \textbf{dead} \\ \xrightarrow{\top} F \text{ is not dead} \end{cases}$$

$$\phi \wedge \neg F \xrightarrow{\text{SAT}} \bot/\top \begin{cases} \xrightarrow{\bot} F \text{ is } \textbf{core} \\ \xrightarrow{\top} F \text{ is not core} \end{cases}$$



X is dead — *Root* and Y are core

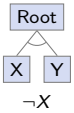
Cardinality of Features (#SAT)

How Many Products Include Feature F?

$$\phi \wedge F \xrightarrow{\#\text{SAT}} |\{S \in C \mid F \in S\}|$$

Commonality: How Constrained is This Feature?

$$\phi \wedge F \xrightarrow{\#\text{SAT}} |\{S \in C \mid F \in S\}| \rightarrow \frac{|\{S \in C | F \in S\}|}{|C|}$$



X: 0 products — *Root* and Y: 1 products

Consistency, Cardinality, and Enumeration – Partial Configurations

Consistency of Partial Configurations (SAT)

Valid Partial Configuration

Is a partial configuration $C = (S, D)$ consistent with the feature model?

$$\phi \wedge \bigwedge_{s \in S} s \wedge \bigwedge_{d \in D} \neg d \xrightarrow{\text{SAT}} \bot/\top \begin{array}{l} \xrightarrow{\bot} C \times \\ \xrightarrow{\top} C \checkmark \end{array}$$

$X \rightarrow Y$

$(\{Root\}, \{X\})$ ✓

Cardinality of Partial Configurations (#SAT)

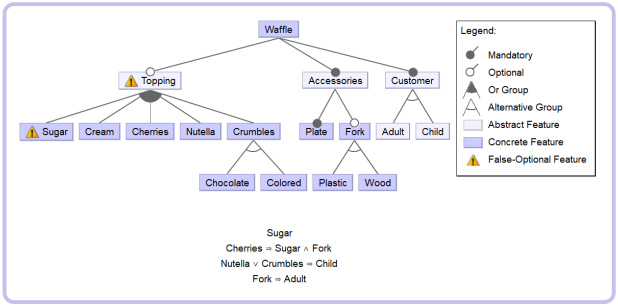
How Many Products Are Still Configurable for C?

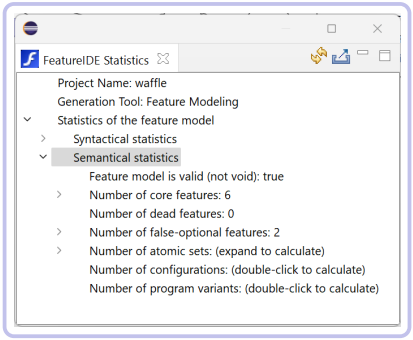
$$\phi \wedge \ldots \xrightarrow{\#\text{SAT}} |\{(S', D') \in C \mid S \subseteq S', D \in D'\}|$$

$X \rightarrow Y$

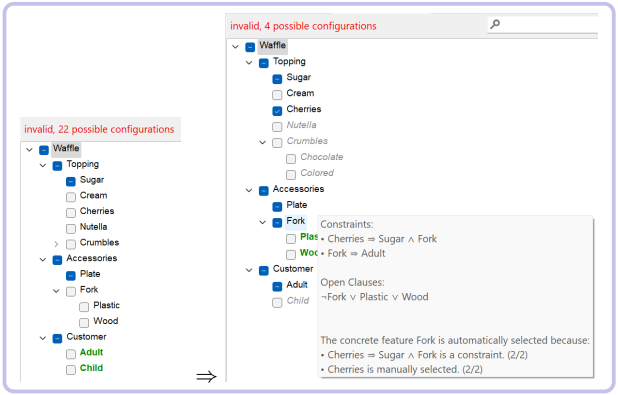
$(\{Root\}, \{X\})$: 2 products

Automated Analyses in FeatureIDE – Feature-Model Editor





Automated Analyses in FeatureIDE – Configuration Editor

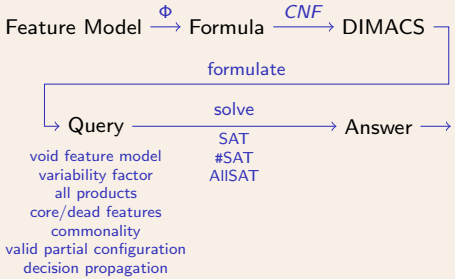


Decision Propagation

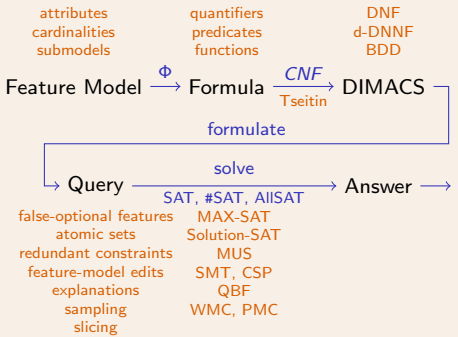
- after each decision (i.e., click) . . .
  - . . . select features that are now **conditionally core**
  - . . . deselect features that are now **conditionally dead**
- this way it is impossible to configure an invalid configuration
- explanations for all propagated decisions

Automated Analysis of Feature Models

The Road So Far ...



... and Beyond



- develop new analyses
- improve efficiency of existing analyses
- investigate correctness and compositionality

Analyzing Feature Models – Summary

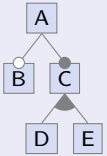
Lessons Learned

- with solvers, we can build reliable configurators for product lines
- SAT-based analyses: void feature model, core/dead features, decision propagation
- #SAT-based analyses: variability factor, feature commonality

Further Reading

- Apel et al. 2013, Section 10.1, pp. 244–254
  — introduction to feature-model analysis
- David Benavides et al. (2010): Automated Analysis of Feature Models 20 Years Later: A Literature Review
  — old but extensive literature survey
- Chico Sundermann et al. (2021): Applications of #SAT Solvers on Feature Models
  — experiments on the scalability of #SAT solvers

Practice



think of a constraint that would make exactly one feature dead

FAQ – 4. Feature Modeling

Lecture 4a

- What is feature modeling? When is it needed?
- How can we specify valid combinations of features?
- What is a complete, partial, valid, invalid configuration?
- What are (dis-)advantages of natural language, configuration map, and feature models?
- What is the graphical syntax and semantics of feature models?
- Give an example feature model!

Lecture 4b

- What representations of feature models are available? Are they equivalent?
- How to represent feature models textually?
- What is UVL (used for)?
- How to identify whether a configuration is valid?
- How to translate feature model into a propositional formula?
- What are DIMACS and KConfig (used for)?
- Would you recommend Excel for feature model? Why (not)?

Lecture 4c

- Why can configuration become challenging?
- How can we identify problems with feature models and configurations?
- How can feature models by analyzed? What analyses are available?
- What solvers can be used to analyze feature models?
- What is the difference between SAT, #SAT, and ALLSAT?
- Why are solvers useful when creating configurations?