CS 228: Introduction to Data Structures Lecture 29

From Postfix to Expression Trees

We can build the expression tree for a postfix expression using a stack (the code is in the `ExpressionTree` class posted on Blackboard).

Algorithm

Scan the expression character by character from left to right. For each character `c`:

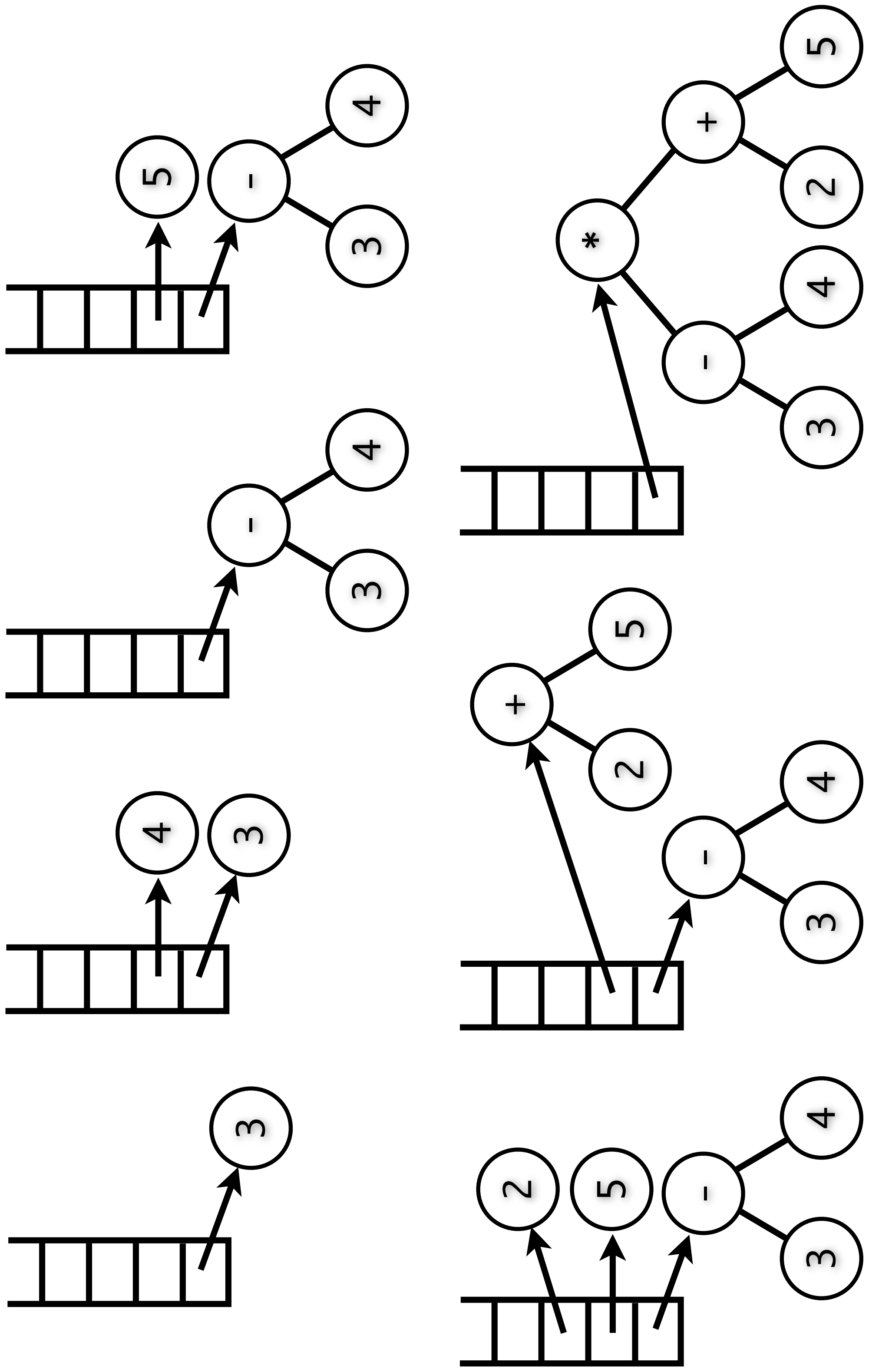
```
1. If c is a blank, skip it.

2. If c is a digit, create a node for it and push it on the
   stack.

3. If c is an operator,

   3.1.create a node n for c,

   3.2.pop the top of stack and make it n's right subtree,

   3.3.pop again, and make it n's left subtree.
```

At the end, the stack contains a reference to the root of the expression tree.

The `makeFromPostfix()` method in `ExpressionTree` implements this algorithm.

Example. The next page shows how the algorithm handles the expression

```
3 4 - 5 2 + *
```



Sets

In mathematics, a **set** is a collection of elements with no duplicates. The Java `Set` interface, which extends `Collection`, models this mathematical abstraction. It defines methods for the standard set operations, such as containment, union, and set difference. Our implementations will build on Java's `AbstractSet`, an abstract class that provides a partial implementation of the `Set` class. `AbstractSet` extends `AbstractCollection`, except that all of the methods and constructors in subclasses of this class must obey the additional constraints imposed by the `Set` interface; for instance, the `add()` method must not permit addition of multiple instances of an object to a set.

Binary Search Trees

We first consider sets where the element type is `Comparable`. Thus, the elements can be ordered so that there is a minimum element, a maximum element, and any element — other than the min and max — has a successor (next) and a predecessor (previous). In sets like this, the elements are often called ***keys***.

A **binary search tree** (BST) is one way to implement a set of comparable elements. A BST is a binary tree whose nodes hold the keys; the keys must satisfy the following.

Binary Search Tree Property: For any node X, every key in the left subtree of X is less than X's key, and every key in the right subtree of X is greater than X's key.

Example. Consider the BST below.

```
      18
     /  \
   12    25
  / \    / \
 4  15  24  32
/  /  \    /
1  13  17  28
\  \       \
 3  14      30
```

For instance, the root is 18, its left subtree (rooted at 12) contains numbers from 1 to 17, and its right subtree (rooted at 25) contains numbers from 24 to 30.

When a node has only one child, that child is either a left child or a right child, depending on whether its key is smaller or larger than its parent's key.

Note. The BST representation of a set is not unique. Here's another way to represent the set of keys from the previous example:

```
        24
       / \
      12  25
     / \   \
    4   17  28
   /   / \   \
  3   15  18  30
 /   /         \
1   13         32
     \
      14
```

You should be able to verify the following:

Fact. An inorder traversal of a binary search tree visits the nodes in sorted order.

In this sense, a search tree maintains a sorted list of entries. However, operations on a binary search tree are usually much faster than the same operations on a sorted linked list because you typically only traverse a small fraction of the nodes of a tree. Before explaining why, we need to provide a few implementation details.

Representing a Set as a BST

Our implementation of sets via BSTs builds on Java's `AbstractSet`, an abstract class that extends `AbstractCollection`, providing a partial implementation of the `Set` class.   The details of the tree representation are hidden within the Node inner class. A Node is like `TreeNode`, except that, in addition to references to the left and right children, it has a `parent` reference, which is used to implement removal and iteration efficiently.

```
public class
BSTSet<E extends Comparable<? super E>>
extends AbstractSet<E>
{
  protected Node root;
  protected int size;

  protected class Node
  {
    public Node left;
    public Node right;
    public Node parent;
    public E data;
    public Node(E key, Node parent)
    {
      this.data = key;
      this.parent = parent;
    }
  }
```