1) <u>Bonus</u> :

Let $A = \begin{pmatrix} 1 & x_1 & x_1^2 & \ln x_1 \\ 1 & x_2 & x_2^2 & \ln x_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{20} & x_{20}^2 & \ln x_{20} \end{pmatrix}$ where $x_1, \dots, x_{20}$ are const

Let $\vec{Y} = \begin{pmatrix} Y_1 \\ \vdots \\ Y_{20} \end{pmatrix}$ where $Y_i = \alpha_1 + \alpha_2 x_i + \alpha_3 x_i^2 + \alpha_4 \ln x_i + \varepsilon_i \quad \forall i \in \{1, \dots, 20\}$

$\left(\alpha_1, \dots, \alpha_4 \text{ are const , and } \varepsilon_i\text{'s} \overset{iid}{\sim} \mathcal{N}(0, \delta_\varepsilon^2) \right)$

We can see that $\vec{Y} = \underbrace{I_{20}}_{\text{const}} \cdot \underbrace{\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{20} \end{pmatrix}}_{\vec{\varepsilon}} + \underbrace{\begin{pmatrix} \alpha_1 + \alpha_2 x_1 + \alpha_3 x_1^2 + \alpha_4 \ln x_1 \\ \vdots \\ \alpha_1 + \alpha_2 x_{20} + \alpha_3 x_{20}^2 + \alpha_4 \ln x_{20} \end{pmatrix}}_{\vec{b}_Y \text{ (const)}}$

$\Rightarrow \vec{Y} \sim \mathcal{N}(\vec{\mu}_Y, \Sigma_Y)$

where $\vec{\mu}_Y = I_{20} \cdot \mathbb{E}(\vec{\varepsilon}) + \vec{b_y} = I_{20} \cdot \vec{0} + \vec{b_y} = \vec{b}_y$

$\Sigma_Y = I_{20} \cdot Var(\vec{\varepsilon})\, I_{20}^T = \begin{pmatrix} 4 & & & 0 \\ & 4 & & \\ & & \ddots & \\ 0 & & & 4 \end{pmatrix}_{20}$

(because ε_i's are iid)

We want to find $\hat{\alpha} = \begin{pmatrix} \hat{\alpha}_1 \\ \vdots \\ \hat{\alpha}_4 \end{pmatrix} = (A^T A)^{-1} A^T . \vec{Y}$

let $B = (A^T A)^{-1} A^T = \text{const}$, we know $\hat{\alpha} \sim \mathcal{N}(\vec{\mu}_{\hat{\alpha}}, \Sigma_{\hat{\alpha}})$

where $\vec{\mu}_\alpha = B \vec{\mu}_Y$ and $\Sigma_{\hat{\alpha}} = B \Sigma_Y B^T$

•) we have $\sum_{i=1}^{4} \hat{\alpha_i} = (1 \quad 1 \quad 1 \quad 1)\ \hat{\alpha} = C \hat{\alpha}$

$\Rightarrow \sum_{i=1}^{4} \hat{\alpha_i} \sim \mathcal{N}(C \hat{\mu}_\alpha, C \Sigma_{\hat{\alpha}} C^T)$

With $x_i = i \quad \forall i \in \{1, \ldots, 20\}$, by using R, we have

- $\mathbb{E}\left(\sum_{i=1}^{4} \hat{\alpha_i}\right) = C \hat{\mu_\alpha} = 6$

- $Var(C \Sigma_{\hat{\alpha}} C^T) = C \Sigma_{\hat{\alpha}} C^T \approx 5{,}326$