4.2) From ① we have $d = 4$

·) let $Y = \begin{pmatrix} \alpha_0 + \alpha_1 x_1 + \dots + \alpha_4 x_1^4 + \varepsilon_1 \\ \vdots \\ \alpha_0 + \alpha_1 x_{100} + \dots \alpha_4 x_{100}^4 + \varepsilon_{100} \end{pmatrix}$

$\Rightarrow Y \sim \mathcal{N}(\vec{\mu}_Y, \Sigma_Y)$

where $\vec{\mu}_Y = \begin{pmatrix} \alpha_0 + \alpha_1 x_1 + \dots + \alpha_4 x_1^4 \\ \vdots \\ \alpha_0 + \alpha_1 x_{100} + \dots + \alpha_4 x_{100}^4 \end{pmatrix}$

$\Sigma_Y = \begin{pmatrix} 100 & & 0 \\ & \ddots & \\ 0 & & 100 \end{pmatrix}$

·) When all the $\varepsilon_i = 0$, then $(A^T A)^{-1} A^T \vec{Y} =$

$(A^T A)^{-1} A^T \vec{\mu}_Y = \vec{\alpha} = \begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_4 \end{pmatrix}$

Therefore, $\hat{\alpha} = (A^T A)^{-1} A^T \vec{Y} = C\vec{Y} \sim \mathcal{N}(\vec{\alpha}, \underbrace{C \Sigma_Y C^T}_{\Sigma_{\hat{\alpha}}})$

Then $\hat{\alpha_0} = (1\ 0\ 0\ 0\ 0)\,\hat{\alpha} = D\hat{\alpha} \sim \mathcal{N}(\alpha_0, \underbrace{D \Sigma_{\hat{\alpha}} D^T}_{\sigma^2_{\alpha_0}})$

$\Rightarrow \dfrac{\hat{\alpha_0} - \alpha_0}{\sqrt{\sigma^2_{\alpha_0}}} \sim \mathcal{N}(0, 1)$

·) $H_0 : \alpha_0 = 0$

$TN = \dfrac{\hat{\alpha_0}}{\sqrt{\sigma^2_{\alpha_0}}} = \dfrac{3,73}{\sqrt{2,34}} \approx 2,44$

$\in$ Rejection Region $= (-\infty, -1{,}96) \cup (1{,}96, \infty)$

$\Rightarrow$ Reject H_0

• p-value $\approx 0{,}015$

•) 95% CI : $P\left(-1{,}96 < \dfrac{\hat{\alpha}_0 - \alpha_0}{\sqrt{s^2_{\alpha_0}}} < 1{,}96\right) = 95\%$

$\Rightarrow \quad P\left(\hat{\alpha}_0 - 1{,}96\, s_{\alpha_0} < \alpha_0 < \hat{\alpha}_0 + 1{,}96 \cdot s_{\alpha_0}\right) = 95\%$

With $\hat{\alpha}_0 = 3{,}73$, $s^2_{\alpha_0} = 2{,}34$

$\Rightarrow$ 95% CI for α_0 : $(0{,}73 ; 6{,}73)$