

Kyffin Williams: Digital Analysis of Paintings

Final Report for CS39440 Major Project

Author: Alexander David Brown (adb9@aber.ac.uk)

Supervisor: Hannah Dee (hmd1@aber.ac.uk)

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Department of Computer Science
Aberystwyth University
Aberystwyth
Ceredigion
SY23 3DB
Wales, UK

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Acknowledgements

To Llyod, Hannah and the Cooper's Arms for setting the scene for this crazy project.

To Jade, Kate, Yarrow and Sam for being great friends in times of need.

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Abstract

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Chapter 1

Background & Objectives

1.1 Sir John “Kyffin” Williams

Sir John “Kyffin” Williams (1918-2006) was a Welsh painter and printmaker, widely regarded as the defining artist of Wales during the 20th century [5]. He was advised to take up art by a doctor after failing a British Army medical examination because of an ‘abnormality’ (epilepsy) as something which would not tax his brain.

He studied at the Slade School of Fine Art and taught art in Highgate School, after which he retired to Anglesey until he died in 2006 after a long battle with cancer.

His most characteristic pictures are of Welsh landscapes, painted with thick layers of oil paint applied with a palette knife [2]. Most of his paintings are highly textural; to the point of being 3-dimensional.

As his life progressed Kyffin’s ‘abnormality’ grew steadily worse, especially when exposed to bright light. As a result most of his paintings are of overcast Welsh landscapes and tend to become visibly darker over time [8]. By eye it is generally quite easy to approximate the time period in which a painting was created.

In 1969 he won a scholarship to study and paint in Y Wladfa; the Welsh settlement in Patagonia. This period of his life is very obvious from his paintings as there is a complete contrast in colour between Patagonian and Welsh landscapes.

1.2 Interdisciplinary work with the National Library of Wales

This project was initially suggested through a conversation between Hannah Dee and Gareth “Llyod” Roderick about image processing and art. Llyod is a PhD student at the National Library of Wales (NLW) researching (TODO: Find out what Llyod’s thesis title is). Their initial idea was to be able to geolocate a Kyffin painting on a map to build up a geographical representation of Kyffin’s work.

Hannah started to create a prototype for performing geographical analysis, this proved to be a difficult task and one which is still being researched.

However, the nature of Kyffin’s illness and painting style allows for a second form of analysis: temporal. As previously stated it is fairly easy to judge by eye a good approximation of the period in which a Kyffin painting was created. It should, therefore, follow that this process can be performed digitally.

When I started this project I was given a “database” (in reality this was just a spreadsheet) Llyod had produced, containing information of Kyffin Williams’ paintings, including: title, year,

category (landscape, portrait, etc.), canvas size and a few additional details which aren't so relevant to the project.

The first meeting held was between Llyod, Hannah and I, in which we discussed the current state of the project, what our aims for the project were and what form of help Llyod could provide to us. As one of the objectives of this project is to, eventually, get a paper published, the relevant details of the process we would need to go through if we wanted to do so.

The second meeting was between Hannah, Llyod, Lorna M. Hughes (Llyod's supervisor) and I. Again we discussed the state of the project. Llyod had also produced a better version of his "database" to be more machine readable and succinct. A lot of information came from this meeting;

- The "cut-off" point between early and late is around 1973.
- The size of the canvas might be a useful data point to use in classification, as Kyffin sold more paintings he would have had the money available for larger canvases and the paint for said canvas.
- It is a little dubious as to whether some dates can be trusted. One painting owned by the NLW was stated to be his last painting, but Lorna believes it was painted much earlier and claimed to be his last to improve the sale price.
- Llyod may have found date markings on some paintings. These again may not be accurate, but may prove to increase the sample size.
- It should be easy to provide a "no later than" estimate for each painting from the art historians.
- Paul (?) should be able to produce some exemplars for us as a ground truth.
- Llyod may be able to find more paintings in the hands of private collectors to increase the sample size.
- Llyod had been playing around with ImageJ to do some basic graph plotting. This might be useful to look at further to expand my own work.

There were also more detailed discussions about publications, particularly in a digital humanities journal.

1.2.1 Continuation of the Kyffin Project

There are several projects that could continue on from the Kyffin Project.

One was to use the Learning/Teaching development fund to produce a web-based front-end for some of my analysis.

Another venture was to look into PhD funding to build up a 3D map of some of Kyffin's paintings and being able to display it (perhaps via HTML5 and WebGL) so they can explore the painting digitally how it is meant to be in real life.

1.3 Existing Work

1.3.1 Edge-Orientated Gradients

1.3.2 Brush-stroke Analysis

1.4 Analysis Objectives

Analysis is one of the biggest sections of this project and involves creating techniques which will allow comparison of paintings in a way which will allow some form of classification to be performed on them.

Typically I would expect this to produce some form of high-dimension state space in which each painting is a point in the state space. From this state space the distance between one painting and another can be easily resolved using a distance measure like Manhattan distance (1), euclidean distance (2) or a distance measure more specific to the state space should it be needed (e.g.: chi-squared for histograms).

$$d_1(\mathbf{p}, \mathbf{q}) = \|\mathbf{p} - \mathbf{q}\|_1 = \sum_{i=0}^n |p_i - q_i| \quad (1)$$

$$d_1(\mathbf{p}, \mathbf{q}) = \sqrt{\sum_{i=0}^n (q_i - p_i)^2} \quad (2)$$

1.4.1 Colour-space Analysis

The simplest way of analysing a digital image is to look at the colours which it consists of. Doing this is relatively simple; each pixel has a set of values defining the colour of that point, getting something meaningful from this is less simple.

The simplest strategy is to perform some form of statistical analysis on each painting then use this for classification. Several good and computationally cheap options exist for this; mean (3) and standard deviation (4), are some good examples which often come predefined in image processing and computer vision libraries.

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i \quad (3)$$

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2} \quad (4)$$

The representation of colour is another important factor, an Red, Green, Blue (RGB) representation will have all three values change if there are many changes in brightness of the colours whilst a Hue, Saturation, Value (HSV) representation will only have a single value change.

Therefore, an object of this section should be to explore different colour models and statistical methods which can be applied to them.

Another useful technique which should be investigated early into the project are image histograms. These histograms plot the distribution of colour across an image and are therefore a very powerful method of analysing an image, especially for comparison. As with statistical analysis, histograms will be largely effective by colour model.

1.4.2 Texture Analysis

As Kyffin Williams' work is very textural, it follows that a main part of the analysis should focus around the texture of his paintings. Unfortunately for this section, it seems unlikely that I will be able to get any 3-dimensional models of Kyffin's paintings. This would have been a nice, if rather large, section of the project.

Instead it is more sensible to look at the orientation of edges in Kyffin's work. Some useful pre-existing techniques have already been discussed in section 1.3.1. Histograms of edge orientation [4] seem like a promising concept which may prove relatively simple to implement.

This section may also help with any work into brush-stroke analysis (see section 1.4.3).

1.4.3 Brush-stroke Analysis

With Kyffin's distinctive style and how obviously this style changes over time, the ultimate aim of this project is to be able to analyse the brush-strokes¹ in a painting.

From looking at the paintings it is very apparent that in his earlier work he made a lot more strokes than in his later works². The strokes in his later work tend to have larger areas and span more of the canvas.

If it is possible to calculate a rough amount and size of strokes made in a given painting it should be a reasonable piece of data to classify on. As previously discussed in section 1.3.2 there has already been a decent amount of research into determining brush-strokes in a painting.

It would be preferable to try and take one of the techniques discussed in that research and change it to suit the needs of the project rather than attempting to create a whole new method of brush-stroke recognition.

1.4.4 Ensemble Techniques

With some of the aforementioned analysis techniques it makes sense to combine two or more techniques together; a good example would be colour histograms and histograms of edge orientation.

This form of analysis is inspired by the concept of the same name in statistics and machine learning which tend to obtain better predictive performance. It may also be worth while trying to weight different techniques so that the techniques which give the best performance affect the result of the ensemble technique more.

1.5 Classification Objectives

The overall objective of classification is to be able to label a painting by Kyffin Williams as being painted in a given year based on analysis performed on all other paintings with known years.

This ties in with the main aim of this project of being able to classify any Kyffin Williams painting, whether it has a known or unknown year, as being from a given year. Evidently for paintings with an unknown year it is difficult to know how accurately the system has been, so, for the most part, these paintings have been ignored and those paintings with a known year have made up the training and validation set.

Because of the small size of paintings with known years it should be computationally viable to perform leave-one-out cross validation (figure 1.1).

¹ A slight misnomer as Kyffin used a palette knife to paint with rather than a traditional brush

² Although this isn't quite true as the canvases he worked on in his later life tended to be larger

```

function LOOCV(data)                                     ▷ data is a set of all data points
  for all item ∈ data do
    classifieditem ← CLASSIFY(item, data \ {item})
  end for
return classified
end function

```

Figure 1.1: Pseudocode for Leave-One-Out Cross Validation

This can be used to evaluate the performance of the analysis technique and classification algorithm. Pearson's product-moment correlation coefficient (5) between actual year and classified year has been suggested to be a good performance measure for this project.

$$\rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y} \quad (5)$$

1.5.1 Classification

One of the simplest methods of classification is k -Nearest Neighbour (figure 1.2) from this one can take a poll of the years for each neighbour and assign the year of the painting to classify to be the average of these years.

Depending which form of average you take (mathematical mean (3), median or mode) will alter the result; although it should be noted that median is very unlikely to give a result on its own due to the sparseness of the data.

```

Require:  $0 < k \leq |data|$                                      ▷ data is a set of all data points
function KNEARESTNEIGHBOUR(k, data)
  for  $i = 1 \rightarrow k$  do
    nni ← NEAREST(data)
    data ← data \ {nni}
     $i = i + 1$ 
  end for
return nn
end function

```

Figure 1.2: Pseudocode for k -Nearest Neighbour

1.5.1.1 Use of Weka

1.5.1.2 Learning Classifier Systems (LCS)

1.5.2 Exemplars

Chapter 2

Development Process

2.1 Introduction

2.2 Modifications

Chapter 3

Design

3.1 Overall Architecture

The basic architecture for any system like this is to load the data in from a source of some form, apply an analysis technique to each data point then pass this data into the classification system.

From the classification system you should then be able to get the classified and actual year for each data point which can then have validation performed on it. This architecture is summed up in figure 3.1.

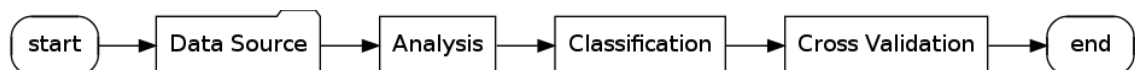


Figure 3.1: Basic Overall Architecture

Building up from this it is apparent that to implement the analysis and classification steps that there is a need to implement the factory method design pattern [6, p. 93-100]. Reading from a data source should be a simple matter of reading from a file, and cross validation has already been decided to use leave-one-out cross validation.

Figure 3.2 shows the design after adding in the factory methods.

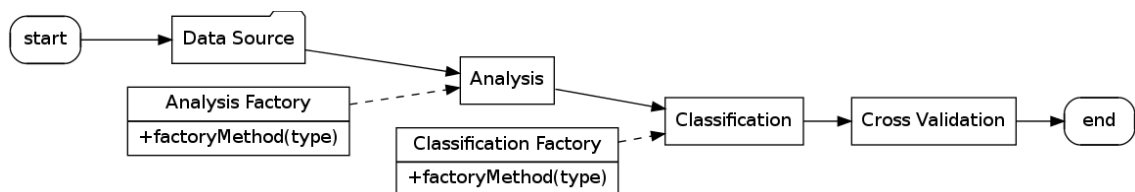


Figure 3.2: Overall Architecture with Factory Methods

From this we then need the two top-level interfaces `Analyser` and `Classifier`. The `Analyser` interface should have a single method which runs analysis on a painting and return some form of object which represents the analysed data.

The `Classifier` class should have a method which takes a single painting and a set of paintings, returning a year which is the classified year of the single painting based on the set of paintings.

At this point it is also required that there is a class to store meta-data of a painting. Figure 3.3 depicts the design after adding in these parts.

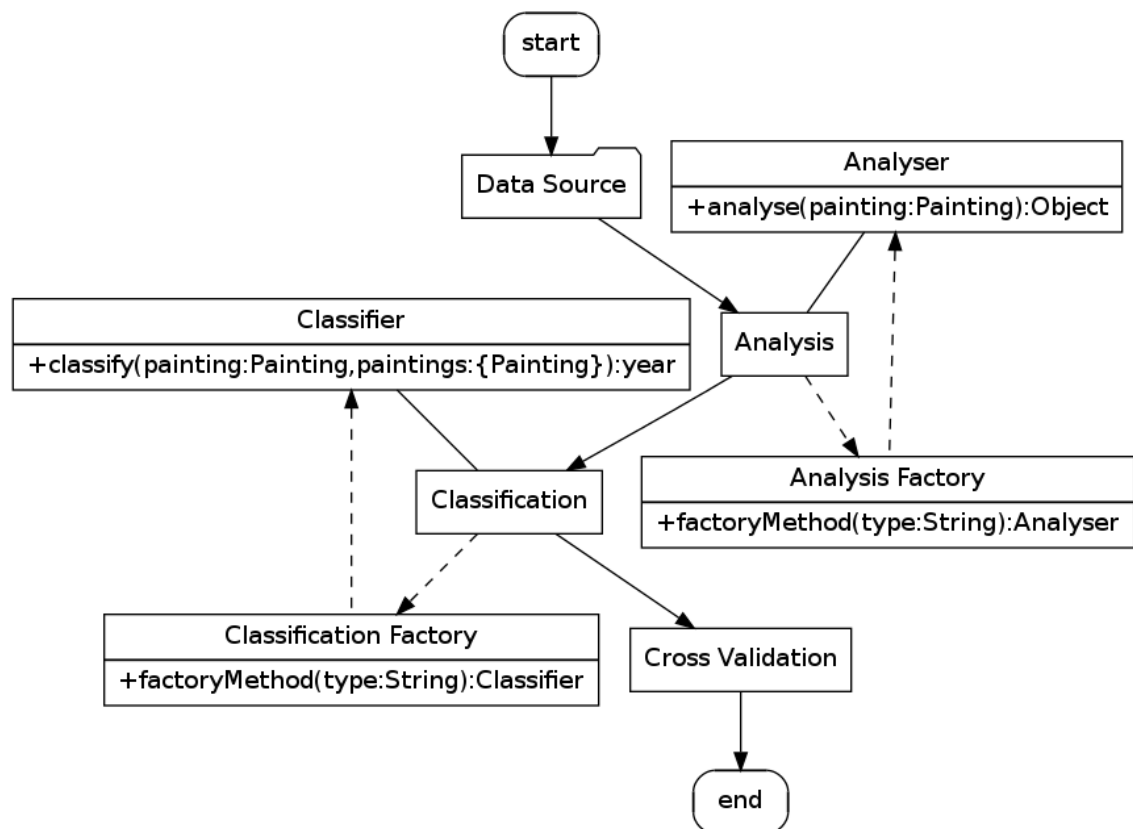


Figure 3.3: Overall architecture with Interfaces for Analysers and Classifiers

Chapter 4

Implementation

4.1 Colour Space Analysis

4.1.1 Colour Spaces

4.1.2 Colour Histograms

4.2 Texture Analysis

4.2.1 Edge Orientation

4.2.1.1 Histogram of Edge Orientation

4.3 Brush-Stroke Analysis

4.4 Classification and Validation

4.4.1 K-Nearest Neighbour

4.4.2 Leave-One-Out Cross Validation

4.4.3 Weka 3

4.4.3.1 Attribute-Relation File Format (ARFF)

4.4.4 Exemplars

4.4.4.1 “Real” Exemplars

4.4.4.2 Theoretical Exemplars

4.5 3rd Party Libraries and Tools

4.5.1 Python

4.5.1.1 Python setuptools

4.5.2 OpenCV

4.5.3 scipy & numpy

4.5.4 matplotlib

4.5.5 Weka 3

4.5.5.1 liac-arff

4.5.6 git & github

Chapter 5

Testing

5.1 Overall Approach to Testing

5.2 Validation

5.2.1 Leave-One-Out Cross Validation

5.2.2 Validation using Weka

Chapter 6

Evaluation

6.1 Evaluation of Requirements

6.2 Evaluation of Design

6.3 Evaluation of Tools

6.3.1 Python

6.3.1.1 setuptools

6.3.2 OpenCV

6.3.3 Weka 3

6.3.3.1 Attribute-Relation File Format (ARFF)

6.3.4 scipy & numpy

Appendices

Appendix A

3rd Party Libraries and Tools

1.1 Python 2.7

1.1.1 setuptools

<http://pypi.python.org/pypi/setuptools>

1.1.2 scipy

<http://www.scipy.org/> [9]

1.1.3 numpy

<http://www.numpy.org/> [9]

1.1.4 matplotlib

<http://matplotlib.org/>

1.1.5 liac-arff

<https://github.com/renatopp/liac-arff>

1.2 OpenCV

<http://opencv.org/> [1]

1.2.1 OpenCV Python

<http://opencv.willowgarage.com/wiki/PythonInterface> [1]

1.3 Weka 3

<http://www.cs.waikato.ac.nz/ml/weka/> [7]

1.4 git

1.4.1 github

Appendix B

Equations

2.1 Statistical Equations

2.1.1 Mean

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

2.1.2 Standard Deviation

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}$$

2.1.3 Pearson's product-moment coefficient

$$\rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

2.2 Distance Equations

2.2.1 Manhattan Distance

$$d_1(\mathbf{p}, \mathbf{q}) = \|\mathbf{p} - \mathbf{q}\|_1 = \sum_{i=0}^n |p_i - q_i|$$

2.2.2 Euclidean Distance

$$d_1(\mathbf{p}, \mathbf{q}) = \sqrt{\sum_{i=0}^n (q_i - p_i)^2}$$

2.3 Filter Equations

2.3.1 Gradient Direction

$$\theta = \text{atan2} \left(\frac{\delta f}{\delta x}, \frac{\delta f}{\delta y} \right)$$

2.3.2 Discrete Derivative Masks

$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

2.3.3 Gabor Filter

$$g(x, y; \lambda, \theta, \psi, \sigma, \gamma) = \exp\left(-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}\right) \cos\left(2\pi \frac{x'}{\lambda} + \psi\right) \quad (1)$$

where:

$$\begin{aligned} x' &= x \cos \theta + y \sin \theta \\ y' &= x \sin \theta + y \cos \theta \end{aligned}$$

Appendix C

Code Samples

3.1 Gabor Filter Example Implementation

Listing C.1: Example implementation of a Gabor Filter in MATLAB from wikipedia [3]

```
function gb=gabor_fn(sigma,theta,lambda,psi,gamma)

sigma_x = sigma;
sigma_y = sigma/gamma;

% Bounding box
nstds = 3;
xmax = max(abs(nstds*sigma_x*cos(theta)),abs(nstds*sigma_y*sin(theta)));
xmax = ceil(max(1,xmax));
ymax = max(abs(nstds*sigma_x*sin(theta)),abs(nstds*sigma_y*cos(theta)));
ymax = ceil(max(1,ymax));
xmin = -xmax; ymin = -ymax;
[x,y] = meshgrid(xmin:xmax,ymin:ymax);

% Rotation
x_theta=x*cos(theta)+y*sin(theta);
y_theta=-x*sin(theta)+y*cos(theta);

gb= exp(-.5*(x_theta.^2/sigma_x^2+y_theta.^2/sigma_y^2)).*cos(2*
    pi/lambda*x_theta+psi);
```

3.2 OpenCV Histogram Example Code

Listing C.2: Example Histogram calculation and displaying code from OpenCV [1].

Taken from: http://opencv.willowgarage.com/documentation/python/imgproc_histograms.html#calchist

Calculating and displaying 2D Hue–Saturation histogram of a color image

```

import sys
import cv

def hs_histogram(src):
    # Convert to HSV
    hsv = cv.CreateImage(cv.GetSize(src), 8, 3)
    cv.CvtColor(src, hsv, cv.CV_BGR2HSV)

    # Extract the H and S planes
    h_plane = cv.CreateMat(src.rows, src.cols, cv.CV_8UC1)
    s_plane = cv.CreateMat(src.rows, src.cols, cv.CV_8UC1)
    cv.Split(hsv, h_plane, s_plane, None, None)
    planes = [h_plane, s_plane]

    h_bins = 30
    s_bins = 32
    hist_size = [h_bins, s_bins]
    # hue varies from 0 (~0 deg red) to 180 (~360 deg red again
    */
    h_ranges = [0, 180]
    # saturation varies from 0 (black–gray–white) to
    # 255 (pure spectrum color)
    s_ranges = [0, 255]
    ranges = [h_ranges, s_ranges]
    scale = 10
    hist = cv.CreateHist([h_bins, s_bins], cv.CV_HIST_ARRAY,
        ranges, 1)
    cv.CalcHist([cv.GetImage(i) for i in planes], hist)
    (_, max_value, _, _) = cv.GetMinMaxHistValue(hist)

    hist_img = cv.CreateImage((h_bins*scale, s_bins*scale), 8,
        3)

    for h in range(h_bins):
        for s in range(s_bins):
            bin_val = cv.QueryHistValue_2D(hist, h, s)
            intensity = cv.Round(bin_val * 255 / max_value)
            cv.Rectangle(hist_img,
                (h*scale, s*scale),
                ((h+1)*scale - 1, (s+1)*scale - 1),
                cv.RGB(intensity, intensity, intensity)
                ,
                cv.CV_FILLED)

    return hist_img

```

```
if __name__ == '__main__':
    src = cv.LoadImageM(sys.argv[1])
    cv.NamedWindow("Source", 1)
    cv.ShowImage("Source", src)

    cv.NamedWindow("H-S_Histogram", 1)
    cv.ShowImage("H-S_Histogram", hs_histogram(src))

    cv.WaitKey(0)
```

Annotated Bibliography

- [1] G. Bradski, "The OpenCV Library," *Dr. Dobb's Journal of Software Tools*, 2000.

Used Python (<http://opencv.willowgarage.com/documentation/python>) and C++ (<http://opencv.willowgarage.com/documentation>) documentation for library reference and some learning on image processing/computer vision. Used since 11 October 2012.

- [2] I. Chilvers, J. Graves-Smith, and I. Chilvers, *A dictionary of modern and contemporary art*. Oxford University Press, 2009. [Online]. Available: <http://www.worldcat.org/isbn/0199239665>

- [3] W. Contributors, "Gabor filter," Online, Oct. 2012. [Online]. Available: http://en.wikipedia.org/w/index.php?title=Gabor_filter&oldid=517342109

- [4] N. Dalal and B. Triggs, "Histograms of oriented gradients for human detection," in *Computer Vision and Pattern Recognition, 2005. CVPR 2005. IEEE Computer Society Conference on*, ser. CVPR '05, vol. 1. Washington, DC, USA: IEEE, June 2005, pp. 886–893 vol. 1. [Online]. Available: <http://dx.doi.org/10.1109/cvpr.2005.177>

Describes a method of producing histograms of edge orientations which may prove to be a useful analysis technique for Kyffin Williams' art. The most interesting part of this paper is the use of segmentation and binning of gradients which seems like it could be useful to differentiate different parts of the image which may be painted in different styles.

- [5] J. Davies and A. Gymreig, *The Welsh Academy encyclopaedia of Wales*. University of Wales Press, 2008, pp. 957–958. [Online]. Available: <http://www.worldcat.org/isbn/9780708319536>

- [6] E. Gamma, *Entwurfsmuster : Elemente wiederverwendbarer objektorientierter Software*, ser. Addison-Wesley professional computing series. Addison-Wesley, 1996. [Online]. Available: <http://www.worldcat.org/isbn/9780201633610>

- [7] M. Hall, E. Frank, G. Holmes, B. Pfahringer, P. Reutemann, and I. H. Witten, "The WEKA data mining software: an update," *SIGKDD Explor. Newsl.*, vol. 11, no. 1, pp. 10–18, Nov. 2009. [Online]. Available: <http://dx.doi.org/10.1145/1656274.1656278>

Citation for the Weka data mining software. Weka is a Java based tool which can be used to run a lot of classifiers to a dataset, making it a very useful tool to apply to the Kyffin Williams project. Weka allows the application of complex machine learning techniques without having to spend a lot of time learning, understand and implementing said techniques.

- [8] R. Harris, "How rolf learnt to paint like sir kyffin williams," BBC Broadcast, Feb. 2011. [Online]. Available: <http://www.bbc.co.uk/programmes/p00f6nyt>

A video on the BBC by Rolf Harris about some of Kyffin Williams' life and about his interesting style of painting.

- [9] E. Jones, T. Oliphant, P. Peterson, *et al.*, "SciPy: Open source scientific tools for python," 2001.