Coverage of Concurrent Single and Pair Operation Cancellations over Triple Operation Cancellations in Refactoring

Let $I(A)$ denote the quantity of inconsistencies associated with dependencies connected to node A. It is defined as:

$$I(A) = \sum_{i=1}^{l_A} r_{A_i} S_{A_i} = \sum_{i=1}^{l_A} r_{A_i} H(A, A(i), d)$$

Where:

- A denotes the node.
- A_i represents the dependency connected to node A, expressed as either $A_i = (A, A(i))$ or $(A(i), A)$.
- $A(i)$ denotes the other endpoint of dependency A_i with $1 \leq i \leq l_A$.
- l_A is the count of dependencies connected to node A.
- r_{A_i} is the weight of dependency A_i, which corresponds to the number of dependencies between A and $A(i)$, and is considered a constant.
- $S_{A_i} = H(A, A(i), d)$, H is the function that assesses whether dependency A_i is consistent with the architecture design, where 0 represents consistency and 1 represents inconsistency, which is equivalent to S_{A_i}. d indicates the direction of the dependency, with values of 1 or -1, representing a dependency or inverse dependency relationship.

Step 1：cancellation of one refactoring operation

Upon the cancellation of operation A, the impact is only on the inconsistency quantity of dependencies connected to node A. If operation A leads to a change for node A from A to A', the change in inconsistency quantity, $\Delta I(A)$, is given by:

$$\Delta I(A) = \sum_{i=1}^{l_A} r_{A_i} \Delta S_{A_i} = \sum_{i=1}^{l_A} r_{A_i} \Delta H(A, A(i), d)$$

$$= \sum_{i=1}^{l_A} r_{A_i} [H(A', A(i), d) - H(A, A(i), d)]$$

Analogous expressions can be derived for $\Delta I(B)$ and $\Delta I(C)$.

$$\Delta I(B)=\sum_{i=1}^{l_B} r_{B_i}\Delta S_{B_i}=\sum_{i=1}^{l_B} r_{B_i}\Delta H(B,B(i),d)$$
$$=\sum_{i=1}^{l_B} r_{B_i}[H(B',B(i),d)-H(B,B(i),d)]$$
$$\Delta I(C)=\sum_{i=1}^{l_C} r_{C_i}\Delta S_{C_i}=\sum_{i=1}^{l_C} r_{C_i}\Delta H(C,C(i),d)$$
$$=\sum_{i=1}^{l_C} r_{C_i}[H(C',C(i),d)-H(C,C(i),d)]$$

Step 2：Cancellation of two refactoring operation concurrently

Assuming dependencies exist between nodes A and B, which are characterized as follows:

- Dependencies connecting A to B are the l_A-th and l_A-1-th dependencies in the set of dependencies connected to A,
- Dependencies connecting B to A are the l_B-2-th and l_B-3-th dependencies in the set of dependencies connected to B

The inconsistency quantity connected to nodes A and B can be formulated as:

$$I(A,B)=\sum_{i=1}^{l_A-2} r_{A_i}S_{A_i}+\sum_{i=1}^{l_B-4} r_{B_i}S_{B_i}+\sum_{i=l_B-1}^{l_B} r_{B_i}S_{B_i}+r_{A_{l_A}}H(A,B,1)$$
$$+r_{A_{l_A-1}}H(A,B,-1)$$

And the change in inconsistency quantity due to the concurrent cancellation of operations A, B, C is expressed as：

$$\Delta I(A,B)=\sum_{i=1}^{l_A-2} r_{A_i}\Delta S_{A_i}+\sum_{i=1}^{l_B-4} r_{B_i}\Delta S_{B_i}+\sum_{i=l_B-1}^{l_B} r_{B_i}\Delta S_{B_i}+r_{A_{l_A}}\Delta H(A,B,1)$$
$$+r_{A_{l_A-1}}\Delta H(A,B,-1)$$

Similarly,

$$\Delta I(B,C)=\sum_{i=1}^{l_B-2} r_{B_i}\Delta S_{B_i}+\sum_{i=1}^{l_C-4} r_{C_i}\Delta S_{C_i}+\sum_{i=l_C-1}^{l_C} r_{C_i}\Delta S_{C_i}+r_{B_{l_B}}\Delta H(B,C,1)$$
$$+r_{B_{l_B-1}}\Delta H(B,C,-1)$$

$$\Delta I(C,A) = \sum_{i=1}^{l_C-2} r_{C_i}\Delta S_{C_i} + \sum_{i=1}^{l_A-4} r_{A_i}\Delta S_{A_i} + \sum_{i=l_A-1}^{l_A} r_{A_i}\Delta S_{A_i} + r_{C_{l_C}}\Delta H(C,A,1) + r_{C_{l_C-1}}\Delta H(C,A,-1)$$

Step 3：Cancellation of three refactoring operation concurrently

Assuming dependencies exist between nodes A, B and C, which are characterized as follows:

- Dependencies connecting A to B are the l_A-th and l_A-1-th dependencies in the set of dependencies connected to A,
- Dependencies connecting A to C are the l_A-2-th and l_A-3-th dependencies in the set of dependencies connected to A,
- Dependencies connecting B to C are the l_B-th and l_B-1-th dependencies in the set of dependencies connected to B,
- Dependencies connecting B to A are the l_B-2-th and l_B-3-th dependencies in the set of dependencies connected to B,
- Dependencies connecting C to A are the l_C-th and l_C-1-th dependencies in the set of dependencies connected to C,
- Dependencies connecting C to B are the l_C-2-th and l_C-3-th dependencies in the set of dependencies connected to C.

The inconsistency quantity connected to nodes A, B and C can be formulated as

$$I(A,B,C) = \sum_{i=1}^{l_A-4} r_{A_i}S_{A_i} + \sum_{i=1}^{l_B-4} r_{B_i}S_{B_i} + \sum_{i=1}^{l_C-4} r_{C_i}S_{C_i}$$
$$+r_{A_{l_A}}H(A,B,1) + r_{A_{l_A-1}}H(A,B,-1)$$
$$+r_{B_{l_B}}H(B,C,1) + r_{B_{l_B-1}}H(B,C,-1)$$
$$+r_{C_{l_C}}H(C,A,1) + r_{C_{l_C-1}}H(C,A,-1)$$

And the change in inconsistency quantity due to the concurrent cancellation of operations A, B and C is expressed as

$$\Delta I(A,B,C) = \sum_{i=1}^{l_A-4} r_{A_i}\Delta S_{A_i} + \sum_{i=1}^{l_B-4} r_{B_i}\Delta S_{B_i} + \sum_{i=1}^{l_C-4} r_{C_i}\Delta S_{C_i}$$
$$+r_{A_{l_A}}\Delta H(A,B,1) + r_{A_{l_A-1}}\Delta H(A,B,-1)$$
$$+r_{B_{l_B}}\Delta H(B,C,1) + r_{B_{l_B-1}}\Delta H(B,C,-1)$$

$$+r_{C_{l_C}}\Delta H(C,A,1)+r_{C_{l_C-1}}\Delta H(C,A,-1)$$

Step 4：Scenario Coverage Confirmation

To prove whether the scenario of concurrently canceling two refactoring operations can cover the scenario of concurrently canceling three refactoring operations. It can be calculated as follows:

$$\begin{aligned}
&\Delta I(A,B)+\ \Delta I(B,C)+\ \Delta I(C,A)-\Delta I(A,B,C)\\
&=(\sum_{i=1}^{l_A-2} r_{A_i}\Delta S_{A_i}+\sum_{i=1}^{l_B-4} r_{B_i}\Delta S_{B_i}+\sum_{i=l_B-1}^{l_B} r_{B_i}\Delta S_{B_i})+(\sum_{i=1}^{l_B-2} r_{B_i}\Delta S_{B_i}+\sum_{i=1}^{l_C-4} r_{C_i}\Delta S_{C_i}\\
&\qquad+\sum_{i=l_C-1}^{l_C} r_{C_i}\Delta S_{C_i})+(\sum_{i=1}^{l_C-2} r_{C_i}\Delta S_{C_i}+\sum_{i=1}^{l_A-4} r_{A_i}\Delta S_{A_i}+\sum_{i=l_A-1}^{l_A} r_{A_i}\Delta S_{A_i})\\
&\qquad-(\sum_{i=1}^{l_A-4} r_{A_i}\Delta S_{A_i}+\sum_{i=1}^{l_B-4} r_{B_i}\Delta S_{B_i}+\sum_{i=1}^{l_C-4} r_{C_i}\Delta S_{C_i})\\
&=\sum_{i=1}^{l_A-2} r_{A_i}\Delta S_{A_i}+\sum_{i=l_B-1}^{l_B} r_{B_i}\Delta S_{B_i}+\sum_{i=1}^{l_B-2} r_{B_i}\Delta S_{B_i}+\sum_{i=l_C-1}^{l_C} r_{C_i}\Delta S_{C_i}+\sum_{i=1}^{l_C-2} r_{C_i}\Delta S_{C_i}\\
&\qquad+\sum_{i=l_A-1}^{l_A} r_{A_i}\Delta S_{A_i}\\
&=\sum_{i=1}^{l_A} r_{A_i}\Delta S_{A_i}+\sum_{i=1}^{l_B} r_{B_i}\Delta S_{B_i}\sum_{i=1}^{l_C} r_{C_i}\Delta S_{C_i}\\
&=\Delta I(A)+\Delta I(B)+\Delta I(C)
\end{aligned}$$

Therefore,
$\Delta I(A,B,C)=\Delta I(A,B)+\ \Delta I(B,C)+\ \Delta I(C,A)+\Delta I(A)+\Delta I(B)+\Delta I(C)$

If canceling operations A, B, and C individually does not cause any inconsistency, and canceling in pairs does not cause any inconsistency, that is $\boldsymbol{\Delta I(A)=\Delta I(B)=\Delta I(C)=\Delta I(A,B)=\ \Delta I(B,C)=\ \Delta I(C,A)=0}$**. Then** $\boldsymbol{\Delta I(A,B,C)=0}$

This result demonstrates that if the individual cancellation of operations A, B and C does not introduce any inconsistencies, and canceling in pairs does not cause any inconsistency, then their cancellation in triple will also not affect the inconsistency quantity.