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# Supplementary Material

Relevant to the four measurement variables (*fmv*) equations

Dear Reviewer,

The following represents the core equations of the article within the pre-Proof and Proof Sections of the *quantum double-field game theory* (QDFGT) *model*. You may refer to the *.pdf documents representing the results and/or click on the following weblinks.  You may change the values of the fmv variables in the Wolfram and Symbolab calculators as exemplified below, according to the article analysis and discussions.

The evaluation of Eq. (34) can be done via the following link as the assigned values to the variables are changed to update the result for the upper and lower bounds of the equation.

|k|*|(4*(d-λ/4))|/(e^(i*2*k*r)*2*((d-λ/4)/(sqrt(a*r))))*e^(i*k*a),  d=1, λ=1, a=2, r=1, k*r=1

where Eq. (31) can be validated retroactively. For Eq. (37),

4\cdot 3\int _{4\pi }^{0\:}\int _{16\pi ^2\:}^{4\pi \:\:}\int _0^{16\pi ^2}\left(\frac{1}{4e^{\alpha \cdot \:\:\:1\cdot \:\:0.999}e^{2\beta \cdot \:0.999-\gamma \cdot \:0.999}}\right)\cdot \:d\alpha \:d\beta \:d\gamma

and for Eq. (38), the following:

sqrt(a*r)*sqrt(|(|(2*(d-λ/4))|^4*(e^(i*2*k*r)))|/(4*pi*r^2)*|16*pi^2/(|(2*(d-λ/4))|^4*k^2*(e^(i*2*k*a)))|)/(3*sqrt(pi)*d),  d=0.999, a=1, r=1, k*r=1

and

sqrt(a*r)*sqrt(|(|(2*(d-λ/4))|^4*(e^(i*2*k*r)))|/(4*pi*r^2)*|16*pi^2/(|(2*(d-λ/4))|^4*k^2*(e^(i*2*k*a)))|)/(3*sqrt(pi)*d),  d=0.9, a=2, r=1, k*r=1

The *spontaneous symmetry breaking* (the Mexican hat potential) scenario satisfying Fig. 5a) or Fig. 4 hyperbolic number projections, can be tested via the following link for Eq. (43). In this case, distance $d$=1 is used to illustrate the geometry of the projections through Eqs. (17)-(19). Other values can be tried. The *.pdf file included within the .zip file, shows the geometry of a typical condition by any other Mexican hat potential model compared to the current ones by our proposed model, which supports a bi-complex field, Ref. [21]:

(|(|phi|^2*2*d/mu)^2 - (3* |psi|)^2*kappa^2|)^(1/4), sqrt(2)*phi^2 = k^2*psi, λ=10^(-34)*6.626, d=0.9, kappa^2=|2*(d-λ/4)|, mu=(N^2-N)/2, N=4

The *transition probability computation*, Eq. (51), can be tested using the following formula, or by visiting the computational intelligence webpage on **Wolfram|Alpha**

4*\pi * (( 2/3*(N-1))^2* ln sqrt(Z^(2/3 * d))) /(4*\pi * (sqrt(\delta))^(-1)* (N*(N-1)/6)+2*\pi*(Z/9)/\delta)  , d=1, Z=3, N=4, \delta=1 Wolfram|Alpha

The low versus high-energy scattering amplitude in Born approximation to unity is one of the factors to simplify Eq. (50) to (51), as discussed for Eq. (33),

(e^(i*k*(d-2*r)))*(e^(i*k*(d-r))),  d=1, r=10^(-34)*6.626, k*r=0 Wolfram|Alpha
And
(e^(i*N*pi*k*r)),  r=10^(-34)*6.626, k*r=1, N=5

The expressions in Eqs. (50)-(53) can be evaluated by (2*pi/9)*\alpha, \alpha = 0.95 (as the alpha value can be changed accordingly) for the lower bound as opposed to the upper bound.

The variables such as *d*, *delta* and *Z* can be changed in value to discuss the results falling into the range specified in Eq. (51) or Example 8. Here is an example where the values have been changed as discussed in the paper:

4*\pi * (( 2/3*(N-1))^2* ln sqrt(Z^(2/3 * d))) /(4*\pi * (sqrt(\delta))^(-1)* (N*(N-1)/6)+2*\pi*(Z/9)/\delta) , d=0.99, Z=3, N=4, \delta=0.99 Wolfram|Alpha

For the lower bound of *delta*=0.5, the following from the simplified expression of Eq. (51) as Eq. (52), is the result satisfying the lower range of the interval in Eq. (52)

4*\pi * (( 2/3*(N-1))^2* ln sqrt(Z^(2/3 *d))) /(4*\pi * (\delta)^(-1)* (N*(N-1)/6)+(Z)/\delta) , d=0.99, Z=3, N=4, \delta=0.5 Wolfram|Alpha

Further simplification results in Eq. (53) and the variables tested is exemplified in the following link:
4/9 *d* \delta*(N-1)^2/(v * (N*(N-1)/2) ) , N=4, v=8/9, d=0.9, \delta=0.99 Wolfram|Alpha

For other calculations, such as the transition probability matrix, Eq. (54), and the plots from Fig. 5, you may refer to the DFGTAlgorithm.nb file that can be run within the Wolfram Mathematica software environment.

Sincerely,

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