
POLYNOMIAL REGRESSION (DEGREE 3)
ONE ITERATION OF GRADIENT DESCENT

Problem Setup

Input feature:
$x = 2$

Polynomial features:
$x_1 = x = 2$
$x_2 = x^2 = 4$
$x_3 = x^3 = 8$

Actual output:
$y = 20$

Initial weights and bias:
$w_1 = 1$
$w_2 = 1$
$w_3 = 1$
$b = 0$

Learning rate:
$\alpha = 0.1$

Step 1: Prediction ($\hat{y} = wx + b$)

Formula:
$\hat{y} = w_1x + w_2x^2 + w_3x^3 + b$

Calculation:
$\hat{y} = (1 \times 2) + (1 \times 4) + (1 \times 8) + 0$
$\hat{y} = 14$

Step 2: Loss (Mean Squared Error for one sample)

Formula:
Loss = $(y - \hat{y})^2$

Calculation:
Loss = $(20 - 14)^2$
Loss = 36

Step 3: Gradients (Partial Derivatives)

Gradient with respect to w_1:

Formula:
$\partial L/\partial w_1 = -2(y - \hat{y})x$

Calculation:
$\partial L/\partial w_1 = -2(20 - 14)(2)$
$\partial L/\partial w_1 = -24$

Gradient with respect to w_2:

Formula:
$\partial L/\partial w_2 = -2(y - \hat{y})x^2$

Calculation:
$\partial L/\partial w_2 = -2(20 - 14)(4)$
$\partial L/\partial w_2 = -48$

Gradient with respect to w_3:

Formula:
$\partial L/\partial w_3 = -2(y - \hat{y})x^3$

Calculation:
$\partial L/\partial w_3 = -2(20 - 14)(8)$
$\partial L/\partial w_3 = -96$

Gradient with respect to bias b:

Formula:
$\partial L/\partial b = -2(y - \hat{y})$

Calculation:
$\partial L/\partial b = -2(20 - 14)$
$\partial L/\partial b = -12$

Step 4: Gradient Descent Update (One Iteration)

Update w_1:

Formula:
$w_1 = w_1 - \alpha(\partial L/\partial w_1)$

Calculation:
$w_1 = 1 - 0.1(-24)$
$w_1 = 3.4$

Update w_2:

Formula:
$w_2 = w_2 - \alpha(\partial L/\partial w_2)$

Calculation:
$w_2 = 1 - 0.1(-48)$
$w_2 = 5.8$

Update w_3:

Formula:
$w_3 = w_3 - \alpha(\partial L/\partial w_3)$

Calculation:
$w_3 = 1 - 0.1(-96)$
$w_3 = 10.6$

Update bias b:

Formula:
$b = b - \alpha(\partial L/\partial b)$

Calculation:
$b = 0 - 0.1(-12)$
$b = 1.2$

Final Parameters After One Iteration

$w_1 = 3.4$
$w_2 = 5.8$
$w_3 = 10.6$
$b = 1.2$
