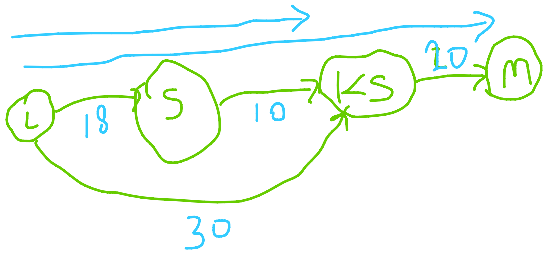
L23/24 - Dijkstra's Algorithm (Single Source Shortest Path) (Greedy Algorithm)

Tuesday, June 16, 2020 7:30 AM

Shortest Distance: Sum of weights on edges from source to destination vertices.

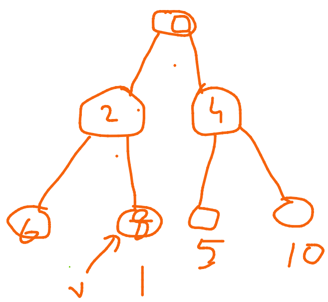
Pre-Conditions:
1. All edge weights must be positive.
2. Directed weighted Graphs

Constraints



Principle of Optimality:

INITIALIZE-SINGLE-SOURCE(G, s)
1. for each vertex v ∈ G. V
2. v. d = ∞
3. v. π = NIL
4.
5. s. d = 0



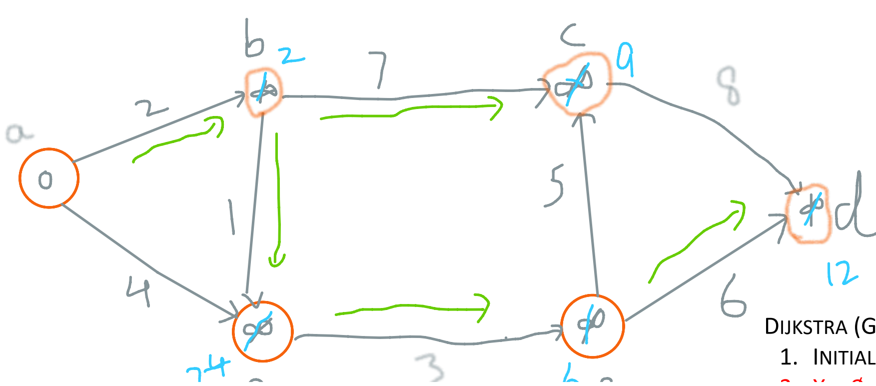
DIJKSTRA (G, w, s)

1. INITIALIZE-SINGLE-SOURCE(G, s)		O(|V|)
2. X = ∅		
3. H = G.V	O(|V|)	O(|V|* log |V|)
4. While H ≠ ∅		O(|V|)
5. u = EXTRACT-MIN(H)		O(|V|* log |V|)
6. X = X U {u}		
7. for each vertex v ∈ G. Adj[u]		O(|E|)
8. if v. d > u. d + w(u, v)		O(|E|)
9. v. d = u. d + w(u, v)		O(|E|* log(|V|)
10. v. π = u		O(|E|)

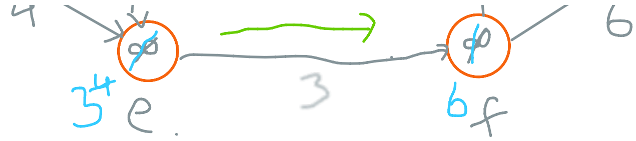
$O((|V|+|E|) * \log(|V|))$

Adjacency List

a	b|2	e|4
b	c|7	e|1
c	d|1	
d		
e	f|3	
f	c|5	d|6



DIJKSTRA (G, w, s)
1. INITIALIZE-SINGLE-SOURCE(G, s) O(|V|)

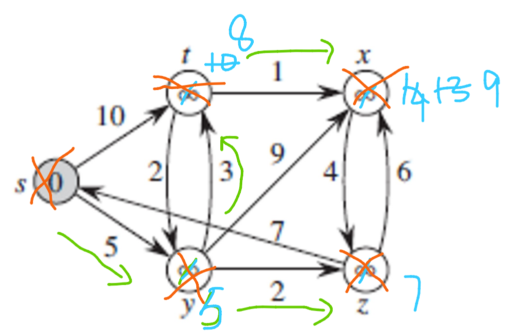


DIJKSTRA (G, w, s)

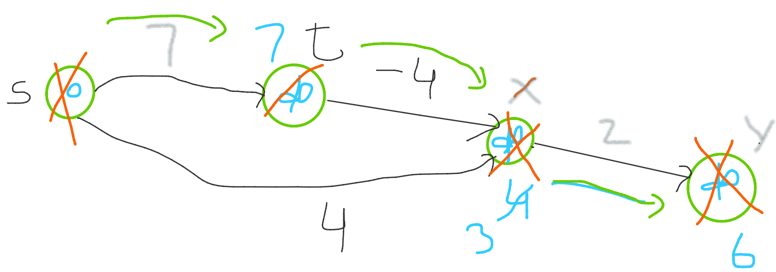
1. INITIALIZE-SINGLE-SOURCE(G, s) — $O(|V|)$
2. X = Ø
3. H = G.V — $O(|V| * \log |V|)$
4. While H ≠ Ø — $O(|V|)$
5. u = EXTRACT-MIN(H) — $O(|V| * \log |V|)$
6. X = X U {u}
7. for each vertex v ∈ G. Adj[u] — $O(|E|)$
8. if v. d > u. d + w(u, v) — $O(|E|)$
9. v. d = u. d + w(u, v) — $O(|E| * \log(|V|))$
10. v. π = u — $O(|E|)$

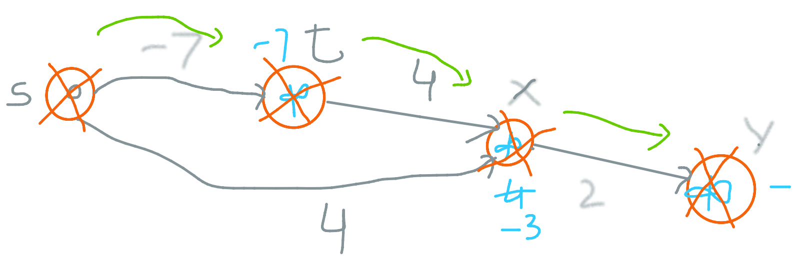
Predecessor	Nil	~~Nil~~ a	~~Nil~~ b	~~Nil~~ f	~~Nil~~ ~~a~~ b	~~Nil~~ e
Node			Distances			
Selected	d(a)	d(b)	d(c)	d(d)	d(`e)	d(f)
	0	∞	∞	∞	∞	∞
a	0	2	∞	∞	4	∞
b	0	2	9	∞	3	∞
e	0	2	9	∞	3	6
f	0	2	9	12	3	6
c	0	2	9	12	3	6
d	0	2	9	12	3	6

Build the Distance Table for the Directed Weighted Graph shown below:



Predecessor	Nil	~~Nil~~ ~~s~~ y	~~Nil~~ s	~~Nil~~ ~~y~~ ~~z~~ t	~~Nil~~ y
Node			Distances		
Selected	s.d	t.d	y.d	x.d	z.d
	0	∞	∞	∞	∞
s	0	10	5	∞	∞
y	0	8	5	14	7
z	0	8	5	13	7
t	0	8	5	9	7
x	0	8	5	9	7





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for each vertex v ∈ G. Adj[u]            O(|E|)
    if v. d > u. d + w(u, v)              O(|E|)
        v. d = u. d + w(u, v)             O(|E|*  log(|V|)
        v. π = u                          O(|E|)
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$$O(2(|V| \times \log|V|)) + O(|E| \times \log(|V|))$$

$$= O((|V| + |E|) \times \log|V|)$$

$$= O(|E| \times \log|V|)$$