

# Greedy Algorithm/Fractional Knapsack

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- Mostly used in **Optimization problems**.
- Problems usually have  $n$  inputs and require to obtain a subset/ordering of inputs that satisfy some **constraints**.
- Any subset that satisfies these constraints is called **feasible solution**.
- Find a feasible solution that maximizes/minimizes an **objective function**.
- A feasible solution that maximizes/minimizes an **objective function** is called an **optimal solution**.

Greedy method usually works in **stages**.

- At each stage an input is selected to be included in the optimal solution.
- The selection procedure is based on some **optimization measure**.
- There may be different optimization measures for a given problem.
- The optimization measure may be the objective function.
- If the inclusion will cause the solution to be infeasible, it is discarded.

```

1  Algorithm Greedy( $a, n$ )
2  //  $a[1 : n]$  contains the  $n$  inputs.
3  {
4       $solution := \emptyset$ ; // Initialize the solution.
5      for  $i := 1$  to  $n$  do
6          {
7               $x := \text{Select}(a)$ ;
8              if Feasible( $solution, x$ ) then
9                   $solution := \text{Union}(solution, x)$ ;
10         }
11     return  $solution$ ;
12 }

```

### **FRACTIONAL KNAPSACK PROBLEM:**

Given  $n$  objects and a knapsack/bag of some capacity. Object  $i$  has weight  $w_i$  and the knapsack has capacity  $m$ . If a fraction  $x_i$ ,  $0 \leq x_i \leq 1$ , is placed into the knapsack, then a profit of  $p_i x_i$  is earned.

The objective is to obtain a filling of the knapsack that maximizes the profit earned.

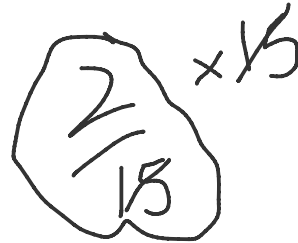
As the knapsack has capacity  $m$ , the sum of weights of objects included in the knapsack must be at most  $m$ .

(The profits and weights are positive numbers.)

Formally,

$$\begin{aligned}
 &\text{maximize } \sum_{1 \leq i \leq n} p_i x_i & (1) \\
 &\text{subject to } \sum_{1 \leq i \leq n} w_i x_i \leq m & (2) \\
 &\text{and } 0 \leq x_i \leq 1, \quad 1 \leq i \leq n & (3)
 \end{aligned}$$

A **feasible solution** (or filling) is any set  $(x_1, \dots, x_n)$  that satisfies eqs. 2 & 3. An **optimal solution** is any feasible solution that maximize eq. 1.



Example:

$n = 3$ ,

$m = 20$ .

$(p_1, p_2, p_3) = (25, 24, 15)$

$(w_1, w_2, w_3) = (18, 15, 10)$

$= (1.38, 1.6, 1.5)$

$$9 + 5 + 2.5$$

$$12.5 + 8 + 3.75$$

	$(x_1, x_2, x_3)$	$\sum w_i x_i$	$\sum p_i x_i$
1. Random <u>Profit</u>	$(1/2, 1/3, 1/4)$	16.5	24.25
2. Maximum weight	$(1, 2/15, 0)$	18+2	$25 \cdot 1 + 24 \cdot 2/15 = 28.2$
3. Minimum Weight	$(0, 2/3, 1)$	10+10	$15 \cdot 1 + 24 \cdot 2/3 = 15 + 16 = 31$
4. Profit/weight	$(0, 1, 1/2)$	15 + 5	$24 + 15 \cdot 1/2 = 24 + 7.5 = 31.5$