L9 - Time Complexity (Recursive Functions)

Monday, April 6, 2020 7:45 PM

Asymptotic Analysis of Recursive Functions

$$Sum(n) = \begin{cases} 0\,; & if\ n = 0; \quad Basis\ step \\ Sum(n-1) + (lastNumber); & if\ n > 0; \quad Recursive\ step \end{cases}$$

Time Complexity of recursive Sum of a list of numbers

Calculate the Cost and Frequency of each line ***as if there is no recursive call***.

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recSum(Arr, n)	Operations	Freq.
1. if n == 0	c1	1
2. return 0	c2	1 (0)
3. return recSum(Arr, n-1) + Arr[n]	c3 + (?)	1

recSum (array, size)

$T_{recSum}(n)$ is a function that represents the Time Complexity of ***recSum*** when invoked on a list of ***n elements***.

$T_{recSum}(n) = (?) + c1+c3$ if $n > 0$

$= (T_{recSum}(n-1)) + c$ $= (T_{recSum}(n-1)) + O(n^0) = (T_{recSum}(n-1)) + O(1)$

$T_{recSum}(n) = c1+c2 = c21$ if n==0

Recurrence Equation/Relation

$$T_{recSum}(n) = \begin{cases} c21; & if\ n = 0 \\ T_{recSum}(n-1) + c; & if\ n > 0 \end{cases}$$

Fibonacci

$$fib(n) = \begin{cases} 0; & if\ n = 0 \quad Basis\ Step \\ 1; & if\ n = 1 \quad Basis\ Step \\ fib(n-1) + fib(n-2); & if\ n > 1 \quad Recursive\ Step \end{cases}$$

Time Complexity of recursive Fibonacci

Calculate the Cost and Frequency of each line ***as if there is no recursive call***.

recFib(n)	Cost	Freq.
1. If n <= 1	c1	1
2. return n	c2	1 (0)
3. return recFib(n-1) + recFib(n-2)	(?) + (?) + c3	1

$T_{recFib}(n)$ = (?) + (?) + c1 + c3 if n > 1

= (T_{recFib}(n-1)) + (T_{recFib}(n-2)) + c

T_{recFib}(n) = c1+c2 = c11 if n <= 1

Recurrence Equation/Relation

$$T_{recFib}(n) = \begin{cases} c11; & if\ n \leq 1 \\ T_{recFib}(n-1) + T_{recFib}(n-2) + c; & if\ n > 1 \end{cases}$$

Time Complexity of recursive Factorial

Calculate the Cost and Frequency of each line ***as if there is no recursive call***.

recFactorial(n)	Cost	Freq.
1. If n <= 1	c1	1
2. return 1	c2	1 (0)
3. return n * recFactorial(n-1)	c3 + (?)	1

$T_{recFactorial}(n) = (T_{recFactorial}(n-1)) + c1+c3 = (?) + c$ $= (?) + O(n^0) = (?) + O(1)$
if n > 1

$T_{recFactorial}(n) = c1 + c2 = c13$ if n <= 1

Recurrence Equation/Relation

$$T_{recFactorial}(n) = \begin{cases} c13; & if\ n \leq 1 \\ T_{recFactorial}(n-1) + c; & if\ n > 1 \end{cases}$$