

# L2-Complexity

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## Finding Timing Complexity of a piece of code.

- Finding Exact Running Time
  - Hardware & Software Dependency
  - Very Time Consuming
  - Error Prone
- Finding Complexity profile as function of input size

What is **input size**?

Operations Counting Technique:

- Assign each primitive operation (+, -, \*, /, etc.) equal cost (1 unit)
- Assign unit cost to object creation
- Assign unit cost to initialization, copy & assignment of primitive objects
- Assign unit cost to return statement of primitive objects
- Assign unit cost to function call excluding argument/parameter passing
  
- Find the number of operations of each line of code
- Find the frequency of execution of each line of code.

Developing Timing Equation

operations(Cost) \* Freq

1.  $a = 2*a + 3*k$

$4 * 1$

2.  $a = a * k;$

$2 * 1$

$$T(n) = 4*1 + 2*1 = 6$$

### Developing Timing Equation

//for (i=1 to n)

1. for (int i=1; i<=n; ++i)
2.      $a = a * k;$
3. }

operations(Cost) \* Freq

$$\begin{array}{ll} (1+1) + 2*n & (\text{end-start}+2) \\ 2 * n & \end{array}$$

$$\begin{aligned} T(n) &= 2*n + 2*n + 1+1 \\ &= 4*n + 2 \end{aligned}$$

//for (i=1 to n-1)

4. for (int i=1; i < n; ++i) {      $(3+1)+2*(n-2)$
5.      $a = a * k;$
6. }

operations(Cost) \* Freq

$$\begin{array}{ll} (1+1) + 2 * (n-1) & (\text{end-start}+1) \\ 2 * (n-1) & \end{array}$$

$$\begin{aligned} T(n) &= 2*(n-1) + 2*(n-1) + 1+1 \\ &= 4*n - 2 \end{aligned}$$

operations(Cost) \* Freq

```
//for (i=1 to n step 2)
1. for (int i=1; i<=n; i=i+2){ (4+1)+3*(n/2 - 1) = (1+1)+ 3*(n/2)
2.     a = a * k;                2*(n/2)
3. }
```

Let n=8  
i=1,3,5,7,9

$$T(n) = 5*n/2 + 2$$

operations(Cost) \* Freq

```
//for (i=1 to n step 3)
1. for (int i=1; i<=n; i=i+3){ (4+1) + 3*(8/3-1) = (1+1) + 3*(n/3)
2.     a = a * k;                2*(n/3)
3. }
```

Let n=9  
i=1,4,7,10

Let n=8  
i=1,4,7,10

$$T(n) = 5*(n/3) + 2$$

Developing Timing Equation

operations(Cost) \* Freq

```
1. for (int i=1; i<=n; i=i*2) { (4+1) + 3* log2(n) (1+1) + 3* (log2(n)+1)
```

```

2.     a = a * k;
3. }

```

$$2 * (\log_2(n)+1)$$

Let n=5  
 i=1,2,4,8  
 Let n=8  
 i=1,2,4,8,16

$$\begin{aligned}
 T(n) &= 3 * (\log_2(n)+1) + 2 * (\log_2(n)+1) + 1+1 \\
 &= 5 * \log_2(n) + 7
 \end{aligned}$$

Home Work: Find the operations cost & Frequency of each Line.

Developing Timing Equation

$$\text{operations(Cost)} * \text{Freq}$$

```

1. for (int i=n; n>=1; i=i/2) {
2.     a = a * k;
3. }

```

$$T(n) =$$