Options Pricing Research

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1 Options

Options are a versatile financial product that are based on the value of the underlying security. These contracts offers the buyer an opportunity to buy or sell, but unlike a future, the contact holder is not required to buy or sell the commodity.Through paying a premium, a buyer can gain the rights granted by the contract.
There are two types of options: call and put options. A call options is the right to buy or take a long position in a given asset. A put options is the right to sell or take a short position in a given asset. The asset to be bought or solder under the terms of the options is the underlying asset. The price at which the underlying will be delivered is called the Strike Price. The date after which the option may no longer be exercised is the expiration date.
The contact specifications contain the following: Underlying asset, expriation date, exercise price and type.

2 Binomial Pricing Model

The binomial asset pricing model is a powerful tool that can be used to understand arbitrage pricing theory. This first section will explore the simplest binomial model before generalising to a more realistic, complex multiperiod binomial model.
For the general one-period model we can call the beginning of the period time zero(T_0) and the end of the period time one(T_1). At time zero, we have a stock whose price per share can be written as S_0, a positive quantity known at T_0. At T_1, the price per share of this stock will be one of two positive values, which we can denote as $S_1(H)$ and $S_1(T)$, the S and H standing for heads and tails respectively. We can assume that the probability of a head is p, therefore making the probability of a tail $q = 1 - p$, both being positive values.
The outcome of the coin toss and hence the value which the stock price will take at time one is known at T_1 but not at T_0. Any quantity not known at T_0 will be known as random henceforth as it depends on the random experiment of tossing a coin.
Two new positive values can be introduced:
$u = \frac{S_1(H)}{S_0)}, d = \frac{S_1(T)}{S_0}$.
It is assumed that $d < u$ where u can be referred to as the up factor and d

as the down factor.
An interest rate r can be introdcued. One dollar invested in the market at T_0 will yield 1+r dollars at T_1.
An essential feature of an efficient market is that if a trading strategy can turn nothing into something, then it must also run the risk of loss. Otherwise there would be an arbitrage. More specifically, an arbitrage can be defined as as a trading strategy that begins with no money, has zero probability of losing money, and has a positive probability of making money.
In the one-period binomial model, to rule out arbitrage we must assume $0 < d < 1 + r < u$.
Let us now consider a European call option which relies on the fact that its owner has the right but not the obligation to buy one share of the stock at $T_1$for the strike price K. We shall assume here that $S_1(T) < K < S_1(H)$. If we get a tail then the option expires worthless. If the outcome is a head then the option can be exercised and yields a profit of $S_1(H) - K$. This can be summarised by saying that the option at T_1 is worth $(S_1 - K)^+$

Markov and Derivative Pricing
Consider a derivative whose payoff at time n is:

$$V_N = v_N(S_N)$$

Risk neutral pricing formula:

$$V_n = \frac{1}{1+r} E_N[V_{N+1}]$$

Hence we have:

$$V_{N-1} = \frac{1}{1+r} E_{N-1}[V_N]$$

$$= \frac{1}{1+r} E_{N-1}[v_N]$$

Stock is markov

$$= v_{N-1}(S_{N-1})$$

In general

$$V_n = v_n(S_n)$$