Analysis of the Quantum Advantages for Deep Hedging

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1 Abstract

Deep hedging

2 Hedging

Reword:

Classical derivative pricing models all rely on mathematical path generators, however these are not designed to be realistic, instead they describe diffusion in a risk-neutral measure, rather than the real-world measure, with variables being kept to a minimal for easy computation.

Mathematical set up for hedging:

3 Quantum Computing

4 Parameterised Circuits

4.1 Born Rule

An essential part of modern quantum computing involves the existence of the Born Rule. Born's measurement rule states that:

$$p(x) = |\langle x|\psi(\theta)|\rangle|^2$$

The state $|\psi(\theta)\rangle$ is generated by evolving the vacuum state $|0\rangle$ according to a Hamiltonian H that is constructed from gates. Once combined, the gates form a paramaterised quantum circuit which is paramaterised by using the variables governing each gate, θ. By tuning the values of θ_i one can allow for an evolution to any state that will serve as a solution to a given problem.
By taking the distribution associated to the state, $|\psi(\theta)\rangle$ we can treat the PQC as a generative model, which upon measuring in a given basis, will generate samples of a target distribution χ. This model is paramaterised by θ, which defines a quantum circuit $U(\theta)$ made up of a set of quantum gates such that:

$$|\psi(\theta)\rangle = U(\theta)|0\rangle^{\otimes n}$$

By measuring the circuit, we can obtain samples. Producing samples that emulate the target distribution involves minimising the parameters of the circuit $U(\theta)$, a process once convergence is reached, will generate accurate samples. [**?**]

4.2 State Preparation

State preperation is a core part of the hybrid quantum computing model. This concerns the most efficient methods to input classical data into a quantum system. Without the use of ancillary qubits, we can expect an exponentional circuit depth to prepare an arbitrary quantum state. Using them we can reduce the depth to be subexponential scaling, with recent advancements reaching $\Theta(n)$ given $O(n^2)$ ancillary qubits. [**?**] [**?**] We require state preperation to transfer the classical data, bits, onto the Hilbert space. This involves a function ϕ that maps the input vector to an output label. There are many encoding schemes, each of which aim to offer high information density and low error rates. The main methods of state preperation include: basis, amplitude, angle encoding, and QRAM.

4.2.1 Amplitude Encoding

$$|\psi_x\rangle = \sum_{i-1}^{N} x_i|i\rangle$$

4.2.2 Angle Encoding

$$|x\rangle = \otimes_{i=1}^{N} cos(x_i)|0\rangle + sin(x_i)|1\rangle$$

5 Quantum Architectures

5.1 Measurements

5.1.1 Complete Measurements

In quantum mechanics, we can define measurement to be any process that probes a given quantum system to obtain information, we may also refer to this process as a measurement in the computational basis when focussing on quantum information science. Let's consider a quantum state $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$ which gives us $|0\rangle$ with probability $|\alpha_0|^2$ and $|1\rangle$ with probability $|\alpha_1|^2$. If measured in the standard basis we would expect the outcome to be $|k\rangle$ (for k = 0,1)with a given probability. This outcome would result in the output state of the measurement gate to also be $|k\rangle$, resulting in the original state $|\psi\rangle$ to be irreversibly lost. We can refer to the process the state undergoes as a collapse of state.

5.2 Weak Measurements

Consider an arbitrary system observable A. Assume a probe $[\hat{q}.\hat{p}] = i$ in a minimum uncertainty state, defined by σ_p^{in},$\bar{p}^{in}$, and $\bar{q}^{in} = 0$. We can then assume a Von Neumann type interaction between the observable of interest and the position of the particle, consequence being the probe receiving a momentum kick when it interacts with the system and that the change in the momentum is exactly equal to the observable to be measured, $\hat{A}$. Mathematically, we can define $\hat{A}(p^t) = p^t - p^{in}$ to be the estimate we get for A from measuring probe $\bar{p}^t$. Then for intial system state $|\psi^{in}\rangle$, $E[\psi_{in}|A(p^t)|\psi^{in}] = \langle|\hat{A}|\psi^{in}\rangle$.
Now consider a final projective measurement on the system too, considering the sub-enemble where the system is found in state $|\phi^t\rangle$. Then we can consider the post-selected average $E[A(p^t)|\psi^{in}, \phi^t]$. In the weak measurement limit, $\sigma_p \to \infty$ we get the expectation value for the estimate of the observable A given by:

$$E[A(p^t)|\psi^{in}, \phi^t] \to Re\frac{\langle\phi^y|\hat{A}|\psi^{in}}{\langle\phi^t|\psi^t\rangle}$$

The amount of disturbance to any system operator due to the coupling of the probe is very small.

Can we use weak measurement to sample the qcbm continuously?
look at the weak measurement protocol. Different to the standard orthogonal projection.

6 Quantum Resevoir Computing

7 Deep Hedging

8 Bibliography

To sort out later:
Born machine: https://link.springer.com/article/10.1007/s42484-022-00063-3
Stateprep1: https://arxiv.org/pdf/2201.11495
Stateprep2: https://arxiv.org/pdf/2108.06150