Analysis of the Quantum Advantages for Deep Hedging

Soham Deshpande
Supervisor: Dr. Srinandan Dasmahapatra
Second Supervisor: Dr. Hansung Kim

April 2025

A project report submitted for the award of MEng Computer Science

Abstract

Parameterised Quantum Circuits (PQCs) have opened many doors, one such being the use in financial markets. In this paper, I look at the problem of developing an accurate market generator through the use of quantum computing for the purposes of hedging. Given a Quantum Circuit Born Machine (QCBM), we are able to exploit the high expressibility to generate synthetic data that mimics the statistical distribution of the original dataset. The market generator is then used to simulate an underlying asset to maturity with the intent of learning an optimal hedging strategy π^*, a showcase of a data-driven approach to hedging exposure. I show that the synthetic data produced by this method has shown to capture the fat tails of the market better than classical methods, as well as demonstrating superiority in out-of-sample testing with COVID data. Different generator architectures have been compared to maximise the quality of the synthetic data and avoid issues with barren plateaus. The findings of this research will contribute to the growing literature on risk management in quantitative finance, with applications of the market generator extending beyond deep hedging.

Contents

1 Problem Statement

The problem of hedging a portfolio of derivatives is an important part of risk management used widely in financial institutions. This involves understanding the exposure to the market, and taking strategic positions to negate some of this risk. In an ideal world we can picture a perfect, frictionless market where transaction costs are negligible and every asset in the space has a price; here we can price and hedge perfectly. Unfortunately in practice, we experience incomplete markets due to frictions, costs that interfere with trades such as transaction costs or imperfect information. In addition, recent years have presented markets with periods of heightened volatility, much that disobey traditional frameworks. This generates the need for complex, realistic market models that can account for these.

Traditional methods of hedging options has shown to be ineffective for equity markets, new information resulting in rapid changes. Much of the available literature models the market as a smooth, continuous stochastic process within a Gaussian space. Such models are sufficient for common market activity but fail when presented with discontinuous moves in price. These can be reactions to geopolitical events or natural disasters; traditional models are incapable of capturing the effects. The introduction of Jump-diffusion models aimed to solve this issue though face similar issues. In reaction, we have recently observed non-parametric models that harness neural networks and machine learning which aim to demonstrate high accuracy on out-of-sample forecasts.

An alternative approach that has recently emerged utilises the power of quantum computing. The introduction of parameterised quantum circuits(PQCs) have opened up new pathways for navigating complex, large scale time series datasets. Through rotation gates and quantum entanglement, we are able to learn complex distributions and relationships.

In this research, I aim to tackle the problem of generating synthetic financial data, addressing issues that come about from using a classical method particularly the estimation of tail risk and skewness. Through comparisons between traditional approaches, I aim to demonstrate an advantage in the expressibility of Quantum Circuit Born Machines (QCBMs); these will be described quantitatively using measures such as Value at Risk (VaR) and Conditional Value at Risk (CVaR). By performing out-of-sample tests on COVID and Oil data, I aim to highlight the weaknesses of traditional models and showcase a quantum superiority. There will also be an exploration into the variety of architectures available for the QCBM, evaluating different ansatz designs. Where unusual behaviours due to quantum physics occur, such as barren plateau, I will explore in greater detail as well as any circuit optimisation techniques that may present themselves as possible solutions.

This paper will aim to add to the existing literature on risk management for financial firms as well as providing a framework for generating synthetic data. In addition to that, I aim to extend to the QCBM research that exists currently, noting down any behaviours that may be of interest to the curious and potential experts in the field.

1.1 Aims & Goals

This section aims to provide a high-level insight into the goals of the project with more detailed explanations being left for their respective sections.

Market data, though appearing entirely stochastic at first glance, contains many hidden patterns and relationships. Classical methods such as Black-Scholes [**blackscholes**] have been proposed to capture some of this behaviour, though built on unrealistic assumptions, these govern our fundamental understanding of quantitative finance. With the move to the new era of electronic trading, the need for more complex markets models became more present. Being able to trade within nanoseconds presented traders with a new, human-free method of trading, moving to a purely mathematical approach. In this approach we strive for optimal strategies to make money, taking calculated positions and limiting exposure to large market movements. It is here where traditional methods started to be insufficient; particua

2 Related Literature

To place this research within the context of existing literature, we can split the project into 2 components: the market generator, and parameterised quantum circuits.

The work around deep hedging has evolved, moving away from Greek-based hedging towards a sturdier framework using machine learning. Here a lot of work is being done, with many papers emphasising on using neural networks for optimising delta and gamma exposure [1, 33]. Buehler introduced an approach, modelling trading decisions as neural networks instead of relying on parameterised models [8]. Subsequent advancements focussed on developing realistic market simulators. Wissel proposed a market model for path generation of options but this still employed risk-neutral diffusion[38]. Wiese then introduced a new dimension by using GANs to convert options into local volatility models with simpler no-arbitrage constraints. This focussed on the local stochastic nature of options [10, 45, 46]. Some approaches suggest using actor-critic reinforcement learning algorithms to solve for an optimal value function, a move towards searching for a global maximum over local risk management [7, 27].

Recent research explores using quantum computing to hedge portfolios, here the authors presented a quantum reinforcement learning method based on policy-search and distributional actor-critic algorithms. They proposed using a Quantum Neural Network to approximate the value of a given value function by predicting the expected utility of returns using compound and orthogonal layers which were built using Hamming-weight unitaries [23].

TO CHANGE: This helped overcome the barren plateau by ensuring the gradient variance does not vanish exponentially with qubit count.

Another method models the entire return distribution, leveraging parameterised circuits to learn categorical distributions and capture variability and tail risk [9, 12].

There is an immense amount of research being done on exploiting the benefits of quantum computing, recent advancements being in quantum algorithms. These claim to provide exponential speed-up over classical methods, though in reality, we see great complexity in state preparation, requiring $\Theta(2^n/n)$ circuit depth with n qubits or $\Theta(n)$ circuit depth with $\Theta(2^n)$ ancillary qubits[48]. Here we see hybrid models such as Born machines and Quantum generative adversarial networks boasting high generalisation ability [18, 20, 22].

There has also been research in harnessing back action from quantum weak measurements to enhance the ability of quantum machine learning algorithms. In quantum reservoir computing, the ability to retain information from past inputs plays a key role in processing temporal series and producing future predictions [16, 17, 19, 28].

This research aims to combine the needs of financial firms in hedging portfolios using realistic market models by utilising QCBMs as a tool for simulating paths in combination with deep hedging engines for learning optimal policies. A comparison will be made against hedging under Merton-Jump diffusion.

3 Markets and Derivatives

The market, though inherently can be thought of as a completely random process, where bids and asks are fulfilled, can be modelled as a stochastic process. The aim of this chapter is to serve as a brief introduction and set up notation for later chapters.

3.1 Market

Consider a market with a finite time horizon T defined on the probability space $(\Omega, \mathcal{F}, P)$ along with a filtration $\mathbf{F} = \{\mathcal{F}|0 \leq t \leq T\}$ This can be thought of as an adapted (n+1) dimensional Itô process $X(t) = (X_0(t), X_1(t), ..., X_n(t))$ which has the form

$$dX_0(t) = \rho(t, \omega)X_0(t)dt; \quad X_0(0) = 1 \tag{3.1}$$

and

$$dX_i = \mu_i(t, \omega)dt + \sigma_i(t, \omega)dB(t); \quad X_i(0) = x_i \tag{3.2}$$

where $X_i(t)$ as the price of asset i at a given time t.
We can define a portfolio in the market as

$$\theta(t, \omega) = (\theta_0(t, \omega), \theta_1(t, \omega), ..., \theta_n(t, \omega)) \tag{3.3}$$

where the components $\theta_n(t, \omega)$ represents the number of units of a given asset held at time t.
Following from that, we can define the value of a given portfolio to be

$$V(t, \omega) = V^{\theta}(t, \omega) = \sum_{i=0}^{n} \theta_i(t)X_i(t) \tag{3.4}$$

Lastly, it is important to state that the portfolio is self-financing, any trading strategy α requires no extra cost beyond the initial capital

$$V(t) = V(0) + \int_0^t \theta(s) \cdot dX(t) \tag{3.5}$$

We can also make the following assumptions about the market:

- The market is liquid, allowing the trade to execute instantaneously
- There is no bid-ask spread, the price to buy and sell is the same
- Trading actions taken have no impact on the price of the asset traded

3.2 Derivatives

A derivative refers to any financial instrument whose value is derived from an underlying security, the most fundamental being futures and options. It is common practice to refer to the given underlying security as just 'underlying'.

3.2.1 Futures

A futures contract is a contract that gives the right and obligation to buy a given asset i a specified time T at price K.

3.2.2 Options

The two types of options going to be explored are Puts and Calls; a Call option gives the owner the right but not the obligation to buy a given asset i at a specified price K at time T. Similar to the Call, a Put option gives the owner the right but not the obligation to sell a given asset i at a price K at time T. If the owner can exercise the option any time up to T, we call this an American option. For the purposes of this research, I will only be dealing with vanilla European options.
It is important to define the payoffs for both options:

$$C_T = max(0, S_T - K) \tag{3.6}$$

$$P_T = max(0, K - S_T) \tag{3.7}$$

3.3 Market Data

In this research I will be focussing on hedging a portfolio consisting of a single asset, hence requiring a simulation of a single underlying.

3.3.1 Euro Stoxx 50

The Euro Stoxx 50 Index (SX5E) and relevant derivatives. This is a stock index of 50 stocks in the Eurozone. This index captures around 60% of the free-float market capitalisation of the Euro Stoxx Total Market Index which covers about 95% of the free-float market in the Eurozone[14]. Rationale behind choosing this index is the availability of data, options traded with SX5E as the underlying and the liquidity of the index.
Derivatives that are held in the portfolio to be hedged will include those that have SX5E as the underlying, examples are weekly, monthly, and quarterly expiration options. These are European-style so can only be exercised upon maturity. Data can be found on Bloomberg[5] and Refinitiv [26].

sx5e.png

Figure 1: Euro Stoxx 50 price chart

3.3.2 Brent Crude Oil

It was appropriate to try and learn a volatile commodity. Sensitive to price movements.

4 Merton-Jump Diffusion Model

My model of choice for comparison is the Merton-Jump Diffusion model, this an elementary model that goes beyond Black-Scholes by trying to capture the negative skewness and excess kurtosis of log price returns. This is done through the addition of a compound Poisson jump process. This aims to represent the jumps we observe in the market in a more realistic fashion rather than assuming constant volatility assumption made by Black-Scholes. As a simplification, I will be referring to Black-Scholes by BS and Merton-Jump Diffusion with MJD.

4.1 Model

A standard derivation of the model will allow us to explore its assumptions and limitations. This model consists of two components, jump and diffusion. The diffusion will be modelled using a Weiner process and log-normal jumps driven by a Poisson process. This gives us the following SDE.

$$dS_t = (\alpha - \lambda k)S_t dt + \sigma S_t dW_t + (y_t - 1)S_t dN_t \tag{4.1}$$

where W_t and N_t are Weiner and Poisson processes respectively. λ represents the intensity of the jumps, α is the drift rate (expected return), and k is the expected jump size. Solving the equation gives us an exponential Lévy model described by

$$S_t = S_0 e^{\mathcal{L}_t} \tag{4.2}$$

where S_t is the stock price at time t, S_0 is the initial stock price. We can also define $\mathcal{L}_t$ to be

$$\mathcal{L}_t = (\alpha - \frac{\sigma^2}{2} - \lambda k)t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \tag{4.3}$$

4.2 Assumptions & Limitations

Through inspection of the equations, we can observe the following assumptions:

1. The asset price experiences continuous, random fluctuations over time, governed by Brownian motion (GBM)

2. The asset price experiences sudden, discontinuous jumps modelled by a Poisson process, occurring at a constant rate λ

3. Jumps sizes are assumed to be log-normal $ln(y_t) \sim \mathcal{N}(\mu, \sigma^2)$

Starting with the first assumption, we can see that assuming GBM may produce unrealistic behaviour, most important being a lack of excess kurtosis. Markets often exhibit fat tails, especially within commodities. One such event may be the release of news from OPEC+, the organisation of petroleum-exporting countries. A restriction in oil production may cause the price of oil to jump rapidly. In recent times, wars and conflict has also become ever present, causing large movements in asset prices; therefore it is not unrealistic to expect extreme price movements to be more frequent than can be modelled by a Gaussian.

The MJD requires calibration of parameters before use, typically done using historical data or implied volatility surfaces. Once calibrated these become assumptions of the data and so do not change even if the market observations move away from it. This would lead us to expect higher overfitting to the historical data, possibly failing in unseen conditions such as the market's reaction to COVID.

We also may expect poor volatility clustering with the MJD; constant volatility is not experienced by the market, instead periods of high volatility followed by periods of low volatility is observed. Though this paper won't be focussing on this phenomenon, it is important to consider.

4.3 Calibration

In this research, I have chosen to use maximum likelihood estimation to estimate the parameters for the MJD model. In the analytical solution we require five parameters: α, σ, μ_j, δ, and λ. These are the expected return, volatility of the given asset, expectation of the jump size, standard deviation of the jump size and lastly the jump intensity. We can then use MLE on the probability density of log returns $S_t = ln(\frac{S_t}{S_0})$

$$P(S_t) = \sum_{i=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^i}{i!} N(S_t; (\alpha - \frac{\sigma^2}{2} - \lambda k)t + i\mu_j, \sigma^2 t + i\delta^2) \tag{4.4}$$

The likelihood function hence becomes

$$L(\theta; S) = \prod_{t=1}^{T} P(S_t) \tag{4.5}$$

We can minimise the negative log-likelihood to obtain

$$-\ln L(\theta; S) = -\sum_{t=1}^{T} \ln P(S_t) \tag{4.6}$$

Another popular option to calibrate the MJD model is by considering the implied volatility surface of existing options.

5 Parameterised Quantum Circuits

5.1 Parameterised Circuits

Consider a set of unitaries...

5.2 Born Rule

An essential part of quantum computing involves the existence of the Born Rule. Born's measurement rule states that:

$$p(x) = |\langle x|\psi(\theta)\rangle|^2 \tag{5.1}$$

where

$$|\psi(\theta)\rangle = U(\theta)|0\rangle^{\otimes n} \tag{5.2}$$

The state $|\psi(\theta)\rangle$ is generated by evolving state $|0\rangle$ according to a Hamiltonian H that is constructed from gates. Once combined, the gates form a parameterised quantum circuit which is parameterised by using the variables governing each gate, θ. By tuning the values of θ_i one can allow for an evolution to any state that will serve as a solution to a given problem.

By taking the distribution associated to the state, $|\psi(\theta)\rangle$ we can treat the PQC as a generative model, upon measurement will generate samples of a target distribution χ. This model is parameterised by θ, which defines a quantum circuit $U(\theta)$ made up of a set of quantum gates. By measuring the circuit, we can obtain samples. Producing samples that emulate the target distribution involves minimising the parameters of the circuit $U(\theta)$, a process once convergence is reached, will generate accurate samples [25].

5.3 State Preparation

We require state preparation to transfer the classical data onto the Hilbert space. This involves a function ϕ that maps the input vector to an output label. There are many encoding schemes, each of which aim to offer high information density and low error rates; main methods include: basis, amplitude, angle encoding, and QRAM.

Without the use of ancillary qubits, we can expect an exponential circuit depth to prepare an arbitrary quantum state. Using them we can reduce the depth to be sub-exponential scaling, with recent advancements reaching $\Theta(n)$ given $O(n^2)$ ancillary qubits [39, 48].

5.4 Measurements

In quantum mechanics, we can define measurement to be any process that probes a given quantum system to obtain information, we may also refer to this process as a measurement in the computational basis when focussing on quantum information science. Let's consider a quantum state $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$ which gives us $|0\rangle$ with probability $|\alpha_0|^2$ and $|1\rangle$ with probability $|\alpha_1|^2$. If measured in the standard basis we would expect the outcome to be $|k\rangle$ with a given probability. This outcome would result in the output state of the measurement gate to also be $|k\rangle$, resulting in the original state $|\psi\rangle$ to be irreversibly lost. We can refer to this as a collapse of state.

6 Quantum Circuit Born Machine

Given a dataset $D = \{x_1, x_2..x_n\}$ consisting of n samples and obeys a given distribution χ_d, we would like the QCBM to learn the distribution and generate synthetic data points that are of the distribution χ_s such that χ_s approximates χ_d. The QCBM is a subclass of parameterised quantum circuits, here the quantum circuit contains parameters which are updated during a training process. The QCBM takes the product state $|0\rangle$ as an input, and through an evolution, transforms into a final state $|\phi_0\rangle$ by a sequence of unitary gates. This can then be measured to obtain a sample of bits $x \sim p_\theta(x_s) = |\langle x| |\phi_\theta\rangle|^2$. By training the model we are aiming to let p_θ approach χ_d.
The ansatz for this quantum circuit consists of 7 layers of 1-qubit gates with entangling layers in between them. These are entangled using the CNOT gates as found in the appendix. The number of wires needed depends on the precision required for the generated data. The estimated precision is 12-bit, so the samples are able to take 2^{12} different values in the range of $(v_{min} - \epsilon, v_{max} + \epsilon)$, where $\epsilon > 0$ allows data to be generated that lie outside the range (v_{min}, v_{max}) of the original data.
The QCBM takes a $n \times m$ matrix of parameters in the range $(-\pi, \pi)$ as input, in the form of a dictionary. Each angle takes one of 2^k discrete values, where k is a model parameter. The resulting space therefore spans to: $(2^m)^{n \cdot m}$.

6.1 Architectures

6.1.1 Brick

6.1.2 Pyramid

6.1.3 Butterfly

6.2 Barren Plateau

A point of concern when searching for the optimal set of θs is the large search space, here we may observe issues such as barren plateau(BP). BP insists that the gradient of the parameters of a given PQC will vanish exponentially w.r.t the search space.
Introduce proof of $\frac{\partial C}{\partial \theta} \to 0$

6.3 ZX Calculus

ZX Calculus as a method to optimise/minimise t gate count. introduce what a non clifford gate is.

7 Results

7.1 Path Generation

7.1.1 VaR & CVaR

7.2 Hedging

7.3 Barren Plateau

7.3.1 ZX Calculus

8 Outlook and Conclusions

9 Appendix

9.1 QCBM Architectures

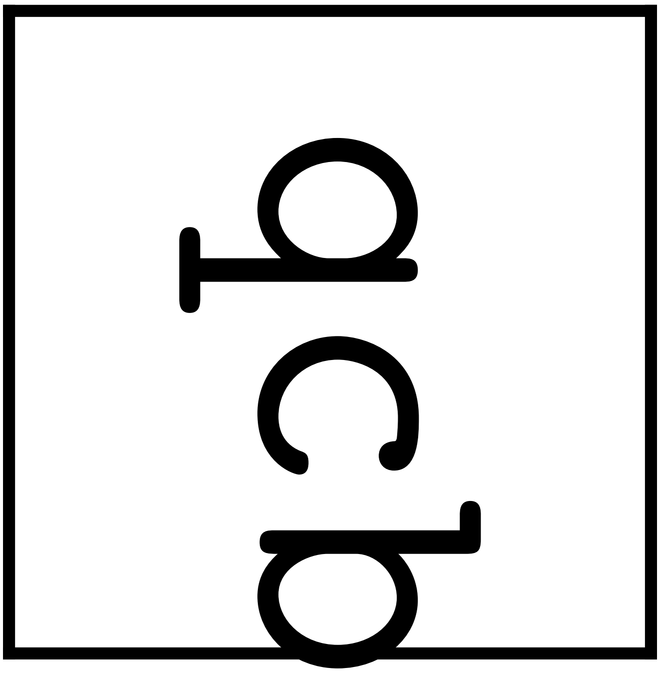

Figure 2: QCBM architecture

References

[1] John Armstrong and George Tatlow. *Deep Gamma Hedging*. Sept. 20, 2024. DOI: 10.48550/arXiv.2409.13567. arXiv: 2409.13567[q-fin]. URL: http://arxiv.org/abs/2409.13567 (visited on 12/04/2024).

[2] Marcello Benedetti et al. "A generative modeling approach for benchmarking and training shallow quantum circuits". In: *npj Quantum Information* 5.1 (May 27, 2019), p. 45. ISSN: 2056-6387. DOI: 10.1038/s41534-019-0157-8. URL: https://www.nature.com/articles/s41534-019-0157-8 (visited on 11/21/2024).

[3] Marcello Benedetti et al. "Erratum: Parameterized quantum circuits as machine learning models (2019 *Quant. Sci. Tech.* **4** 043001)". In: *Quantum Science and Technology* 5.1 (Dec. 4, 2019), p. 019601. ISSN: 2058-9565. DOI: 10.1088/2058-9565/ab5944. URL: https://iopscience.iop.org/article/10.1088/2058-9565/ab5944 (visited on 11/21/2024).

[4] Michael R. Berthold, Ad Feelders, and Georg Krempl, eds. *Advances in Intelligent Data Analysis XVIII: 18th International Symposium on Intelligent Data Analysis, IDA 2020, Konstanz, Germany, April 27–29, 2020, Proceedings*. Vol. 12080. Lecture Notes in Computer Science. Cham: Springer International Publishing, 2020. ISBN: 978-3-030-44583-6 978-3-030-44584-3. DOI: 10.1007/978-3-030-44584-3. URL: http://link.springer.com/10.1007/978-3-030-44584-3 (visited on 12/04/2024).

[5] Bloomberg. *Bloomberg*. Bloomberg.com, 2023. URL: https://www.bloomberg.com/.

[6] Nicolas Boursin, Carl Remlinger, and Joseph Mikael. "Deep Generators on Commodity Markets Application to Deep Hedging". In: *Risks* 11.1 (Dec. 23, 2022), p. 7. ISSN: 2227-9091. DOI: 10.3390/risks11010007. URL: https://www.mdpi.com/2227-9091/11/1/7 (visited on 11/21/2024).

[7] Hans Buehler, Murray Phillip, and Ben Wood. "Deep Bellman Hedging". In: *SSRN Electronic Journal* (2022). ISSN: 1556-5068. DOI: 10.2139/ssrn.4151026. URL: https://www.ssrn.com/abstract=4151026 (visited on 11/21/2024).

[8] Hans Buehler et al. "Deep Hedging: Hedging Derivatives Under Generic Market Frictions Using Reinforcement Learning". In: *SSRN Electronic Journal* (2019). ISSN: 1556-5068. DOI: 10.2139/ssrn.3355706. URL: https://www.ssrn.com/abstract=3355706 (visited on 11/21/2024).

[9] El Amine Cherrat et al. "Quantum Deep Hedging". In: *Quantum* 7 (Nov. 29, 2023), p. 1191. ISSN: 2521-327X. DOI: 10.22331/q-2023-11-29-1191. arXiv: 2303.16585[quant-ph]. URL: http://arxiv.org/abs/2303.16585 (visited on 12/04/2024).

[10] Vedant Choudhary, Sebastian Jaimungal, and Maxime Bergeron. *FuNVol: A Multi-Asset Implied Volatility Market Simulator using Functional Principal Components and Neural SDEs*. Dec. 26, 2023. DOI: 10.48550/arXiv.2303.00859. arXiv: 2303.00859[q-fin]. URL: http://arxiv.org/abs/2303.00859 (visited on 12/04/2024).

[11] Eliahu Cohen. “What Weak Measurements and Weak Values Really Mean: Reply to Kastner”. In: *Foundations of Physics* 47.10 (Oct. 2017), pp. 1261–1266. ISSN: 0015-9018, 1572-9516. DOI: 10.1007/s10701-017-0107-2. URL: http://link.springer.com/10.1007/s10701-017-0107-2 (visited on 11/21/2024).

[12] Kalyan Dasgupta and Binoy Paine. *Loading Probability Distributions in a Quantum circuit*. Aug. 29, 2022. arXiv: 2208.13372[quant-ph]. URL: http://arxiv.org/abs/2208.13372 (visited on 11/21/2024).

[13] Julien Dudas et al. “Quantum reservoir computing implementation on coherently coupled quantum oscillators”. In: *npj Quantum Information* 9.1 (July 7, 2023), p. 64. ISSN: 2056-6387. DOI: 10.1038/s41534-023-00734-4. URL: https://www.nature.com/articles/s41534-023-00734-4 (visited on 11/21/2024).

[14] *EURO STOXX 50*. Wikipedia, Aug. 2021. URL: https://en.wikipedia.org/wiki/EURO_STOXX_50.

[15] Simon Fecamp, Joseph Mikael, and Xavier Warin. “Deep learning for discrete-time hedging in incomplete markets”. In: *The Journal of Computational Finance* (2021). ISSN: 14601559, 17552850. DOI: 10.21314/JCF.2021.006. URL: https://www.risk.net/journal-of-computational-finance/7871526/deep-learning-for-discrete-time-hedging-in-incomplete-markets (visited on 12/04/2024).

[16] Giacomo Franceschetto et al. *Harnessing quantum back-action for time-series processing*. Nov. 6, 2024. arXiv: 2411.03979[quant-ph]. URL: http://arxiv.org/abs/2411.03979 (visited on 11/21/2024).

[17] Keisuke Fujii and Kohei Nakajima. *Quantum reservoir computing: a reservoir approach toward quantum machine learning on near-term quantum devices*. Nov. 10, 2020. arXiv: 2011.04890[quant-ph]. URL: http://arxiv.org/abs/2011.04890 (visited on 11/21/2024).

[18] Santanu Ganguly. “Implementing Quantum Generative Adversarial Network (qGAN) and QCBM in Finance”. In: ().

[19] Jorge García-Beni et al. “Squeezing as a resource for time series processing in quantum reservoir computing”. In: *Optics Express* 32.4 (Feb. 12, 2024), p. 6733. ISSN: 1094-4087. DOI: 10.1364/OE.507684. arXiv: 2310.07406[quant-ph]. URL: http://arxiv.org/abs/2310.07406 (visited on 11/21/2024).

[20] Kaitlin Gili et al. *Do Quantum Circuit Born Machines Generalize?* arXiv.org, 2022. URL: https://arxiv.org/abs/2207.13645.

[21] L. C. G. Govia et al. “Quantum reservoir computing with a single nonlinear oscillator”. In: *Physical Review Research* 3.1 (Jan. 25, 2021), p. 013077. ISSN: 2643-1564. DOI: 10.1103/PhysRevResearch.3.013077. URL: https://link.aps.org/doi/10.1103/PhysRevResearch.3.013077 (visited on 11/21/2024).

[22] Haim Horowitz, Pooja Rao, and Santosh Kumar Radha. *A quantum generative model for multi-dimensional time series using Hamiltonian learning*. Apr. 13, 2022. DOI: 10.48550/arXiv.2204.06150. arXiv: 2204.06150[quant-ph]. URL: http://arxiv.org/abs/2204.06150 (visited on 12/04/2024).

[23] Iordanis Kerenidis, Jonas Landman, and Natansh Mathur. *Classical and Quantum Algorithms for Orthogonal Neural Networks*. Dec. 23, 2022. arXiv: 2106.07198[quant-ph]. URL: http://arxiv.org/abs/2106.07198 (visited on 11/21/2024).

[24] Alex Kondratyev. "Non-Differentiable Leaning of Quantum Circuit Born Machine with Genetic Algorithm". In: *Wilmott* 2021.114 (July 2021), pp. 50–61. ISSN: 1540-6962, 1541-8286. DOI: 10.1002/wilm.10943. URL: https://onlinelibrary.wiley.com/doi/10.1002/wilm.10943 (visited on 11/21/2024).

[25] Jin-Guo Liu and Lei Wang. "Differentiable Learning of Quantum Circuit Born Machine". In: *Physical Review A* 98.6 (Dec. 19, 2018), p. 062324. ISSN: 2469-9926, 2469-9934. DOI: 10.1103/PhysRevA.98.062324. arXiv: 1804.04168[quant-ph]. URL: http://arxiv.org/abs/1804.04168 (visited on 11/21/2024).

[26] *LSEG Workspace*. www.lseg.com. URL: https://www.lseg.com/en/data-analytics/products/workspace.

[27] Ali Asghar Movahed and Houshyar Noshad. "Introducing a new approach for modeling a given time series based on attributing any random variation to a jump event: jump-jump modeling". In: *Scientific Reports* 14.1 (Jan. 12, 2024), p. 1234. ISSN: 2045-2322. DOI: 10.1038/s41598-024-51863-5. URL: https://www.nature.com/articles/s41598-024-51863-5 (visited on 11/21/2024).

[28] Pere Mujal et al. "Time-series quantum reservoir computing with weak and projective measurements". In: *npj Quantum Information* 9.1 (Feb. 23, 2023), p. 16. ISSN: 2056-6387. DOI: 10.1038/s41534-023-00682-z. URL: https://www.nature.com/articles/s41534-023-00682-z (visited on 11/21/2024).

[29] Bernt Øksendal. "Application to Mathematical Finance". In: *Stochastic Differential Equations: An Introduction with Applications*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2003, pp. 269–313. ISBN: 978-3-642-14394-6. DOI: 10.1007/978-3-642-14394-6_12. URL: https://doi.org/10.1007/978-3-642-14394-6_12.

[30] Anupama Padha and Anita Sahoo. "Quantum deep neural networks for time series analysis". In: *Quantum Information Processing* 23.6 (May 24, 2024), p. 205. ISSN: 1573-1332. DOI: 10.1007/s11128-024-04404-y. URL: https://link.springer.com/10.1007/s11128-024-04404-y (visited on 11/21/2024).

[31] Annie E. Paine, Vincent E. Elfving, and Oleksandr Kyriienko. "Quantum Quantile Mechanics: Solving Stochastic Differential Equations for Generating Time-Series". In: *Advanced Quantum Technologies* 6.10 (Oct. 2023), p. 2300065. ISSN: 2511-9044, 2511-9044. DOI: 10.1002/qute.202300065. arXiv: 2108.03190[quant-ph]. URL: http://arxiv.org/abs/2108.03190 (visited on 11/21/2024).

[32] Obdulia Pichardo Lagunas, Juan Martínez-Miranda, and Bella Martínez Seis, eds. *Advances in Computational Intelligence: 21st Mexican International Conference on Artificial Intelligence, MICAI 2022, Monterrey, Mexico, October 24–29, 2022, Proceedings, Part I*. Vol. 13612. Lecture Notes in Computer Science. Cham: Springer Nature Switzerland, 2022. ISBN: 978-3-031-19492-4 978-3-031-19493-1. DOI: 10.1007/978-3-031-19493-1. URL: https://link.springer.com/10.1007/978-3-031-19493-1 (visited on 11/21/2024).

[33] Chunhui Qiao and Xiangwei Wan. *Enhancing Black-Scholes Delta Hedging via Deep Learning*. Aug. 24, 2024. DOI: 10.48550/arXiv.2407.19367. arXiv: 2407.19367[q-fin]. URL: http://arxiv.org/abs/2407.19367 (visited on 12/04/2024).

[34] Lupei Qin, Wei Feng, and Xin-Qi Li. "Simple understanding of quantum weak values". In: *Scientific Reports* 6.1 (Feb. 3, 2016), p. 20286. ISSN: 2045-2322. DOI: 10.1038/srep20286. URL: https://www.nature.com/articles/srep20286 (visited on 11/21/2024).

[35] Michael Ragone et al. "A Lie algebraic theory of barren plateaus for deep parameterized quantum circuits". In: *Nature Communications* 15 (Aug. 2024). DOI: 10.1038/s41467-024-49909-3. (Visited on 11/19/2024).

[36] Samir Saissi Hassani and Georges Dionne. "The New International Regulation of Market Risk: Roles of VaR and CVaR in Model Validation". In: *SSRN Electronic Journal* (2021). ISSN: 1556-5068. DOI: 10.2139/ssrn.3766511. URL: https://www.ssrn.com/abstract=3766511 (visited on 12/04/2024).

[37] Samir Saissi Hassani and Georges Dionne. "The New International Regulation of Market Risk: Roles of VaR and CVaR in Model Validation". In: *SSRN Electronic Journal* (2021). DOI: 10.2139/ssrn.3766511. (Visited on 01/16/2022).

[38] Martin Schweizer and Johannes Wissel. "Arbitrage-free market models for option prices: the multi-strike case". In: *Finance and Stochastics* 12.4 (Oct. 2008), pp. 469–505. ISSN: 0949-2984, 1432-1122. DOI: 10.1007/s00780-008-0068-6. URL: http://link.springer.com/10.1007/s00780-008-0068-6 (visited on 12/04/2024).

[39] Ali Shaib et al. "Efficient noise mitigation technique for quantum computing". In: *Scientific Reports* 13.1 (Mar. 8, 2023), p. 3912. ISSN: 2045-2322. DOI: 10.1038/s41598-023-30510-5. URL: https://www.nature.com/articles/s41598-023-30510-5 (visited on 11/21/2024).

[40] Lina Song. "Dynamic Modeling and Simulation of Option Pricing Based on Fractional Diffusion Equations with Double Derivatives". In: *Computational Economics* (May 26, 2024). ISSN: 0927-7099, 1572-9974. DOI: 10.1007/s10614-024-10628-y. URL: https://link.springer.com/10.1007/s10614-024-10628-y (visited on 12/04/2024).

[41] Apimuk Sornsaeng, Ninnat Dangniam, and Thiparat Chotibut. "Quantum next generation reservoir computing: an efficient quantum algorithm for forecasting quantum dynamics". In: *Quantum Machine Intelligence* 6.2 (Dec. 2024), p. 57. ISSN: 2524-4906, 2524-4914. DOI: 10.1007/s42484-024-00188-7. URL: https://link.springer.com/10.1007/s42484-024-00188-7 (visited on 11/21/2024).

[42] Nikitas Stamatopoulos and William J. Zeng. "Derivative Pricing using Quantum Signal Processing". In: *Quantum* 8 (Apr. 30, 2024), p. 1322. ISSN: 2521-327X. DOI: 10.22331/q-2024-04-30-1322. arXiv: 2307.14310[quant-ph]. URL: http://arxiv.org/abs/2307.14310 (visited on 11/21/2024).

[43] György Steinbrecher and William T. Shaw. "Quantile mechanics". In: *European Journal of Applied Mathematics* 19.2 (Apr. 2008). ISSN: 0956-7925, 1469-4425. DOI: 10.1017/S0956792508007341. URL: http://www.journals.cambridge.org/abstract_S0956792508007341 (visited on 12/04/2024).

[44] Andrew Vlasic. *Quantum Circuits, Feature Maps, and Expanded Pseudo-Entropy: A Categorical Theoretic Analysis of Encoding Real-World Data into a Quantum Computer*. Oct. 29, 2024. DOI: 10.48550/arXiv.2410.22084. arXiv: 2410.22084[quant-ph]. URL: http://arxiv.org/abs/2410.22084 (visited on 12/04/2024).

[45] Magnus Wiese et al. "Deep Hedging: Learning to Simulate Equity Option Markets". In: *SSRN Electronic Journal* (2019). ISSN: 1556-5068. DOI: 10.2139/ssrn.3470756. URL: https://www.ssrn.com/abstract=3470756 (visited on 11/21/2024).

[46] Magnus Wiese et al. *Multi-Asset Spot and Option Market Simulation*. Dec. 13, 2021. DOI: 10.48550/arXiv.2112.06823. arXiv: 2112.06823[q-fin]. URL: http://arxiv.org/abs/2112.06823 (visited on 12/04/2024).

[47] Neil A. Wilmot and Charles F. Mason. "Jump Processes in the Market for Crude Oil". In: *The Energy Journal* 34.1 (Jan. 2013), pp. 33–48. ISSN: 0195-6574, 1944-9089. DOI: 10.5547/01956574.34.1.2. URL: https://journals.sagepub.com/doi/10.5547/01956574.34.1.2 (visited on 11/21/2024).

[48] Xiao-Ming Zhang, Tongyang Li, and Xiao Yuan. "Quantum State Preparation with Optimal Circuit Depth: Implementations and Applications". In: *Physical Review Letters* 129.23 (Nov. 30, 2022), p. 230504. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.129.230504. arXiv: 2201.11495[quant-ph]. URL: http://arxiv.org/abs/2201.11495 (visited on 11/21/2024).

[49] Christa Zoufal, Aurélien Lucchi, and Stefan Woerner. "Quantum Generative Adversarial Networks for learning and loading random distributions". In: *npj Quantum Information* 5.1 (Nov. 22, 2019), p. 103. ISSN: 2056-6387. DOI: 10.1038/s41534-019-0223-2. URL: https://www.nature.com/articles/s41534-019-0223-2 (visited on 11/21/2024).