Analysis of the Quantum Advantages for Deep Hedging

Soham Deshpande
Srinandan Dasmahapatra

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1 Abstract

Deep hedging

2 Hedging

Reword:

Classical derivative pricing models all rely on mathematical path generators, however these are not designed to be realistic, instead they describe diffusion in a risk-neutral measure, rather than the real-world measure, with variables being kept to a minimal for easy computation.

Mathematical set up for hedging:

3 Quantum Computing

4 Parameterised Circuits

4.1 Born Rule

An essential part of modern quantum computing involves the existence of the Born Rule. Born's measurement rule states that:

$$p(x) = |\langle x|\psi(\theta)|\rangle|^2$$

The state $|\psi(\theta)\rangle$ is generated by evolving the vacuum state $|0\rangle$ according to a Hamiltonian H that is constructed from gates. Once combined, the gates form a paramaterised quantum circuit which is paramaterised by using the variables governing each gate, θ. By tuning the values of θ_i one can allow for an evolution to any state that will serve as a solution to a given problem.
By taking the distribution associated to the state, $|\psi(\theta)\rangle$ we can treat the PQC as a generative model, which upon measuring in a given basis, will generate samples of a target distribution χ. This model is paramaterised by θ, which defines a quantum circuit $U(\theta)$ made up of a set of quantum gates such that:

$$|\psi(\theta)\rangle = U(\theta)|0\rangle^{\otimes n}$$

By measuring the circuit, we can obtain samples. Producing samples that emulate the target distribution involves minimising the parameters of the circuit $U(\theta)$, a process once convergence is reached, will generate accurate samples. [?]

4.2 State Preparation

State preperation is a core part of the hybrid quantum computing model. This concerns the most efficient methods to input classical data into a quantum system. Without the use of ancillary qubits, we can expect an exponentional circuit depth to prepare an arbitrary quantum state. Using them we can reduce the depth to be subexponential scaling, with recent advancements reaching $\Theta(n)$ given $O(n^2)$ ancillary qubits. [?] [?] We require state preperation to transfer the classical data, bits, onto the Hilbert space. This involves a function ϕ that maps the input vector to an output label. There are many encoding schemes, each of which aim to offer high information density and low error rates. The main methods of state preperation include: basis, amplitude, angle encoding, and QRAM.

4.2.1 Amplitude Encoding

$$|\psi_x\rangle = \sum_{i-1}^{N} x_i|i\rangle$$

4.2.2 Angle Encoding

$$|x\rangle = \otimes_{i=1}^{N} cos(x_i)|0\rangle + sin(x_i)|1\rangle$$

5 Quantum Architectures

5.1 Measurement and Weak Measurement

Can we use weak measurement to sample the qcbm continuously?
look at the weak measurement protocol. Different to the standard orthogonal projection.

6 Quantum Resevoir Computing

7 Deep Hedging

8 Bibliography

To sort out later:
Born machine: https://link.springer.com/article/10.1007/s42484-022-00063-3
Stateprep1: https://arxiv.org/pdf/2201.11495
Stateprep2: https://arxiv.org/pdf/2108.06150