Analysis of the Quantum Advantages in Deep Hedging

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1 Background

The problem of hedging a portfolio of derivatives is an important part of risk management used widely in financial institutions. We can picture a perfect, frictionless market where transaction costs are negligible and every asset in the space has a price. Here we can price and hedge perfectly. Unfortunately in practice, we experience incomplete markets due to frictions. This generates the need for complex, realistic market models that can account for these. Hedging can be thought of as a stochastic control problem where we have 3 actors, a market state, randomness from market returns and a hedging strategy. By making observations on the market state we are required to make adjustments to one's hedging policy.
We can formally define hedging as :

$$\min_{\theta \in \Theta} \rho(H^\theta - P)$$

where Θ is the set of admissible hedging strategies, P is the uncertain payoff of derivative at final time, H^θ is the uncertain payoff of hedging portfolio if strategy θ is used at final time, and ρ is a risk measure.

In recent times, the rise of quantum computing has become prevalent, with new methods being engineered at an unparalleled pace. As is the pace at which data is being consumed, particularly within the financial space. The number of variables being used to make the "perfect" model grows exponentially, therefore does the need for more high-quality data. Here quantum computing is being exploited for its power to accurately represent probability distributions despite a scarcity of data.

2 Brief

In the deep hedging framework, we are able to construct a dynamic optimal hedging policy by following market simulations to maturity. This is a policy that involves a mapping from market states to actions. Instead of solving analytical equations under unrealistic market assumptions, such as Greek hedging, we can instead model the trading decisions made as an optimisation problem, particularly suitable for neural networks with a rich feature set. This will be done by focusing on the accuracy of the market simulation itself; I will aim to utilise quantum computing techniques to enhance market simulations so that a more optimal hedging strategy can be learnt than prior techniques. In this, an exploration into quantum circuit architectures will be conducted, as well as exploiting quantum machine learning for the simulation and policy search.

3 Benchmarks

Conditional Value-at-Risk(CVaR) is a mathematical tool used to quantify the risk of an investment portfolio. CVaR gives us the expected loss on the portfolio conditioned on the event that a catastrophic tail event occurs. We can define this:

$$\frac{1}{1-\alpha} \int_{-\infty}^{VaR(\alpha)} x \cdot p(x) dx$$

where $p(x)$ is the pdf of getting a return with value x, α is the confidence level.
Another intuitive tool we can use is Profit & Loss (PnL). This would give an objective function that displays the overall use to the business. We can compare the PnL generated from hedging under Black-Scholes and classical deep-learning approaches, to those generated from quantum circuits.

4 Scope

The scope of this project is limited to deep hedging and derivative volatility, with a focus on market simulation. Hedging is a vast subject and so is not feasible to explore the entirety of the space; basic assumptions will be considered to narrow the problem space.