GATE 2023 IN 37Q

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Question: The Laplace transform of the continuous-time signal $x(t) = e^{-3t}u(t-5)$ is ______, where $u(t)$ denotes the continuous-time unit step signal.

Solution: Laplace transform of $e^{-at}u(t)$ is:

$$X(s) = \int_0^\infty x(t)\, e^{-st} dt \tag{1}$$

$$X(s) = \int_0^\infty u(t)\, e^{-at} e^{-st} dt \tag{2}$$

$$X(s) = \int_0^\infty e^{-(s+a)t} dt \tag{3}$$

$$X(s) = \frac{-1}{s+a}\left(\lim_{t\to\infty} e^{-(s+a)t} - 1\right) \tag{4}$$

$$\therefore X(s) = \frac{1}{s+a} \quad \Re(s) > -a \tag{5}$$

Let,

$$g(t) \xleftrightarrow{\mathcal{Z}} G(s) \tag{6}$$

$$h(t) = g(t - t_0) \tag{7}$$

$$h(t) \xleftrightarrow{\mathcal{Z}} H(s) \tag{8}$$

then,

$$H(s) = \int_0^\infty g(t - t_0)\, e^{-st} dt \tag{9}$$

$$H(s) = \int_{-t_0}^\infty g(t)\, e^{-s(t+t_0)} dt \tag{10}$$

$$H(s) = e^{-st_0} \int_0^\infty g(t)\, e^{-s(t)} dt \tag{11}$$

$$H(s) = e^{-st_0} G(s) \tag{12}$$

Parameter	Description	Value
$x(t)$	Given Function	$x(t) = e^{-3t}u(t)$
$X(s)$	Laplace Transform of $x(t)$	$\frac{-e^{-5(s+3)}}{s+3}$

TABLE 1: Table of parameters

$$x(t) = e^{-15} e^{-3(t-5)} u(t-5) \tag{13}$$

if $g(t) = e^{-3(t)}u(t)$ then by (5) and (12),

$$x(t) = e^{-15} g(t-5) \tag{14}$$

$$X(s) = e^{-15} e^{-5s} G(s) \tag{15}$$

$$\therefore X(s) = \frac{e^{-5(s+3)}}{s+3} \quad \Re(s) > -3 \tag{16}$$