GATE 2022 BM 14 Q

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Question: $x(t)$ is a real continuous-time signal whose magnitude frequency response $|X(j\Omega)|$ is shown below. After sampling $x(t)$ at 100 $rad.s^{-1}$, the spectral point P is down-converted to _____ $rad.s^{-1}$ in the spectrum of the sampled signal. (GATE 2022 BM 14 Q)

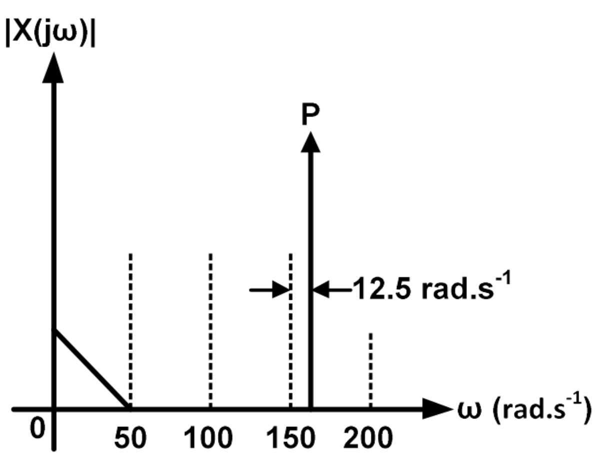

Fig. 1: Plot of $|X(j\omega)|$

Solution:

Parameter	Description
$w(t)$	Sampling Function
$W(s)$	Fourier Transform of $w(t)$
$x(t)$	Input Signal
$X(s)$	Input Signal Frequency Spectrum
$x_s(t)$	Sampled Input Signal
$X_s(s)$	Sampled Signal Frequency Spectrum

TABLE 1: Table of parameters

The sampling function is:

$$w(t) = \sum_{k=-\infty}^{\infty} \delta\left(t - \frac{2\pi k}{100}\right) \tag{1}$$

$$W(j\omega) = 100 \sum_{k=-\infty}^{\infty} \delta\left(j\left(\omega - 100k\right)\right) \tag{2}$$

then the sampled function:

$$x_s(t) = x(t)\, w(t) \tag{3}$$

$$X_s(j\omega) = X(j\omega) * W(j\omega) \tag{4}$$

$$X_s(j\omega) = \int_{-\infty}^{\infty} X(j\theta)\, W(j(\omega - \theta))\, d\theta \tag{5}$$

$$X_s(j\omega) = 100 \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} X(j\theta)\, \delta\left(j\left(\omega - 100k - \theta\right)\right) d\theta \tag{6}$$

$$X_s(j\omega) = 100 \sum_{k=-\infty}^{\infty} X\left(j\left(\omega - 100k\right)\right) \tag{7}$$

Thus, The down sampled point is at:

$$\omega = |162.5 - 100k| \tag{8}$$

where k is the nearest integer to $\frac{162.5}{100}$, which is 2
Thus,

$$\omega = 37.5\; rad\; s^{-1} \tag{9}$$

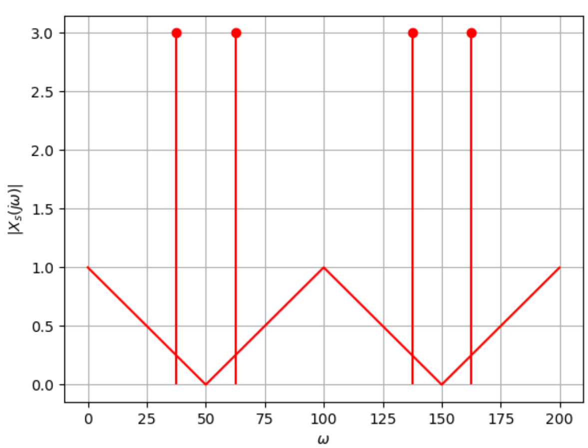

Fig. 2: Plot of $|X_s(j\omega)|$