GATE 2023 IN 37Q

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Question: The Laplace transform of the continuous-time signal $\bar{x}(t) = e^{-3t}u(t-5)$ is _____, where u(t) denotes the continuous-time unit step signal.

Solution: Laplace Transfrom is given by:

$$X(s) = \int_0^\infty x(t) e^{-st} dt \tag{1}$$

$$X(s) = \int_0^\infty u(t-5) e^{-3t} e^{-st} dt$$
 (2)

$$X(s) = \int_{5}^{\infty} e^{-(s+3)t} dt \tag{3}$$

$$X(s) = \frac{-1}{s+3} \left(\lim_{t \to \infty} e^{-(s+3)t} - e^{-5(s+3)} \right)$$
(4)
$$\therefore X(s) = \frac{e^{-5(s+3)}}{s+3} \quad \Re(s) > -3$$
(5)

$$\therefore X(s) = \frac{e^{-5(s+3)}}{s+3} \quad \Re(s) > -3 \tag{5}$$

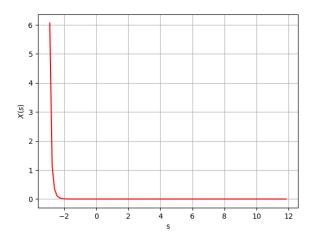


Fig. 1: Plot of X(s) vs s. See Table 1

Parameter	Description	Value
x(t)	Given Function	$x(t) = e^{-3t}u(t)$
X(s)	Laplace Transform of $x(t)$	$\frac{-e^{-5(s+3)}}{s+3}$

TABLE 1: Table of parameters