

# NCERT 11.9.5

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## Question:

Which term of the following sequences:

(a)  $2, 2\sqrt{2}, 4, \dots$  is 128 (b)  $\sqrt{3}, 3, 3\sqrt{3}, \dots$  is 729

(c)  $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$  is  $\frac{1}{19683}$

**Answer:** (a) Let  $x_1(0) = 2$ ,  $x_1(1) = 2\sqrt{2}$ ,  $x_1(2) = 4$ . Since,  $\frac{x_1(1)}{x_1(0)} = \frac{x_1(2)}{x_1(1)}$ , the sequence  $x_1(n)$  is a G.P Series. Let  $r_1 = \frac{x_1(2)}{x_1(1)} = \sqrt{2}$ , then the general term is  $x_1(n) = x_1(0) r_1^{n-1}$ .

Assume  $n^{\text{th}}$  term is 128, which gives:

$$x_1(n) = x_1(0) r_1^{n-1} = 128 \quad (1)$$

$$\Rightarrow r_1^{n-1} = \frac{128}{x_1(0)} \quad (2)$$

$$\Rightarrow n - 1 = \log_{r_1} \frac{128}{x_1(0)} \quad (3)$$

Using values from Table 1,

$$\Rightarrow n - 1 = \log_{\sqrt{2}} \frac{128}{2} \quad (4)$$

$$\Rightarrow n - 1 = \log_{\sqrt{2}} 64 \quad (5)$$

$$\Rightarrow n - 1 = \log_{\sqrt{2}} \sqrt{2}^{12} \quad (6)$$

$$\Rightarrow n - 1 = 12 \quad (7)$$

$$\therefore n = 13 \quad (8)$$

Thus the  $13^{\text{th}}$  term of the G.P  $x_1(n)$  is 128.

$$x_1(n) = x_1(0) r_1^n u[n] \quad (9)$$

where  $u[0] = 1$ .

Let  $x(n) = x(0) r^n u[n]$  be a general sequence then it's Z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \quad (10)$$

$$\Rightarrow X(z) = \sum_{n=0}^{\infty} x(0) r^n z^{-n} \quad (11)$$

$$\Rightarrow X(z) = x(0) \sum_{n=0}^{\infty} r^n z^{-n} \quad (12)$$

$$\Rightarrow X(z) = \frac{x(0)}{r} \left( \frac{1}{1 - \frac{r}{z}} \right) \quad (13)$$

$$\therefore X(z) = \frac{x(0)z}{r(z-r)} \forall |z| > |r| \quad (14)$$

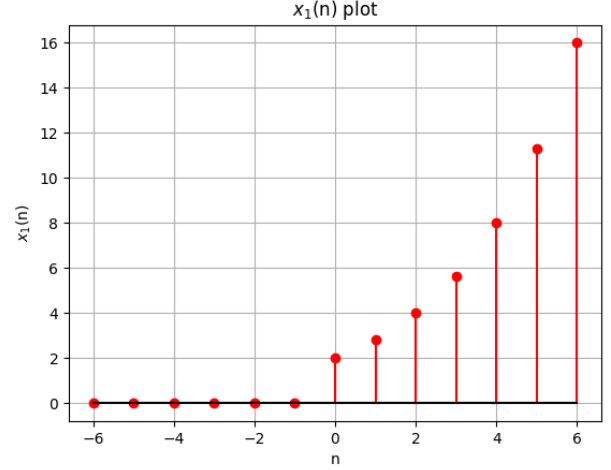


Fig. 1: Plot of  $x_1(n)$  from  $n = -6$  to 6

$$\therefore X_1(z) = \frac{\sqrt{2}z}{z - \sqrt{2}} \forall |z| > \sqrt{2} \quad (15)$$

with ROC:

$$|z| > \sqrt{2}$$

(b) Let  $x_2(0) = \sqrt{3}$ ,  $x_2(1) = 3$ ,  $x_2(2) = 3\sqrt{3}$ .

Since  $\frac{x_2(1)}{x_2(0)} = \frac{x_2(2)}{x_2(1)}$ , the sequence  $x_2(n)$  is a G.P Series. Let  $r_2 = \frac{x_2(2)}{x_2(1)} = \sqrt{3}$ , then the general term is  $x_2(n) = x_2(0) r_2^{n-1}$ .

Assume  $n^{\text{th}}$  term is 729, which gives:

$$x_2(n) = x_2(0) r_2^{n-1} = 729 \quad (16)$$

$$\Rightarrow r_2^{n-1} = \frac{729}{x_2(0)} \quad (17)$$

$$\Rightarrow n - 1 = \log_{r_2} \frac{729}{x_2(0)} \quad (18)$$

Using values from Table 1,

$$\Rightarrow n - 1 = \log_{\sqrt{3}} \frac{729}{\sqrt{3}} \quad (19)$$

$$\Rightarrow n - 1 = \log_{\sqrt{3}} \frac{3^6}{\sqrt{3}} \quad (20)$$

$$\Rightarrow n - 1 = \log_{\sqrt{3}} \sqrt{3}^{11} \quad (21)$$

$$\Rightarrow n - 1 = 11 \quad (22)$$

$$\therefore n = 12 \quad (23)$$

Thus the 12<sup>th</sup> term of the G.P  $x_2(n)$  is 729.

$$x_2(n) = x_2(0)r_2^n u[n] \quad (24)$$

By eqn 14, the Z-transform of  $x_b(n)$ :

$$X_2(z) = \frac{z}{z - \sqrt{3}} \quad (25)$$

with ROC:

$$|z| > \sqrt{3}$$

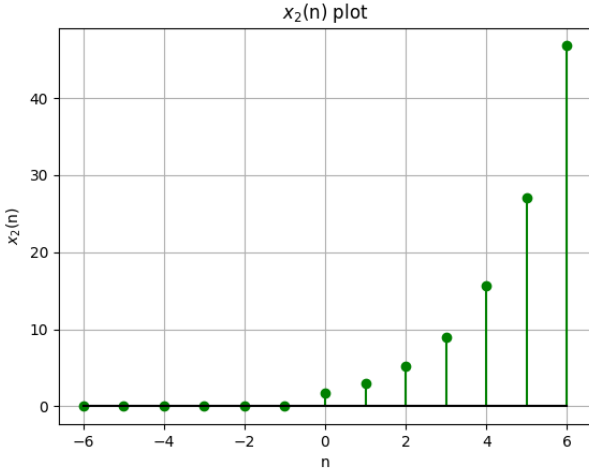


Fig. 2: Plot of  $x_b(n)$  from  $n = -6$  to 6

Thus the 9<sup>th</sup> term of the G.P  $x_3(n)$  is  $\frac{1}{19683}$ .

$$x_3(n) = x_3(0)r_3^n u[n] \quad (34)$$

By eqn 14, the Z-transform of  $x_3(n)$ :

$$X_3(z) = \frac{z}{z - \frac{1}{3}} \implies X_3(z) = \frac{3z}{3z - 1} \quad (35)$$

with ROC:

$$|z| > \frac{1}{3}$$

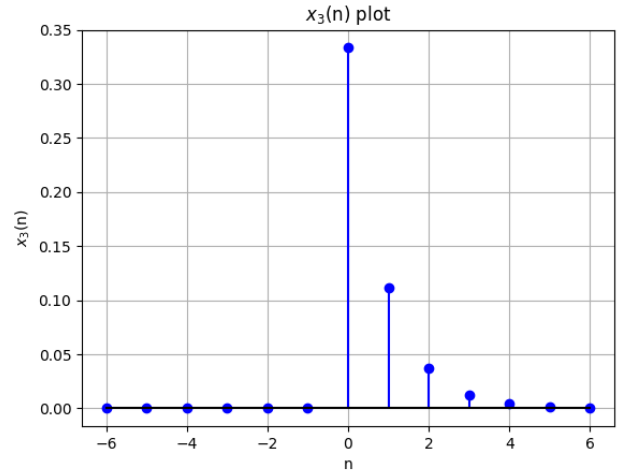


Fig. 3: Plot of  $x_3(n)$  from  $n = -6$  to 6

(c) Let  $x_3(0) = \frac{1}{3}$ ,  $x_3(1) = \frac{1}{9}$ ,  $x_3(2) = \frac{1}{27}$ .

Since  $\frac{x_3(1)}{x_3(0)} = \frac{x_3(2)}{x_3(1)}$ , the sequence  $x_3(n)$  is a G.P Series. Let  $r_3 = \frac{x_3(1)}{x_3(0)} = \frac{1}{3}$ , then the general term is  $x_3(n) = x_3(0)r_3^{n-1}$ .

Assume  $n^{\text{th}}$  term is  $\frac{1}{19683}$ , which gives:

$$x_3(n) = x_3(0)r_3^{n-1} = \frac{1}{19683} \quad (26)$$

$$\implies r_3^{n-1} = \frac{1}{19683x_3(0)} \quad (27)$$

$$\implies n - 1 = \log_{r_3} \frac{1}{19683x_3(0)} \quad (28)$$

Using values from Table 1,

$$\implies n - 1 = \log_{\frac{1}{3}} \frac{1}{19683 \cdot \frac{1}{3}} \quad (29)$$

$$\implies n - 1 = \log_{\frac{1}{3}} \frac{1}{6561} \quad (30)$$

$$\implies n - 1 = \log_{\frac{1}{3}} 3^{-8} \quad (31)$$

$$\implies n - 1 = 8 \quad (32)$$

$$\therefore n = 9 \quad (33)$$

Parameter	Description	Value
$r_1, r_2, r_3$	Common ratio of G.P (a),(b),(c)	$\sqrt{2}, \sqrt{3}, \frac{1}{3}$
$x_i(n)$	Sequence	$x_i(0)r_i^n u[n]$
$X_i(z)$	Transform of $x_i(n)$	—

TABLE 1: Table of parameters