

NCERT 11.9.5

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Question:

Which term of the following sequences:

(a) $2, 2\sqrt{2}, 4, \dots$ is 128 (b) $\sqrt{3}, 3, 3\sqrt{3}, \dots$ is 729

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$ is $\frac{1}{19683}$

Answer: For a general GP series and $k > 0$,

$$x(k) = x(0) r^k \quad (1)$$

$$\therefore k = \log_r \frac{x(k)}{x(0)} \quad (2)$$

And the Z-transform $X(z)$:

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r| \quad (3)$$

(a) By Table 1, (2) and Table 1:

$$x_1(n) = x_1(0) r_1^n u(n) \quad (4)$$

$$k_1 = \log_{r_1} \frac{128}{x_1(0)} \quad (5)$$

$$\therefore k_1 = 12 \quad (6)$$

$$X_1(z) = \frac{2}{1 - \sqrt{2}z^{-1}} \quad |z| > \sqrt{2} \quad (7)$$

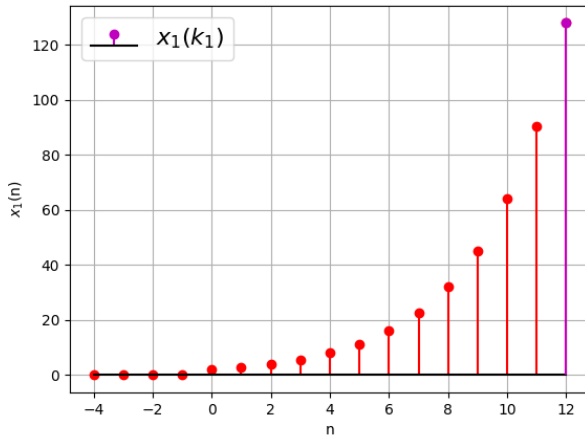


Fig. 1: Plot of $x_1(n)$ vs n . See Table 1

(b) By (2), (3) and Table 1:

$$x_2(n) = x_2(0) r_2^n u(n) \quad (8)$$

$$k_2 = \log_{r_2} \frac{729}{x_2(0)} \quad (9)$$

$$\therefore k_2 = 11 \quad (10)$$

$$X_2(z) = \frac{\sqrt{3}}{1 - \sqrt{3}z^{-1}} \quad |z| > \sqrt{3} \quad (11)$$

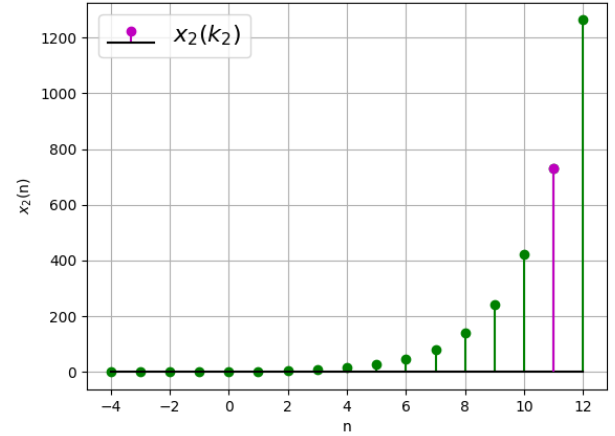


Fig. 2: Plot of $x_2(n)$ vs n . See Table 1

(c) By (2), (3) and Table 1:

$$x_3(n) = x_3(0) r_3^n u(n) \quad (12)$$

$$k_3 = \log_{r_3} \frac{1}{19683 x_3(0)} \quad (13)$$

$$\therefore k_3 = 8 \quad (14)$$

$$X_3(z) = \frac{1}{3 - z^{-1}} \quad |z| > \frac{1}{3} \quad (15)$$

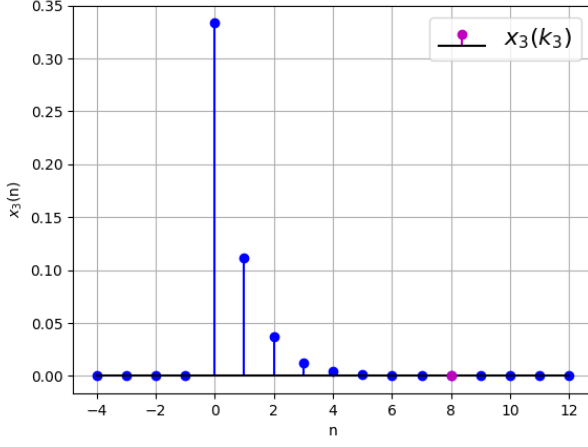


Fig. 3: Plot of $x_3(n)$ vs n . See Table 1

Parameter	Description	Value
r_i	Common ratio of G.P (a),(b),(c)	$\sqrt{2}, \sqrt{3}, \frac{1}{3}$
$x_i(0)$	Initial Values	$2, \sqrt{3}, \frac{1}{3}$
$x_i(k_i)$	Given Values	$128, 729, \frac{1}{19683}$
k_i	Desired index	12, 11, 8
$x_i(n)$	Series	$x_i(0) r_i^n u(n)$
$X_i(z)$	Z-Transform of $x_i(n)$	$\frac{x_i(0)}{1-r_i z^{-1}}$

TABLE 1: Table of parameters