

GATE 2023 IN 37Q

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Question: The Laplace transform of the continuous-time signal $x(t) = e^{-3t}u(t-5)$ is _____, where $u(t)$ denotes the continuous-time unit step signal.

Solution: Laplace transform of $e^{-at}u(t)$ is:

$$X(s) = \int_0^{\infty} x(t) e^{-st} dt \quad (1)$$

$$X(s) = \int_0^{\infty} u(t) e^{-at} e^{-st} dt \quad (2)$$

$$X(s) = \int_0^{\infty} e^{-(s+a)t} dt \quad (3)$$

$$X(s) = \frac{-1}{s+a} \left(\lim_{t \rightarrow \infty} e^{-(s+a)t} - 1 \right) \quad (4)$$

$$\therefore X(s) = \frac{1}{s+a} \quad \Re(s) > -a \quad (5)$$

Laplace transform of $h(t) = g(t-t_0)$ is given by:

$$H(s) = \int_0^{\infty} g(t-t_0) e^{-st} dt \quad (6)$$

$$H(s) = \int_{-t_0}^{\infty} g(t) e^{-s(t+t_0)} dt \quad (7)$$

$$H(s) = e^{-st_0} \int_0^{\infty} g(t) e^{-st(t)} dt \quad (8)$$

$$H(s) = e^{-st_0} G(s) \quad (9)$$

$$g(t) \xleftrightarrow{\mathcal{Z}} G(s) \quad (10)$$

$$x(t) = e^{-15} e^{-3(t-5)} u(t-5) \quad (11)$$

if $g(t) = e^{-3(t)}u(t)$ then by (5) and (9),

$$x(t) = e^{-15} g(t-5) \quad (12)$$

$$X(s) = e^{-15} e^{-5s} G(s) \quad (13)$$

$$\therefore X(s) = \frac{e^{-5(s+3)}}{s+3} \quad \Re(s) > -3 \quad (14)$$

Parameter	Description	Value
$x(t)$	Given Function	$x(t) = e^{-3t}u(t)$
$X(s)$	Laplace Transform of $x(t)$	$\frac{-e^{-5(s+3)}}{s+3}$

TABLE 1: Table of parameters