NCERT 11.9.5

ee23btech11223 - Soham Prabhakar More

Question:
Which term of the following sequences:
(a) $2, 2\sqrt{2}, 4 \dots$ is 128 (b) $\sqrt{3}, 3, 3\sqrt{3} \dots$ is 729
(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27} \dots$ is $\frac{1}{19683}$

Derivations: For a general GP series:

$$x(k) = x(0)\, r^k u(k) \tag{1}$$

Assuming $x(k) = v$ $(k > 0)$,

$$x(k) = x(0)\, r^k = v \tag{2}$$

$$\therefore k = \log_r \frac{v}{x(0)} \tag{3}$$

And the Z-transform $X(z)$:

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad \forall \quad |z| > |r| \tag{4}$$

Answer: (a) Let $x_1(0) = 2$, $r_1 = \sqrt{2}$, then:

$$x_1(n) = x_1(0)\, r_1^n u(n) \tag{5}$$

By eqn 3, Table 1 and $v = 128$:

$$k = \log_{r_1} \frac{128}{x_1(0)} \tag{6}$$

$$\therefore k = 12 \tag{7}$$

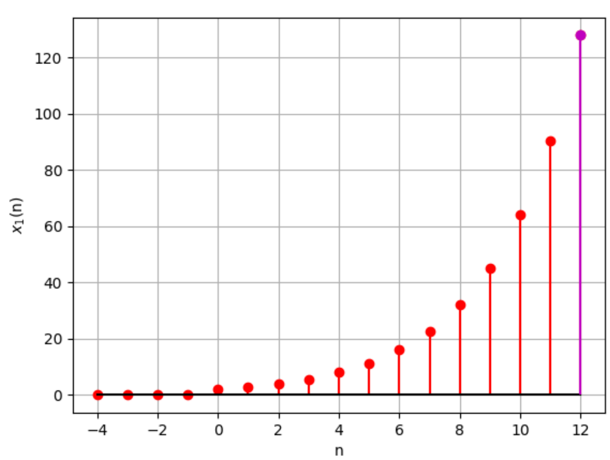

Fig. 1: Plot of $x_1(n)$ vs n. See Table 1

By eqn 4:

$$\therefore X_1(z) = \frac{2}{1 - \sqrt{2}z^{-1}} \quad \forall \quad |z| > \sqrt{2} \tag{8}$$

(b) Let $x_2(0) = \sqrt{3}$, $r_2 = \sqrt{3}$, then:

$$x_2(n) = x_2(0)\, r_2^n u(n) \tag{9}$$

By eqn 3, Table 1 and $v = 729$:

$$k = \log_{r_2} \frac{729}{x_2(0)} \tag{10}$$

$$\therefore k = 11 \tag{11}$$

By eqn 4, the Z-transform of $x_2(n)$:

$$X_2(z) = \frac{\sqrt{3}}{1 - \sqrt{3}z^{-1}} \quad \forall \quad |z| > \sqrt{3} \tag{12}$$

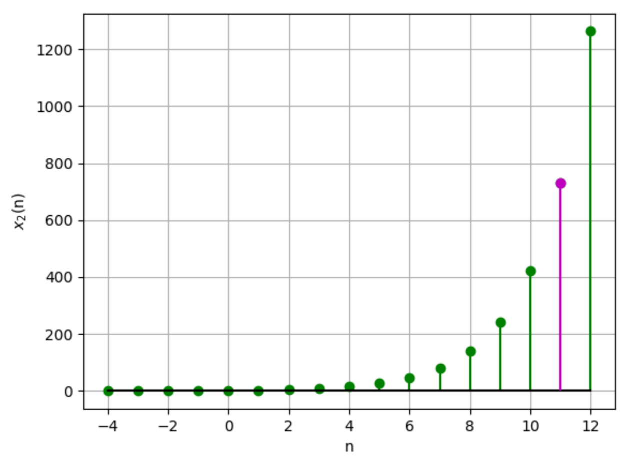

Fig. 2: Plot of $x_2(n)$ vs n. See Table 1

(c) Let $x_3(0) = \frac{1}{3}$, $r_3 = \frac{1}{3}$, then:

$$x_3(n) = x_3(0)\, r_3^n u(n) \tag{13}$$

By eqn 3, Table 1 and $v = \frac{1}{19683}$:

$$k = \log_{r_3} \frac{1}{19683 x_3(0)} \tag{14}$$

$$\therefore k = 8 \tag{15}$$

By eqn 4, the Z-transform of $x_3(n)$:

$$\therefore X_3(z) = \frac{1}{3 - z^{-1}} \quad \forall \quad |z| > \frac{1}{3} \tag{16}$$

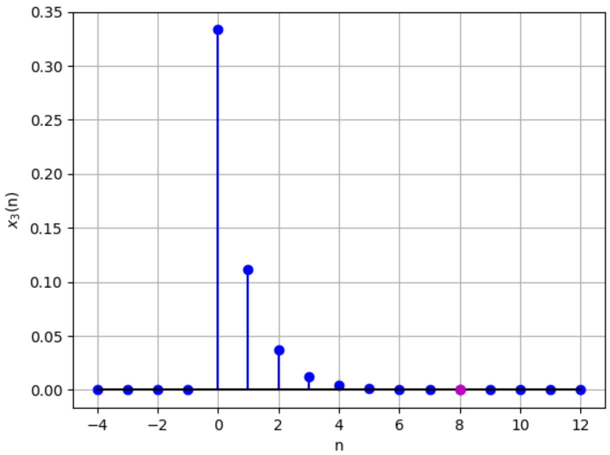

Fig. 3: Plot of $x_3(n)$ vs n. See Table 1

Parameter	Description	Value
r_i	Common ratio of G.P (a),(b),(c)	$\sqrt{2},\ \sqrt{3},\ \frac{1}{3}$
$x_i\left(n\right)$	Sequence	$x_i\left(0\right)r_i^n u[n]$
$X_i\left(z\right)$	Z-Transform of $x_i\left(n\right)$	$\frac{x(0)}{1-rz^{-1}}$

TABLE 1: Table of parameters