

NCERT 11.9.5

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Question:

Which term of the following sequences:

(a) $2, 2\sqrt{2}, 4, \dots$ is 128 (b) $\sqrt{3}, 3, 3\sqrt{3}, \dots$ is 729

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$ is $\frac{1}{19683}$

Answer: (a) Let $x_1(0) = 2$, $r_1 = \frac{2\sqrt{2}}{2} = \sqrt{2}$, then the general term is:

$$x_1(n) = x_1(0) r_1^n u[n] \quad (1)$$

where $u[0] = 1$. Assume n^{th} ($n > 0$) term is 128:

$$x_1(n) = x_1(0) r^n = 128 \quad (2)$$

$$\Rightarrow r^n = \frac{128}{x_1(0)} \quad (3)$$

$$\Rightarrow n = \log_{r_1} \frac{128}{x_1(0)} \quad (4)$$

Using values from Table 1,

$$\Rightarrow n = \log_{\sqrt{2}} \frac{128}{2}$$

$$\Rightarrow n = \log_{\sqrt{2}} 64$$

$$\Rightarrow n = \log_{\sqrt{2}} \sqrt{2}^{12}$$

$$\therefore n = 12$$

Thus the 13th term of the G.P $x_1(n)$ is 128.

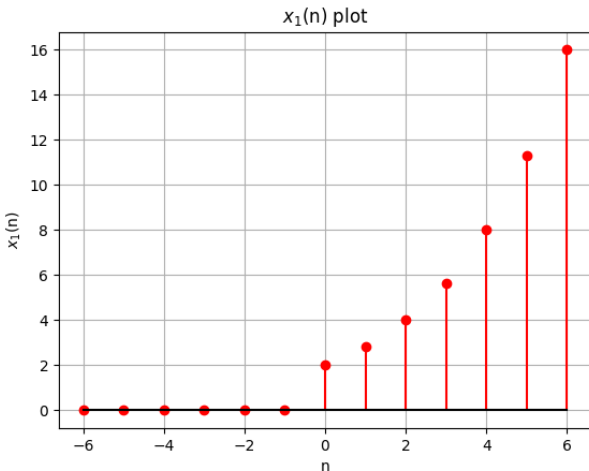


Fig. 1: Plot of $x_1(n)$ from $n = -6$ to 6

Let $x(n) = x(0) r^n u[n]$ be a general sequence then it's Z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \quad (9)$$

$$\Rightarrow X(z) = \sum_{n=0}^{\infty} x(0) r^n z^{-n} \quad (10)$$

$$\Rightarrow X(z) = x(0) \sum_{n=0}^{\infty} r^n z^{-n} \quad (11)$$

$$\Rightarrow X(z) = \frac{x(0)}{r} \left(\frac{1}{1 - \frac{r}{z}} \right) \quad (12)$$

$$\therefore X(z) = \frac{x(0)z}{r(z-r)} \quad \forall |z| > |r| \quad (13)$$

$$\therefore X_1(z) = \frac{\sqrt{2}z}{z - \sqrt{2}} \quad (14)$$

(5) with ROC:

$$|z| > \sqrt{2}$$

(6)

(7) (b) Let $x_2(0) = \sqrt{3}$, $r_2 = \frac{3}{\sqrt{3}} = \sqrt{3}$, then the

(8) general term is:

$$x_2(n) = x_2(0) r_2^n u[n] \quad (15)$$

where $u[0] = 1$. Assume n^{th} ($n > 0$) term is 729, which gives:

$$x_2(n) = x_2(0) r_2^n = 729 \quad (16)$$

$$\Rightarrow r_2^n = \frac{729}{x_2(0)} \quad (17)$$

$$\Rightarrow n = \log_{r_2} \frac{729}{x_2(0)} \quad (18)$$

Using values from Table 1,

$$\Rightarrow n = \log_{\sqrt{3}} \frac{729}{\sqrt{3}} \quad (19)$$

$$\Rightarrow n = \log_{\sqrt{3}} \frac{3^6}{\sqrt{3}} \quad (20)$$

$$\Rightarrow n = \log_{\sqrt{3}} \sqrt{3}^{11} \quad (21)$$

$$\therefore n = 11 \quad (22)$$

Thus the 12th term of the G.P $x_2(n)$ is 729.

By eqn 13, the Z-transform of $x_2(n)$:

$$X_2(z) = \frac{z}{z - \sqrt{3}} \quad (23)$$

with ROC:

$$|z| > \sqrt{3}$$

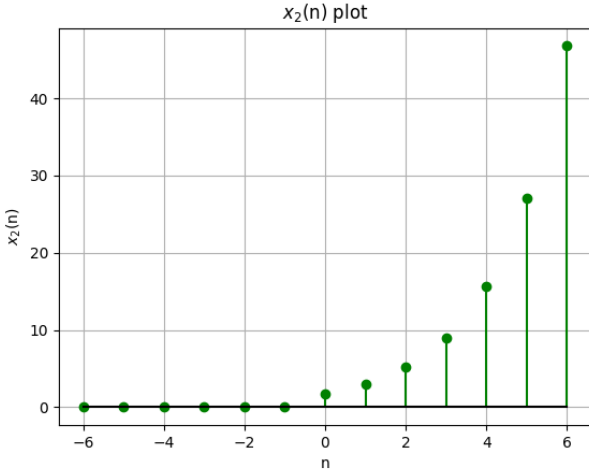


Fig. 2: Plot of $x_2(n)$ from $n = -6$ to 6

(c) Let $x_3(0) = \frac{1}{3}$, $r_3 = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$, then the general term is:

$$x_3(n) = x_3(0) r_3^n u[n] \quad (24)$$

where $u[0] = 1$. Assume n^{th} ($n > 0$) term is $\frac{1}{19683}$, which gives:

$$x_3(n) = x_3(0) r_3^n = \frac{1}{19683} \quad (25)$$

$$\Rightarrow r_3^n = \frac{1}{19683 x_3(0)} \quad (26)$$

$$\Rightarrow n = \log_{r_3} \frac{1}{19683 x_3(0)} \quad (27)$$

Using values from Table 1,

$$\Rightarrow n = \log_{\frac{1}{3}} \frac{1}{19683 \frac{1}{3}} \quad (28)$$

$$\Rightarrow n = \log_{\frac{1}{3}} \frac{1}{6561} \quad (29)$$

$$\Rightarrow n = \log_{\frac{1}{3}} 3^{-8} \quad (30)$$

$$\therefore n = 8 \quad (31)$$

Thus the 9th term of the G.P $x_3(n)$ is $\frac{1}{19683}$.
By eqn 13, the Z-transform of $x_3(n)$:

$$X_3(z) = \frac{z}{z - \frac{1}{3}} \Rightarrow X_3(z) = \frac{3z}{3z - 1} \quad (32)$$

with ROC:

$$|z| > \frac{1}{3}$$

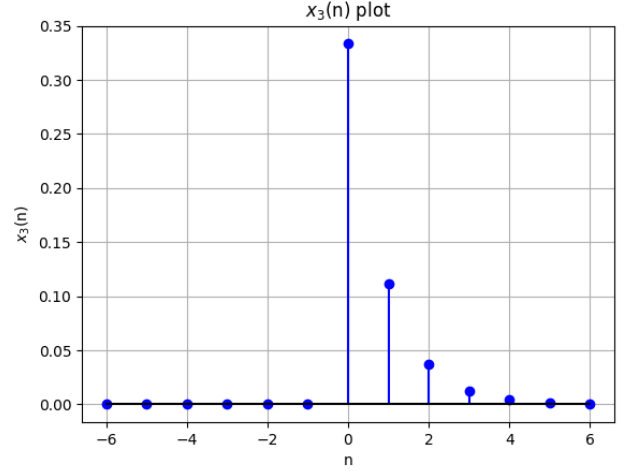


Fig. 3: Plot of $x_3(n)$ from $n = -6$ to 6

Parameter	Description	Value
r_1, r_2, r_3	Common ratio of G.P (a),(b),(c)	$\sqrt{2}, \sqrt{3}, \frac{1}{3}$
$x_i(n)$	Sequence	$x_i(0) r_i^n u[n]$
$X_i(z)$	Transform of $x_i(n)$	—

TABLE 1: Table of parameters