NCERT 11.9.5

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Question:
Which term of the following sequences:
(a) $2,2\sqrt{2},4\ldots$ is 128 (b) $\sqrt{3},3,3\sqrt{3}\ldots$ is 729
(c) $\frac{1}{3},\frac{1}{9},\frac{1}{27}\ldots$ is $\frac{1}{19683}$

Answer: For a general GP series and $k > 0$,

$$x(k) = x(0)\, r^k \tag{1}$$

$$\therefore k = \log_r \frac{x(k)}{x(0)} \tag{2}$$

And the Z-transform $X(z)$:

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r| \tag{3}$$

(a) By Table 1, (2) and Table 1:

$$x_1(n) = x_1(0)\, r_1^n u(n) \tag{4}$$

$$k_1 = \log_{r_1} \frac{128}{x_1(0)} \tag{5}$$

$$\therefore k_1 = 12 \tag{6}$$

$$X_1(z) = \frac{2}{1 - \sqrt{2}z^{-1}} \quad |z| > \sqrt{2} \tag{7}$$

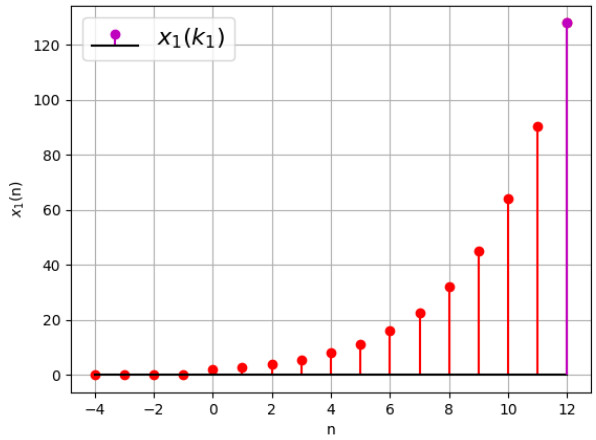

Fig. 1: Plot of $x_1(n)$ vs n. See Table 1

(b) By (2), (3) and Table 1:

$$x_2(n) = x_2(0)\, r_2^n u(n) \tag{8}$$

$$k_2 = \log_{r_2} \frac{729}{x_2(0)} \tag{9}$$

$$\therefore k_2 = 11 \tag{10}$$

$$X_2(z) = \frac{\sqrt{3}}{1 - \sqrt{3}z^{-1}} \quad |z| > \sqrt{3} \tag{11}$$

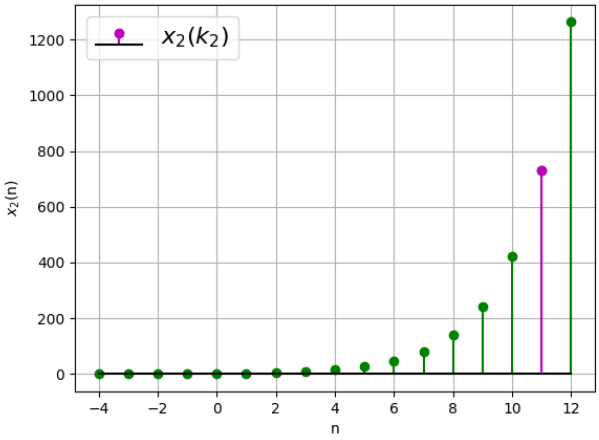

Fig. 2: Plot of $x_2(n)$ vs n. See Table 1

(c) By (2), (3) and Table 1:

$$x_3(n) = x_3(0)\, r_3^n u(n) \tag{12}$$

$$k_3 = \log_{r_3} \frac{1}{19683 x_3(0)} \tag{13}$$

$$\therefore k_3 = 8 \tag{14}$$

$$X_3(z) = \frac{1}{3 - z^{-1}} \quad |z| > \frac{1}{3} \tag{15}$$

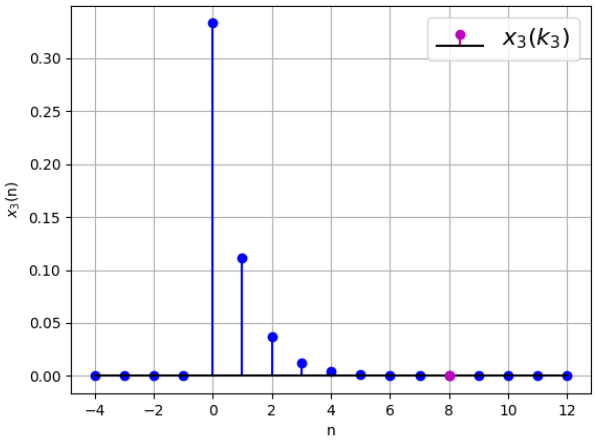

Fig. 3: Plot of $x_3(n)$ vs n. See Table 1

Parameter	**Description**	**Value**
r_i	Common ratio of G.P (a),(b),(c)	$\sqrt{2},\ \sqrt{3},\ \frac{1}{3}$
$x_i(0)$	Initial Values	$2,\ \sqrt{3},\ \frac{1}{3}$
$x_i(k_i)$	Given Values	$128, 729, \frac{1}{19683}$
k_i	Desired index	$12, 11, 8$
$x_i\,(n)$	Series	$x_i\,(0)\,r_i^n u\,(n)$
$X_i\,(z)$	Z-Transform of $x_i\,(n)$	$\frac{x(0)}{1-rz^{-1}}$

TABLE 1: Table of parameters