GATE 2023 IN 37Q

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Question: The Laplace transform of the continuous-time signal $x(t) = e^{-3t}u(t-5)$ is ______, where u(t) denotes the continuous-time unit step signal.

Solution:

Parameter	Description	Value
x(t)	Given Function	$x(t) = e^{-3t}u(t)$
X(s)	Laplace Transform of $x(t)$	$\frac{e^{-5(s+3)}}{s+3}$

TABLE 1: Table of parameters

$$e^{-3t}u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+3} \quad \Re(s) > -3$$
 (1)

Using time shifting,

$$e^{-3(t-5)}u(t-5) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{e^{-5s}}{s+3} \tag{2}$$

$$e^{-15}e^{-3(t-5)}u(t-5) \stackrel{\mathcal{L}}{\longleftrightarrow} e^{-15}\frac{e^{-5s}}{s+3}$$

$$e^{-3t}u(t-5) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{e^{-5(s+3)}}{s+3}$$
(3)

$$e^{-3t}u(t-5) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{e^{-5(s+3)}}{s+3} \tag{4}$$

$$\therefore x(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{e^{-5(s+3)}}{s+3} \quad \Re(s) > -3 \quad (5)$$

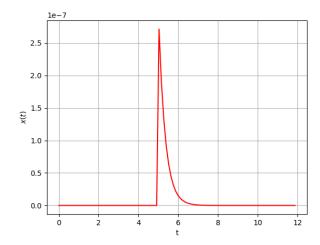


Fig. 3: Plot of x(t) vs t. See Table 1