GATE 2023 IN 37Q

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Question: The Laplace transform of the continuous-time signal $x(t) = e^{-3t}u(t-5)$ is _____, where $u(t)$ denotes the continuous-time unit step signal.

Solution: Laplace Transfrom is given by:

$$X(s) = \int_0^\infty x(t)\, e^{-st} dt \tag{1}$$

$$X(s) = \int_0^\infty u(t-5)\, e^{-3t} e^{-st} dt \tag{2}$$

$$X(s) = \int_5^\infty e^{-(s+3)t} dt \tag{3}$$

$$X(s) = \frac{-1}{s+3}\left(\lim_{t\to\infty} e^{-(s+3)t} - e^{-5(s+3)}\right) \tag{4}$$

$$\therefore X(s) = \frac{e^{-5(s+3)}}{s+3} \quad \Re(s) > -3 \tag{5}$$

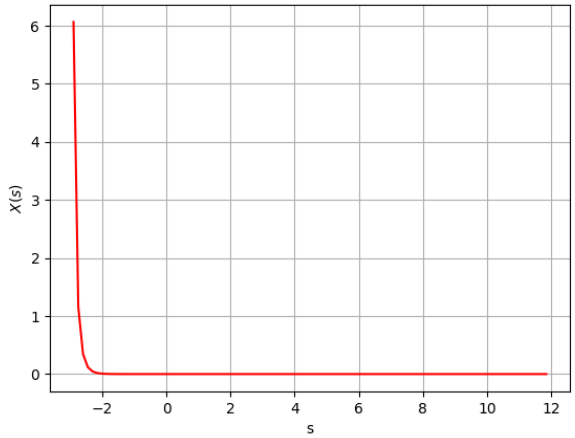

Fig. 1: Plot of $X(s)$ vs s. See Table 1

Parameter	Description	Value
$x(t)$	Given Function	$x(t) = e^{-3t}u(t)$
$X(s)$	Laplace Transform of $x(t)$	$\frac{-e^{-5(s+3)}}{s+3}$

TABLE 1: Table of parameters