

# GATE 2021 ME 3Q

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**Question:** The Dirac-delta function ( $\delta(t - t_0)$ ) for  $t, t_0 \in \mathbb{R}$ , has the following property

$$\int_a^b \phi(t) \delta(t - t_0) dt = \begin{cases} \phi(t_0) & a < t_0 < b \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The Laplace Transform of the Dirac-delta function  $\delta(t - a)$  for  $a > 0$ ;  $\mathcal{L}(\delta(t - a)) = F(s)$  is

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**Solution:**

Parameter	Description
$F(s)$	Laplace transform of $\delta(t - a)$
$G(f)$	Fourier transform of $\delta(t - a)$
$H(t)$	Fourier transform of a function with period $T$
$w_T(t)$	Delta Comb, $\sum_{k=-\infty}^{\infty} \delta(t - kT)$
$W_T(f)$	Fourier transform of $w_T(t)$

TABLE 1: Table of parameters

By (1) and  $a > 0$ ,

$$F(s) = \int_0^{\infty} \delta(t - a) e^{-st} dt \quad (2)$$

$$\therefore F(s) = e^{-as} \quad (3)$$

The fourier transform,

$$G(f) = \int_{-\infty}^{\infty} \delta(t - a) e^{-2\pi jft} dt \quad (4)$$

$$\therefore G(f) = e^{-j2\pi fa} \quad (5)$$

For a periodic signal the fourier transform is defined as:

$$H(f) = \sum_{k=-\infty}^{\infty} c_k \delta\left(f - \frac{k}{T}\right) \quad (6)$$

where  $c_k$  are the fourier series coefficients and  $T$  is the period. Thus,

$$W_T(f) = \sum_{k=-\infty}^{\infty} c_k \delta\left(f - \frac{k}{T}\right) \quad (7)$$

$$c_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} w_T(t) e^{-j2\pi \frac{k}{T} f} dt \quad (8)$$

$$c_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left( \sum_{k=-\infty}^{\infty} \delta(t - kT) \right) e^{-j2\pi \frac{k}{T} f} dt \quad (9)$$

$$(10)$$

$$c_k = \frac{1}{T} \sum_{k=-\infty}^{\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t - kT) e^{-j2\pi \frac{k}{T} f} dt \quad (11)$$

$$c_k = \frac{1}{T} \quad (12)$$

$$W_T(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right) \quad (13)$$

$$\therefore W_T(f) = \frac{1}{T} w_{\frac{1}{T}}(f) \quad (14)$$

Thus, the fourier transform of impulse train is another impulse train.