Assignment 3
Image Segmentation

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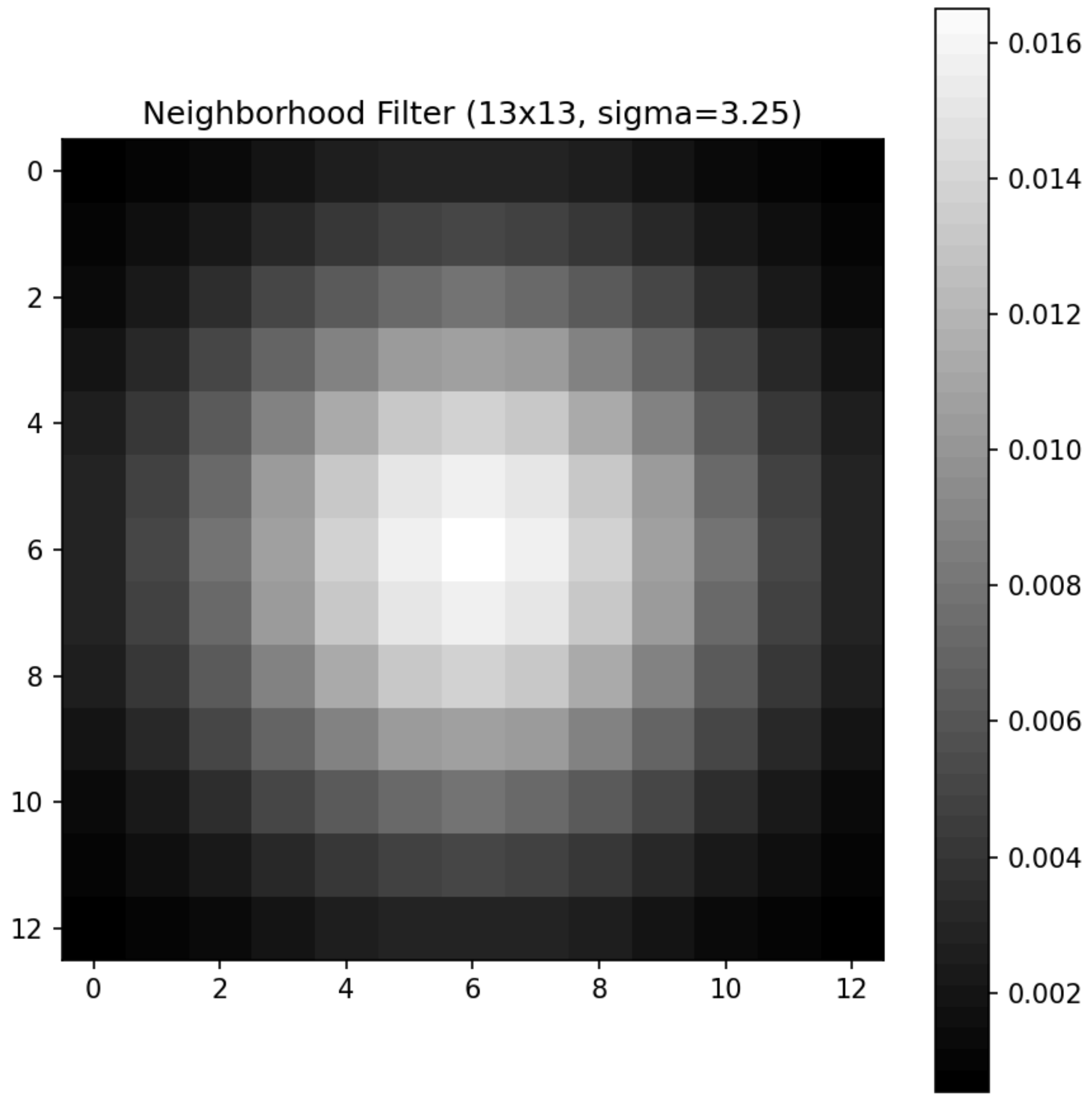
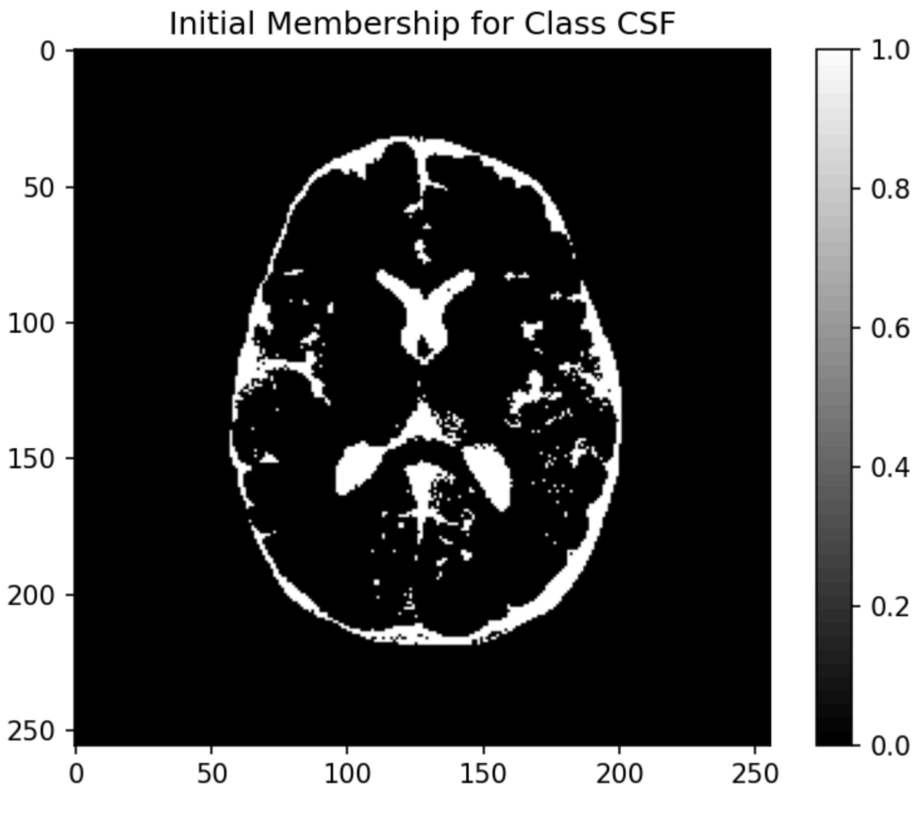
Question 1:

a) Chosen value of q = 2

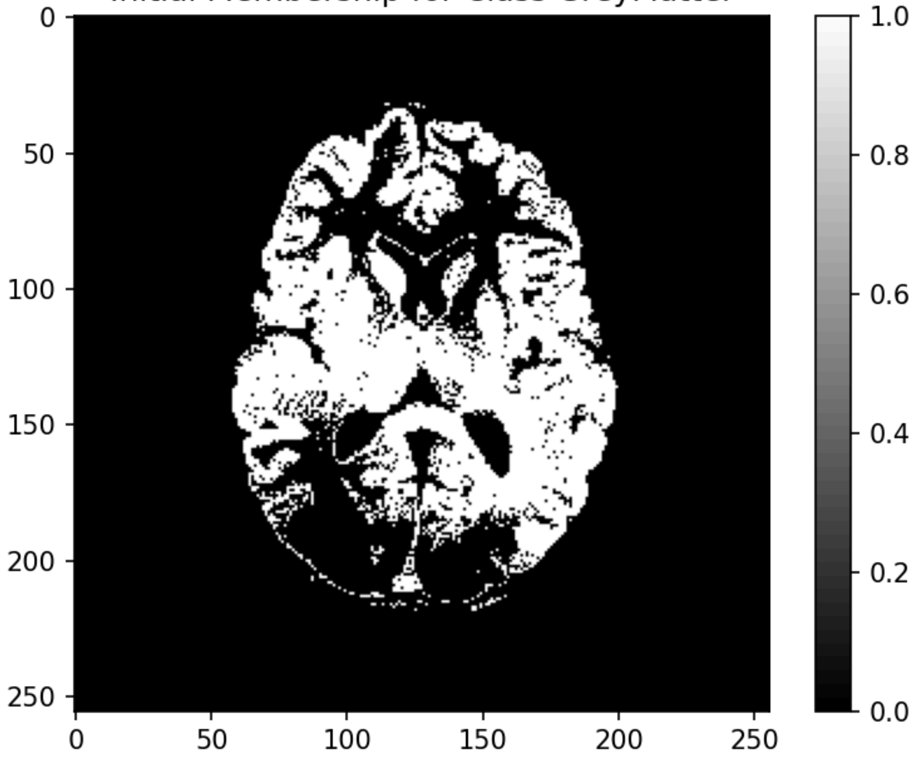
b) The mask is 13 X 13 with standard deviation of 3.25



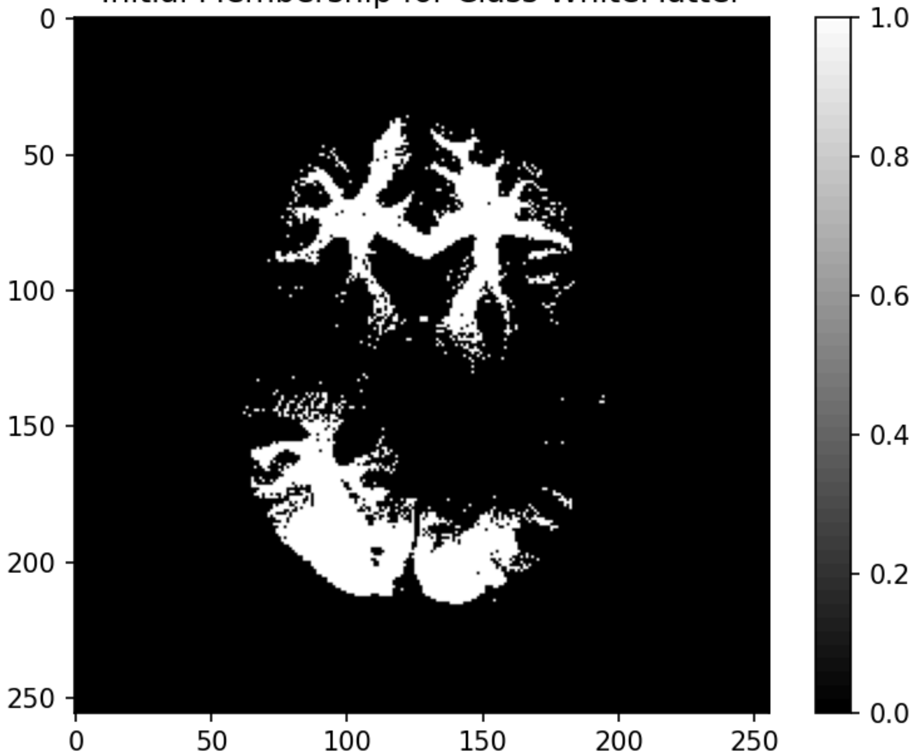
c) We initialise means with K means algorithm. Then we used these means to make initialise membership of each pixel to be crisp such that it is 1 for closest mean and 0 for others. We thought this was a good starting point instead of initialising them from the random.



Initial Membership for Class GreyMatter

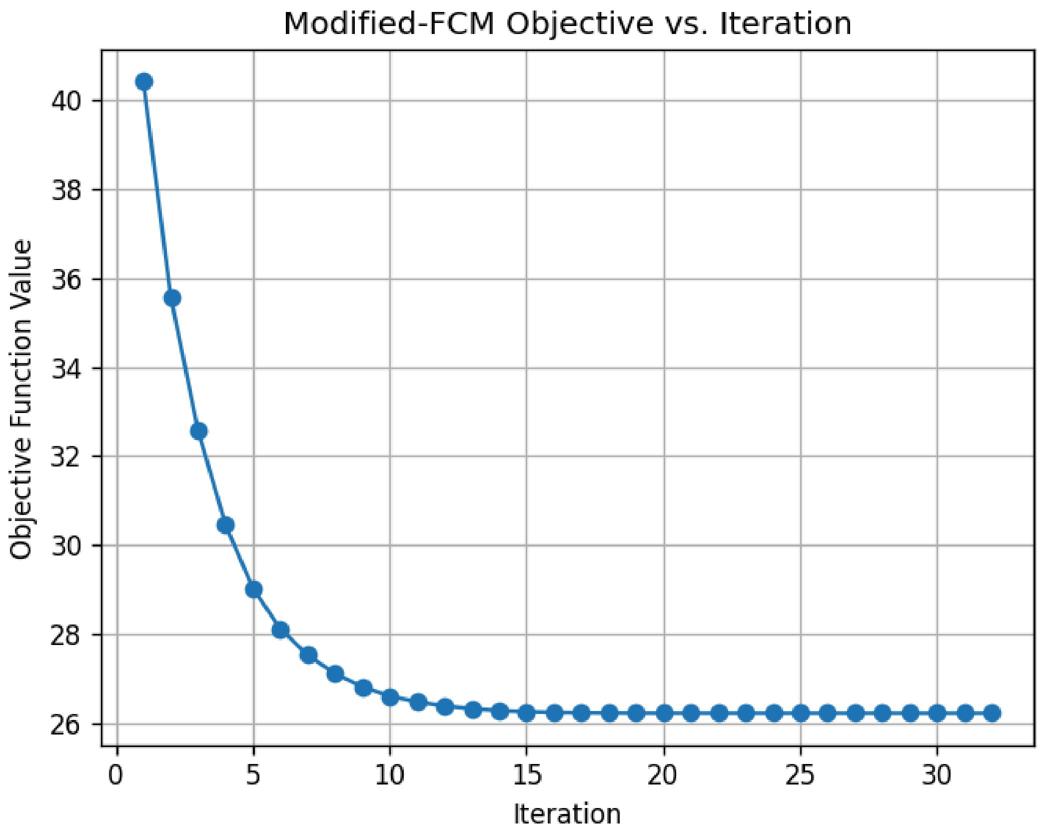


Initial Membership for Class WhiteMatter



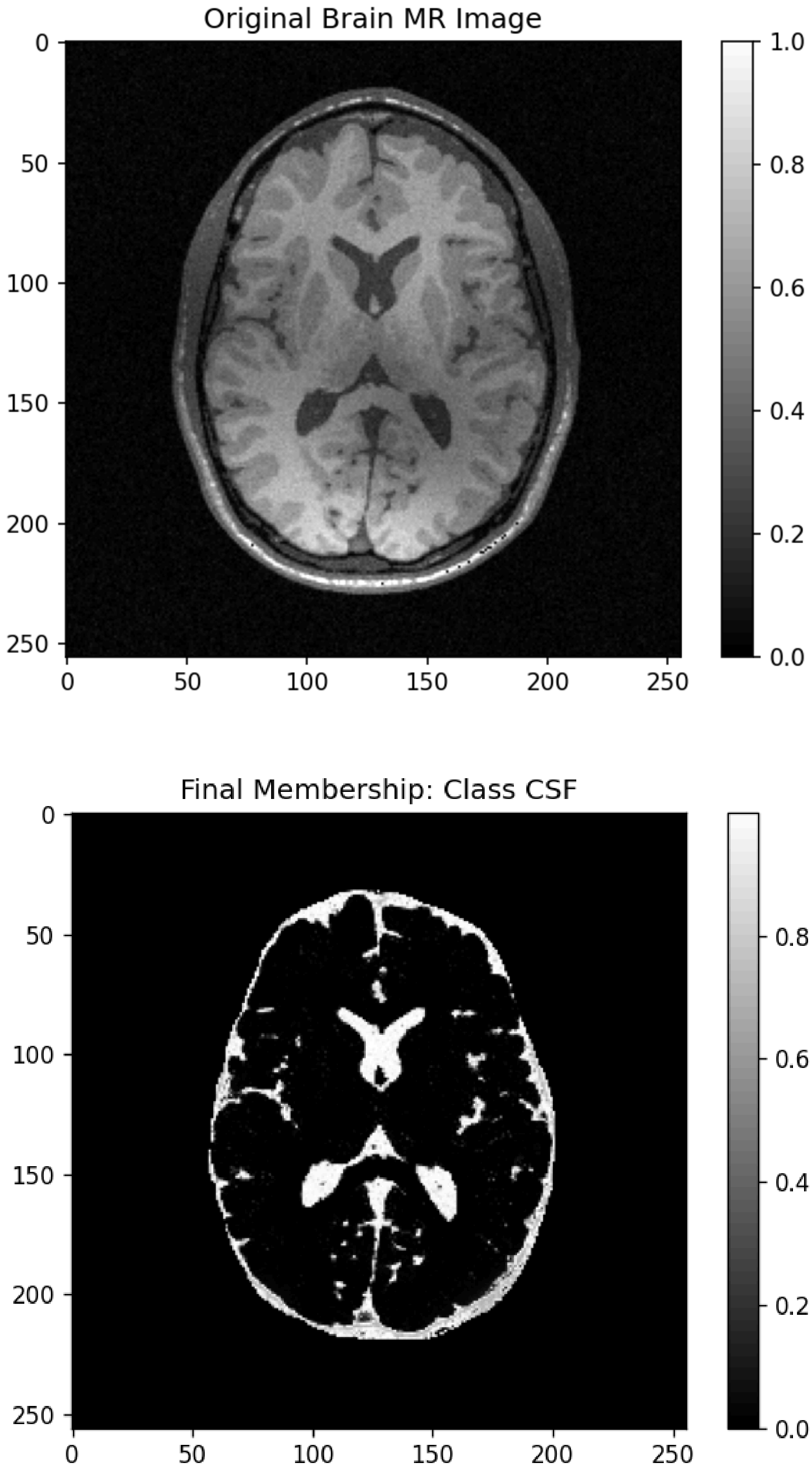
d) Initial class means were found using K means algorithm as it is relatively simple and fast algorithm.
Means :
for CSF = 0.22
for Grey Matter = 0.45
for White Matter = 0.63

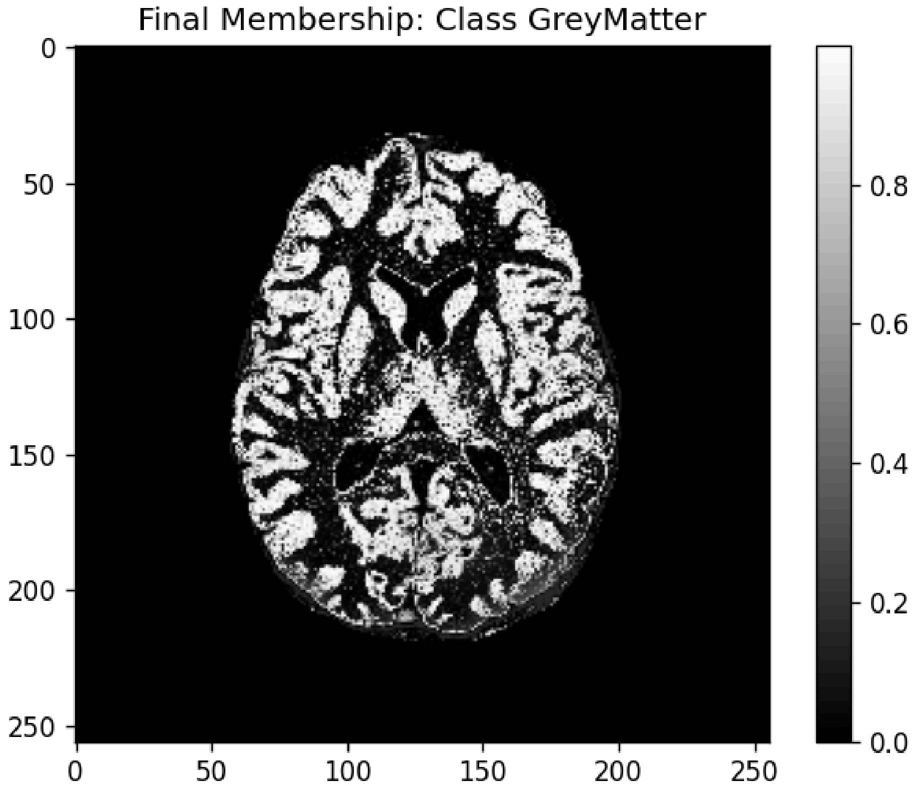
e) The value of objective function:

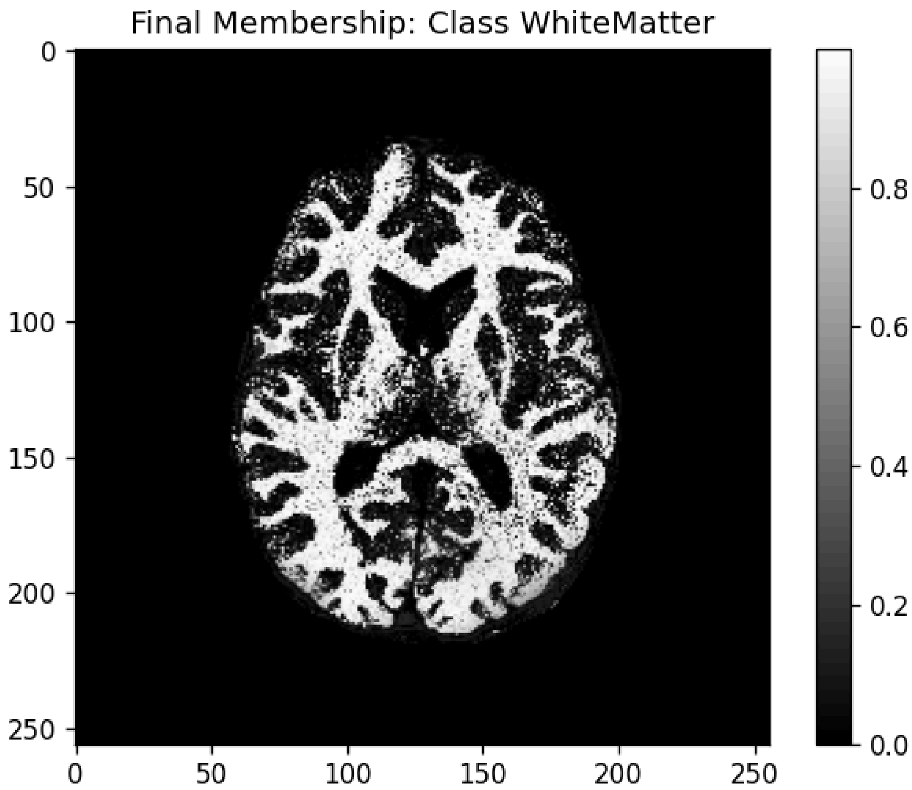


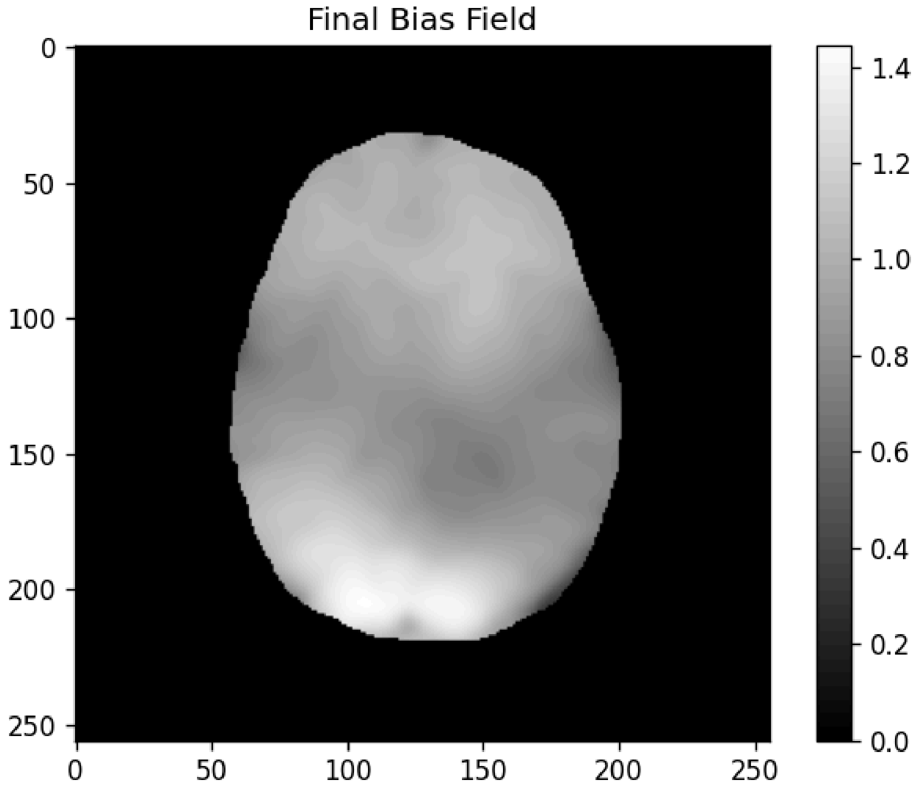
As we can see the objective function decreases significantly upto 15 iteration then it stalls.

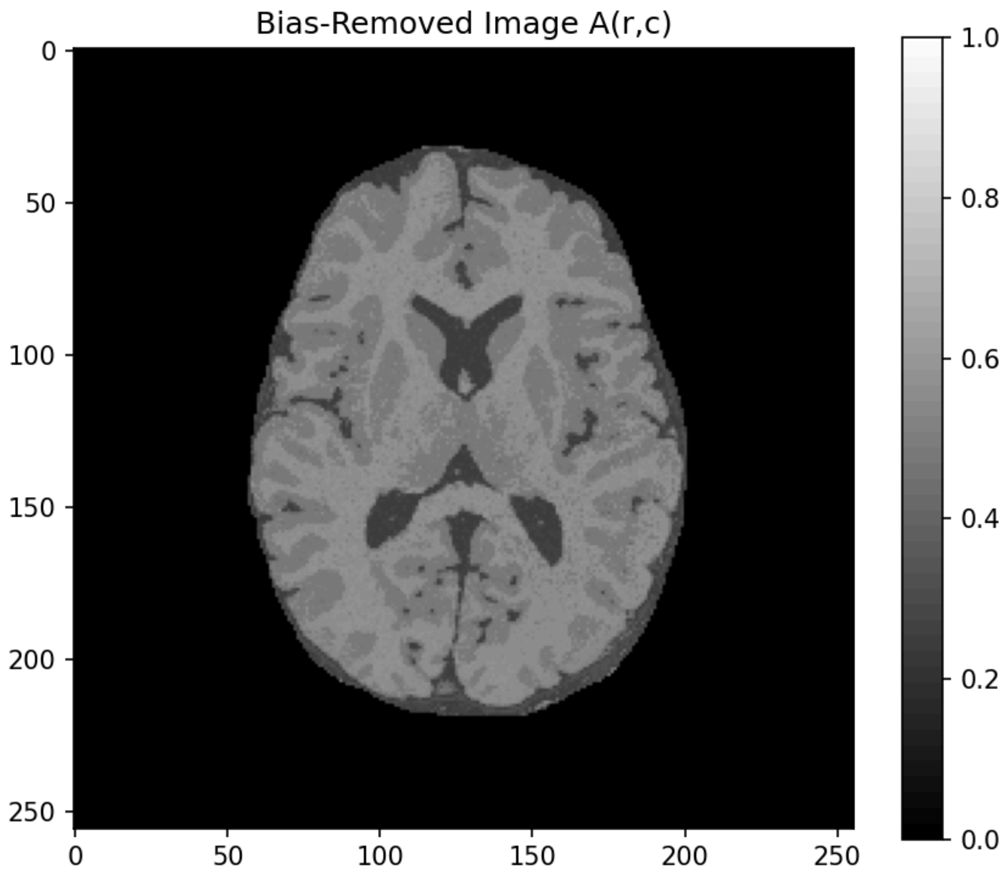
f)

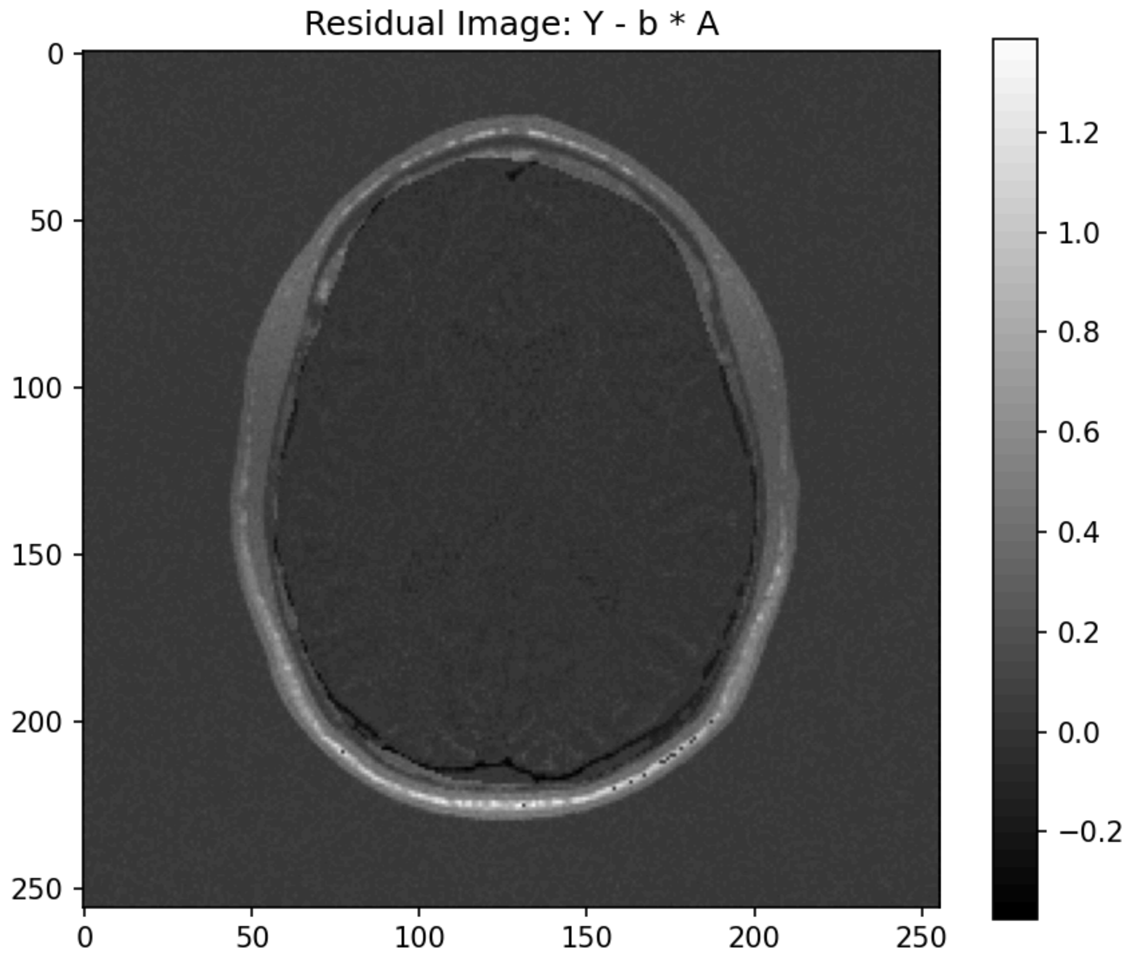

Original Brain MR Image
0
50
100
150
200
250
0 50 100 150 200 250
1.0
0.8
0.6
0.4
0.2
0.0
Final Membership: Class CSF
0
50
100
150
200
250
0 50 100 150 200 250
0.8
0.6
0.4
0.2
0.0


Final Membership: Class GreyMatter
0
50
100
150
200
250
0.8
0.6
0.4
0.2
0.0


Final Membership: Class WhiteMatter
0
50
100
150
200
250
0.8
0.6
0.4
0.2
0.0


Final Bias Field
0
50
100
150
200
250
0
50
100
150
200
250
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0


Bias-Removed Image A(r,c)
0
50
100
150
200
250
0
50
100
150
200
250
1.0
0.8
0.6
0.4
0.2
0.0



g) Class Means:
for CSF = 0.2475255
for Grey Matter = 0.4721438
for White Matter = 0.5716902
Note the initial estimates were pretty close

h) This will note give unique solution:

as Yi = Bi Sum(i) Uik Ck, Note our final solution can also be:
Bi* alpha and Ci/alpha
Which will give same Y(Bias removed image). Thus infinitely many solution.

The uniqueness can be ensured by:

A common theoretical remedy is to **normalize the average bias** to 1 after each iteration. Concretely:

$$\bar{b} = \frac{1}{N}\sum_{n=1}^{N} b_n \quad \Longrightarrow \quad b_n \leftarrow \frac{b_n}{\bar{b}}, \quad c_k \leftarrow c_k \times \bar{b}.$$

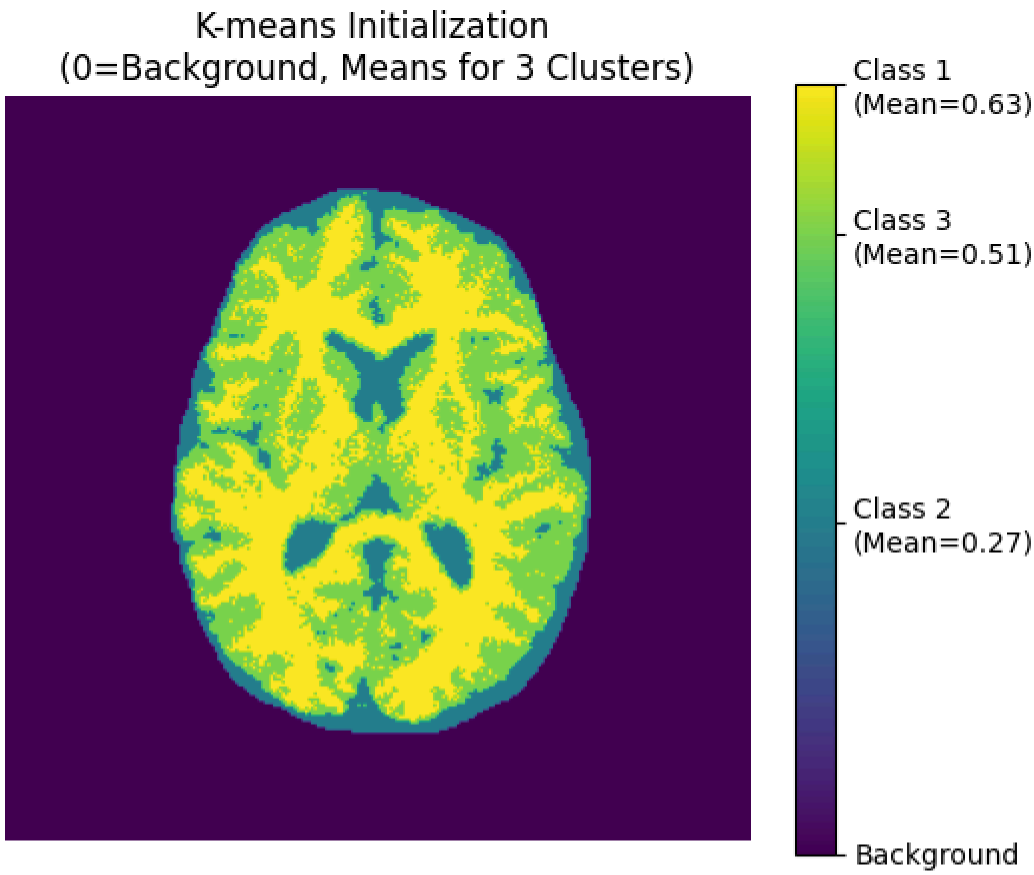
By forcing $\frac{1}{N}\sum_n b_n = 1$, we eliminate the free scaling degree of freedom, thereby **ensuring a unique solution** (up to minor numerical tolerance) for $\{b_n\}, \{c_k\}$, and the membership $\{u_{n,k}\}$.

The following is implemented in the code

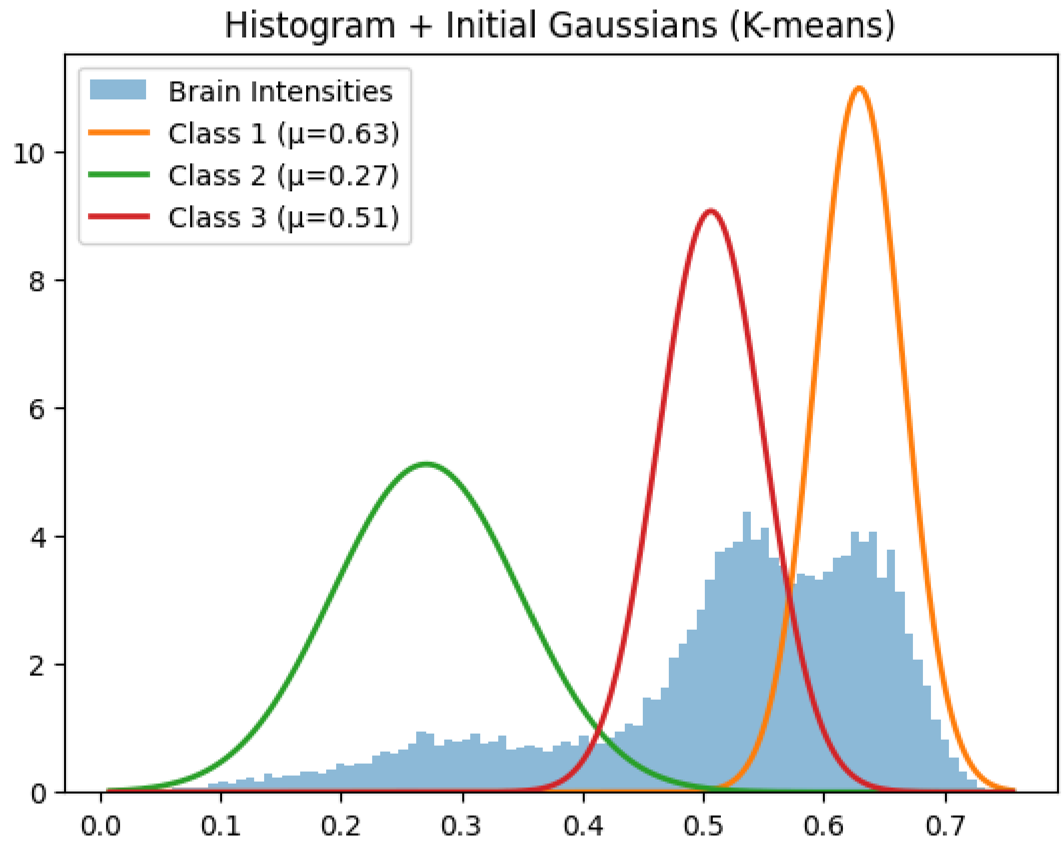
Question 2:

a) After playing around with multiple values we found that the beta = 0.6 works best.

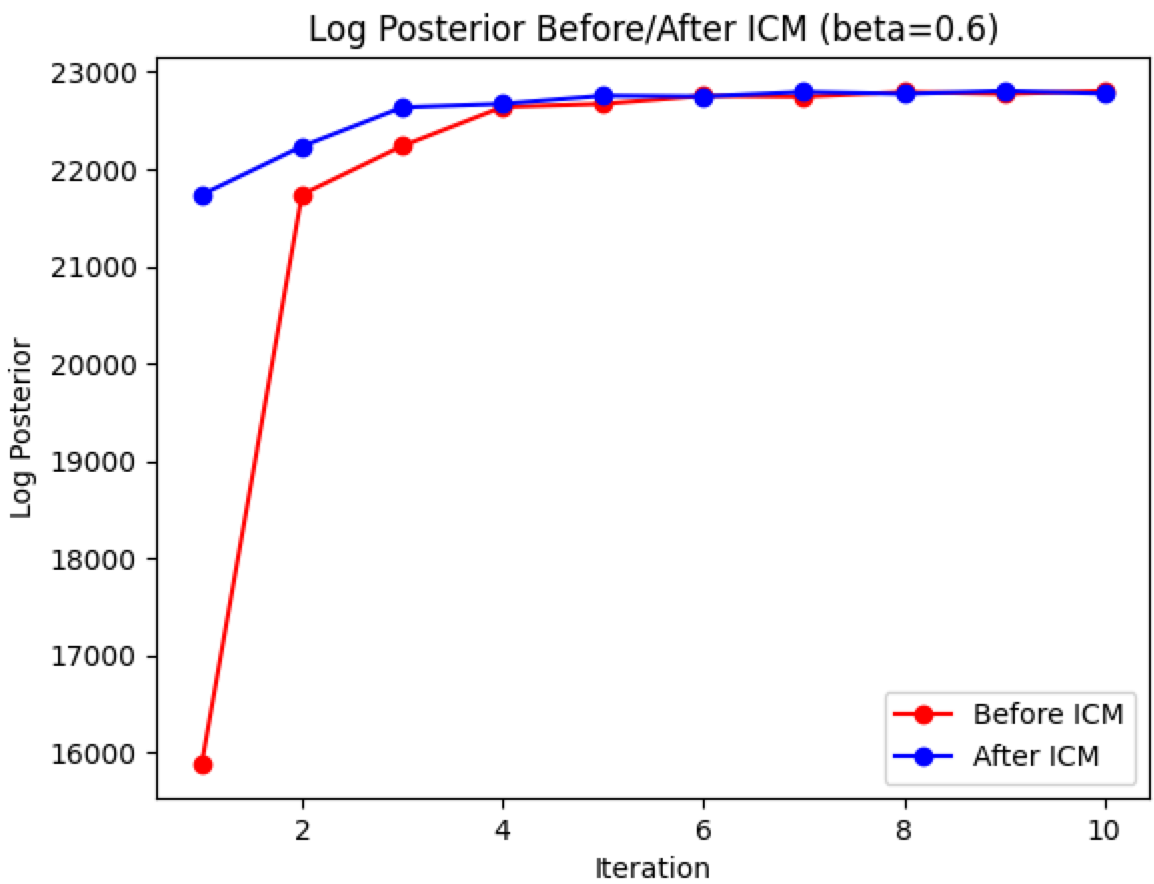
b) For initial estimates of labels we used K means algorithm as it is fairly simple and gives close enough initialisation to final desired values.



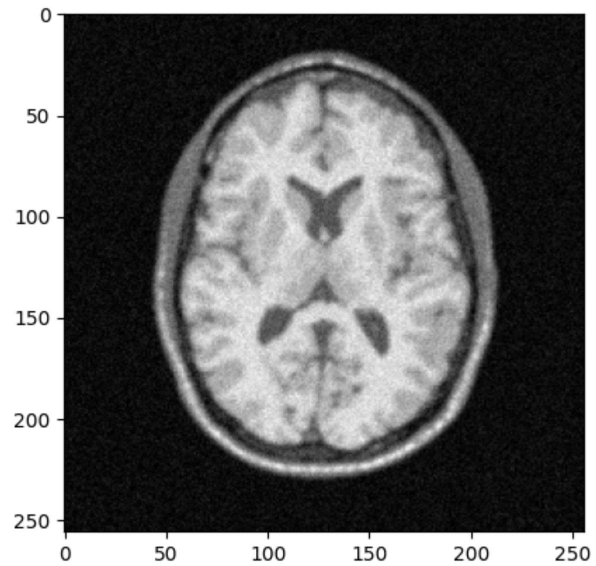
c) We got means from the K means and the we crisp assigned the pixels to classes. Finally we used the value of intensity of each pixel in given class to find standard deviation for that class.

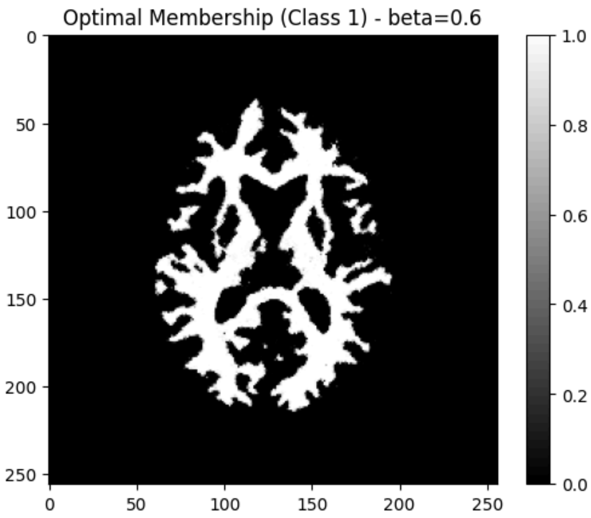


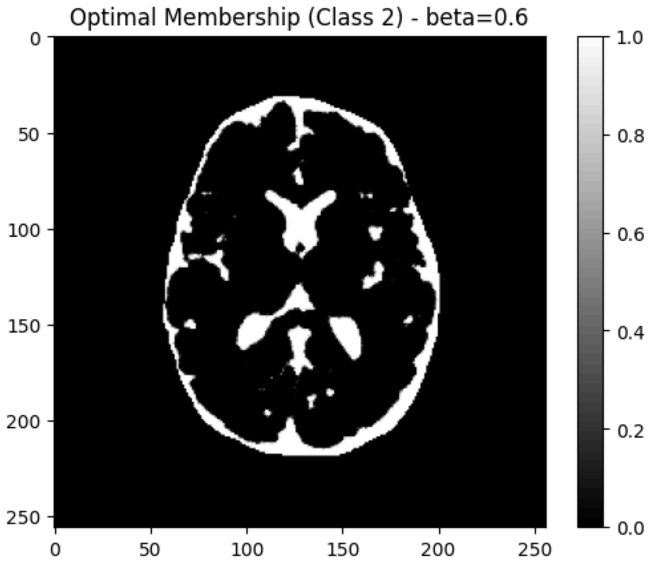
d) For the modified ICM segmentation, the values of the log posterior probability for the labels, before and after the ICM update:

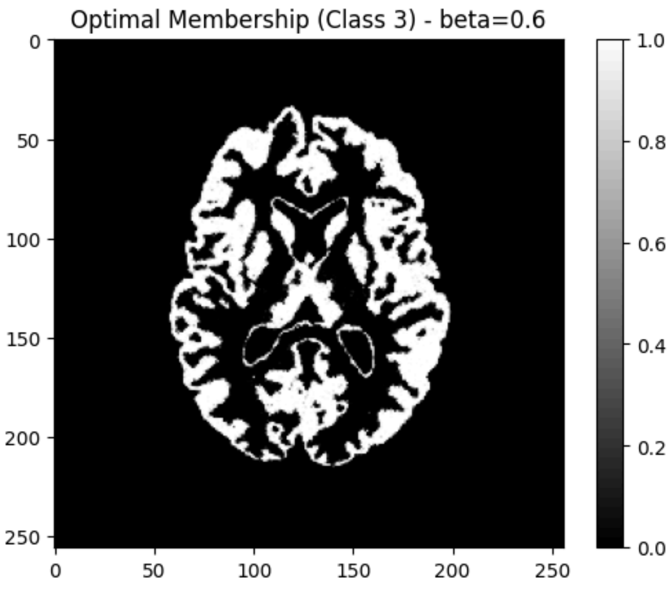


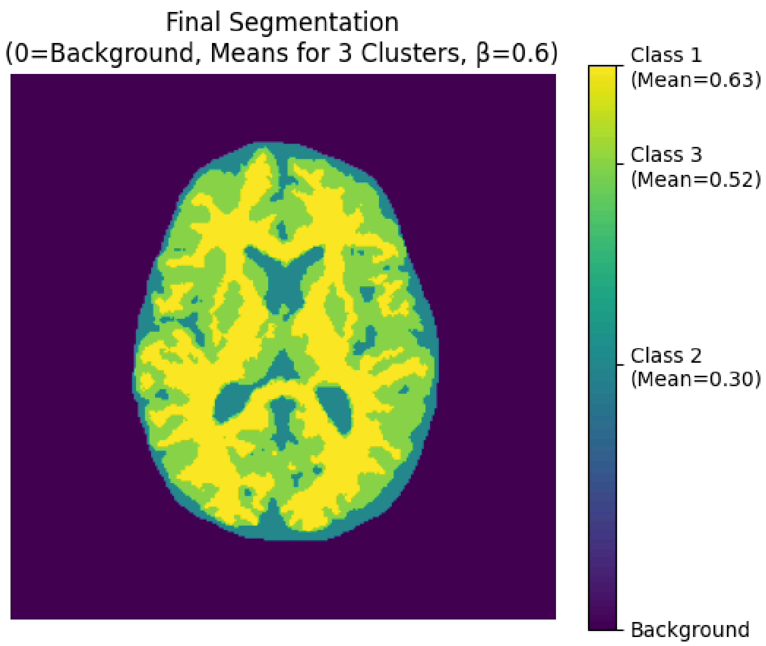
e) Kednvk

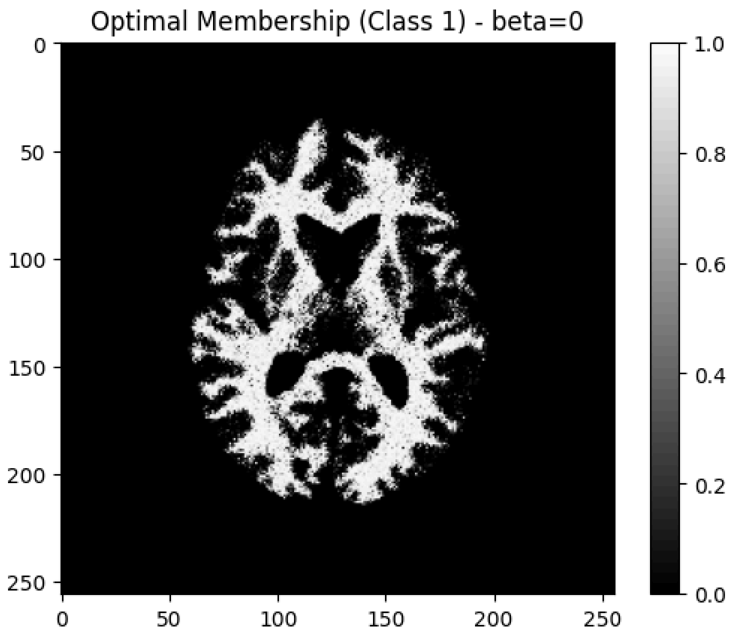


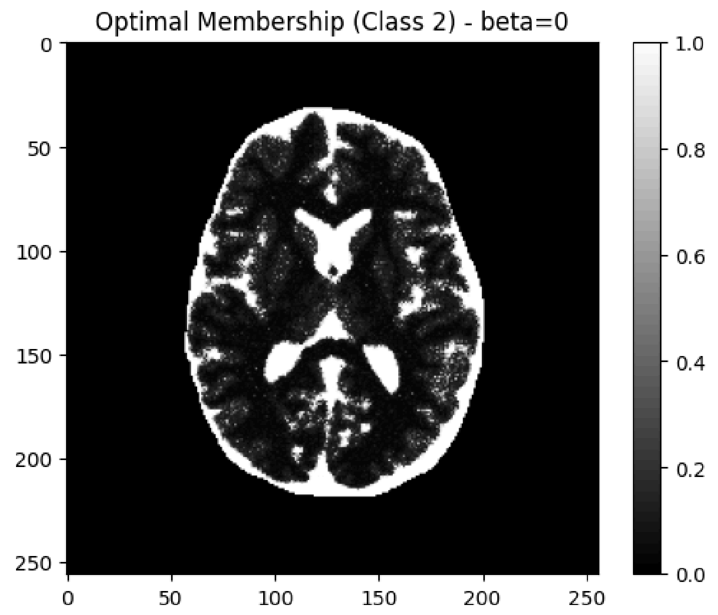

Optimal Membership (Class 1) - beta=0.6
0
50
100
150
200
250
1.0
0.8
0.6
0.4
0.2
0.0

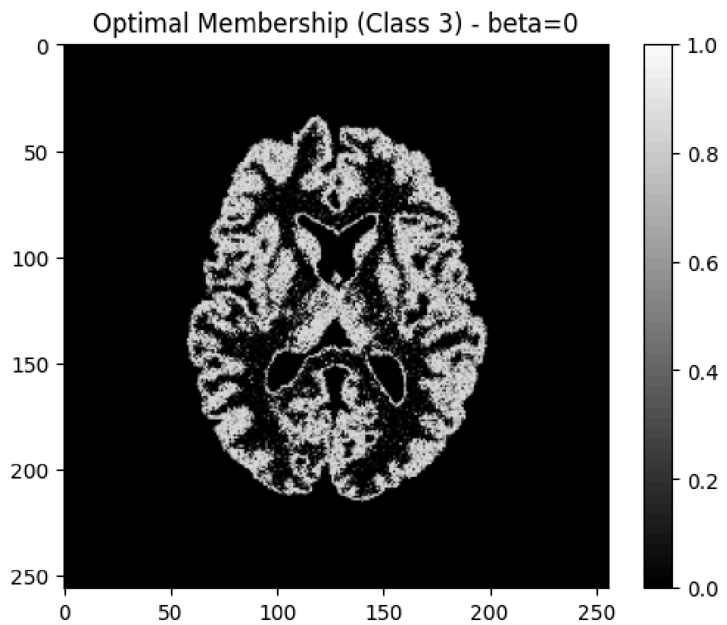

Optimal Membership (Class 2) - beta=0.6
0
50
100
150
200
250
1.0
0.8
0.6
0.4
0.2
0.0

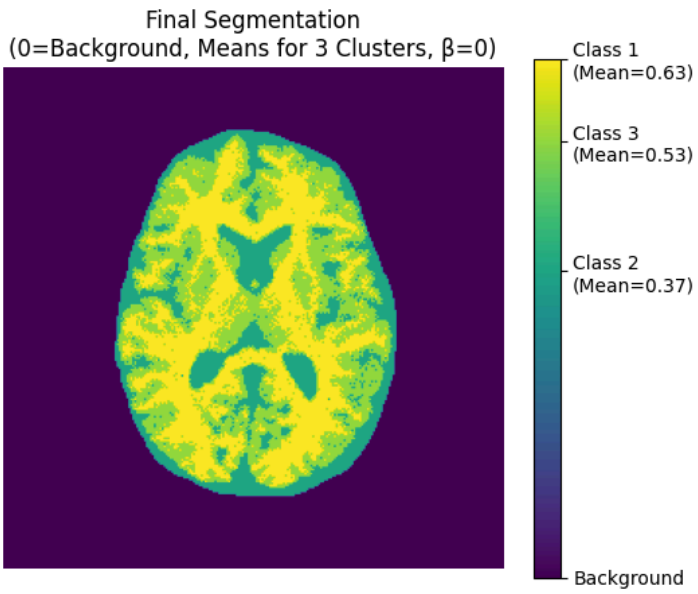

Optimal Membership (Class 3) - beta=0.6
0
50
100
150
200
250
1.0
0.8
0.6
0.4
0.2
0.0


Final Segmentation
(0=Background, Means for 3 Clusters, β=0.6)
Class 1
(Mean=0.63)
Class 3
(Mean=0.52)
Class 2
(Mean=0.30)
Background


Optimal Membership (Class 1) - beta=0
0
50
100
150
200
250
0
50
100
150
200
250
1.0
0.8
0.6
0.4
0.2
0.0


Optimal Membership (Class 2) - beta=0
0
50
100
150
200
250
0
50
100
150
200
250
1.0
0.8
0.6
0.4
0.2
0.0


Optimal Membership (Class 3) - beta=0
0
50
100
150
200
250
0
50
100
150
200
250
1.0
0.8
0.6
0.4
0.2
0.0



f) Class mean = 0.63, 0.52, 0.30