Two-Machine System: Dynamic Simulation

For Simulation as well as Linearized Analysis

1 Two Machine System Data

- Purely resistive constant impedance type loads are considered.
- Three phase balanced conditions are considered.
- Both the synchronous machines are identical.
- The machine parameters are given below:
  R_a = 0.001 pu, H = 3 MJ/MVA,
  x_d = 2 pu, x'_d = 0.32 pu, x''_d = 0.2 pu,
  T_{do} = 5 s, T''_{do} = 0.05 s
  x_q = 1.9 pu, x'_q = 0.75 pu, x''_q = 0.2 pu,
  T'_{qo} = 1 s, T''_{qo} = 0.05 s
- The exciter parameters are given below:
  k_A = 200, T_A = 0.02 s
  $E_{fd(max)}$ = 6.0, $E_{fd(min)}$ = -6.0
- The governor parameters are given below:
  T_{g1} = 2 s, T_{g2} = 6 s, K_g = 20 pu/pu,
  $0.6 \leq P_m \leq 1.1$ pu.

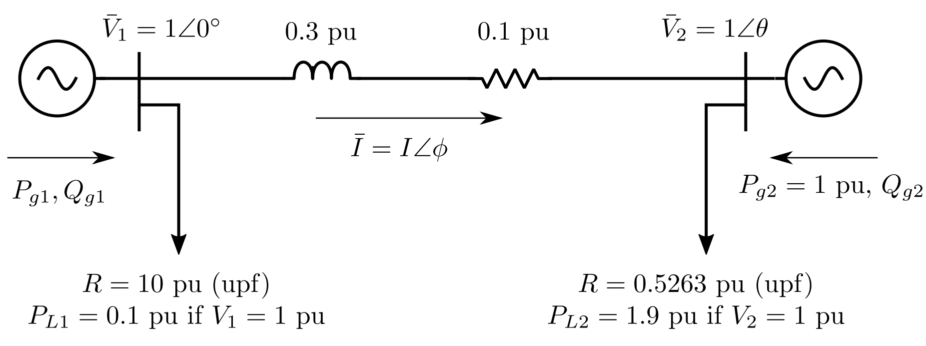

Figure 1: Two-machine system

2 System Equations

2.1 Synchronous Machine Model

2.1.1 Flux Equations

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}\left(-\psi_H + \psi_d\right) \tag{1}$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}\left(-\psi_F + \psi_d + \frac{x'_d}{x_d - x'_d}E_{fd}\right) \tag{2}$$

$$\frac{d\psi_K}{dt} = \frac{1}{T''_q}\left(-\psi_K + \psi_q\right) \tag{3}$$

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}\left(-\psi_G + \psi_q\right) \tag{4}$$

$$\psi_d = x''_d i_d + \frac{(x'_d - x''_d)}{x'_d}\psi_H + \frac{(x_d - x'_d)}{x_d}\frac{x''_d}{x'_d}\psi_F \tag{5}$$

$$\psi_q = x''_q i_q + \frac{\left(x'_q - x''_q\right)}{x'_q}\psi_K + \frac{\left(x_q - x'_q\right)}{x_q}\frac{x''_q}{x'_q}\psi_G \tag{6}$$

If network transients are neglected and $\omega \approx \omega_B$, then

$$-\psi_q - R_a i_d = v_d \tag{7}$$

$$\psi_d - R_a i_q = v_q \tag{8}$$

2.1.2 Mechanical Equations

$$\frac{d\delta}{dt} = \omega - \omega_o \tag{9}$$

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d) \approx P_m - (\psi_d i_q - \psi_q i_d) \tag{10}$$

2.2 Exciter Model

The exciter model is given as

$$E_{fd}(s) = \frac{k_A}{1 + sT_A}(V_{ref} - V(s)), \quad \text{where } V = \sqrt{v_d^2 + v_q^2}$$

A state space representation of this exciter is given as

$$\frac{dX_E}{dt} = \frac{1}{T_A}\left(-X_E + k_A\left(V_{ref} - V\right)\right) \tag{11}$$

$$V = \sqrt{v_d^2 + v_q^2} = \sqrt{v_D^2 + v_Q^2} \tag{12}$$

where

$$E_{fd} = \begin{cases} E_{fd(min)} & X_E < E_{fd(min)} \\ X_E & E_{fd(min)} \le X_E \le E_{fd(max)} \\ E_{fd(max)} & X_E > E_{fd(max)} \end{cases} \tag{13}$$

2.3 Speed Governor Model

The speed governor model is given as

$$\frac{\Delta P_m(s)}{\Delta\bar{\omega}(s)} = K_g \frac{1 + sT_{g1}}{1 + sT_{g2}}, \quad \text{where } \Delta P_m = P_m - P_{mo}, \Delta\bar{\omega} = \frac{\omega_{ref} - \omega}{\omega_B}$$

Try to find out a state space realization of the speed governor (along with the limiters given in the data). Let the state variable be x_G.

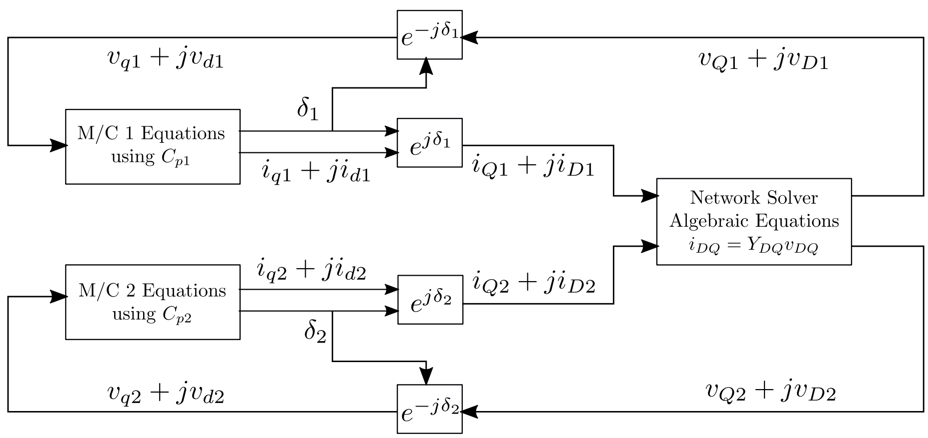

Figure 2: Schematic of dynamic simulation

2.4 Kron Transformation

$$f_{abc} = C_K f_{DQo}, \quad C_K = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\omega_o t & \sin\omega_o t & \sqrt{\frac{1}{2}} \\ \cos\left(\omega_o t - \frac{2\pi}{3}\right) & \sin\left(\omega_o t - \frac{2\pi}{3}\right) & \sqrt{\frac{1}{2}} \\ \cos\left(\omega_o t + \frac{2\pi}{3}\right) & \sin\left(\omega_o t + \frac{2\pi}{3}\right) & \sqrt{\frac{1}{2}} \end{bmatrix}$$

The D-Q variables and the d-q variables are related as

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = [C_{p1}] \begin{bmatrix} f_{d1} \\ f_{q1} \\ f_o \end{bmatrix} = [C_K] \begin{bmatrix} f_D \\ f_Q \\ f_o \end{bmatrix}$$

The transformed variables are related as

$$\begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} f_d \\ f_q \end{bmatrix} = \begin{bmatrix} f_D \\ f_Q \end{bmatrix}, \quad f_Q + jf_D = (f_q + jf_d)e^{j\delta}$$

For the two machines, it can be written that

$$\begin{aligned} f_{Q1} + jf_{D1} &= (f_{q1} + jf_{d1})e^{j\delta_1} \quad \text{for Machine 1} \\ f_{Q2} + jf_{D2} &= (f_{q2} + jf_{d2})e^{j\delta_2} \quad \text{for Machine 2} \end{aligned}$$

The differential equations are only associated with the machines as the network transients are neglected here. If you wish to consider the stator transients, you will also have to model the network transients here. For the model given here, there are 16 differential states i.e. states that are associated with differential equations. They are:
$\psi_{F1}, \psi_{H1}, \psi_{G1}, \psi_{K1}, \delta_1, \omega_1, x_{E1}, x_{G1}, \psi_{F2}, \psi_{H2}, \psi_{G2}, \psi_{K2}, \delta_2, \omega_2, x_{E2}, x_{G2}$.

2.5 Algebraic Equations

2.5.1 Synchronous Machine

The algebriac equations corresponding to each machine in d-q variables are

$$\psi_d = x_d'' i_d + \frac{(x_d' - x_d'')}{x_d'}\psi_H + \frac{(x_d - x_d')}{x_d}\frac{x_d''}{x_d'}\psi_F \tag{14}$$

$$\psi_q = x_q'' i_q + \frac{(x_q' - x_q'')}{x_q'}\psi_K + \frac{(x_q - x_q')}{x_q}\frac{x_q''}{x_q'}\psi_G \tag{15}$$

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d \tag{16}$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q \tag{17}$$

Transforming the algebraic equations to D-Q variables for both the machines we get

$$\psi_{D1} = x_d'' i_{D1} + \mathcal{F}_{D1}(\psi_{F1}, \psi_{H1}, \psi_{G1}, \psi_{K1}, \delta_1) \tag{18}$$

$$\psi_{D2} = x_d'' i_{D2} + \mathcal{F}_{D2}(\psi_{F2}, \psi_{H2}, \psi_{G2}, \psi_{K2}, \delta_2) \tag{19}$$

$$\psi_{Q1} = x_q'' i_{Q1} + \mathcal{F}_{Q1}(\psi_{F1}, \psi_{H1}, \psi_{G1}, \psi_{K1}, \delta_1) \tag{20}$$

$$\psi_{Q2} = x_q'' i_{Q2} + \mathcal{F}_{Q2}(\psi_{F2}, \psi_{H2}, \psi_{G2}, \psi_{K2}, \delta_2) \tag{21}$$

$$0 = -\omega_B \psi_{Q1} - \omega_B R_a i_{D1} - \omega_B v_{D1} \tag{22}$$

$$0 = \omega_B \psi_{D1} - \omega_B R_a i_{Q1} - \omega_B v_{Q1} \tag{23}$$

$$0 = -\omega_B \psi_{Q2} - \omega_B R_a i_{D2} - \omega_B v_{D2} \tag{24}$$

$$0 = \omega_B \psi_{D2} - \omega_B R_a i_{Q2} - \omega_B v_{Q2} \tag{25}$$

Note: You can arrive at these equations as $x''_d = x''_q$. The network algebraic equations in the D-Q domain can be written as

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{r\omega_B}{x} & -\omega_B \\ \omega_B & -\frac{r\omega_B}{x} \end{bmatrix} \begin{bmatrix} i_{lD} \\ i_{lQ} \end{bmatrix} + \frac{\omega_B}{x} \begin{bmatrix} v_{D1} - v_{D2} \\ v_{Q1} - v_{Q2} \end{bmatrix} \tag{26}$$

$$v_{Q1} = R_{L1}(i_{Q1} - i_{lQ}) \tag{27}$$

$$v_{D1} = R_{L1}(i_{D1} - i_{lD}) \tag{28}$$

$$v_{Q2} = R_{L2}(i_{Q2} + i_{lQ}) \tag{29}$$

$$v_{D2} = R_{L2}(i_{D2} + i_{lD}) \tag{30}$$

Total number of algebraic variables: 14 and they are
$\psi_{D1}, \psi_{Q1}, \psi_{D2}, \psi_{Q2}, i_{D1}, i_{Q1}, i_{D2}, i_{Q2}, v_{D1}, v_{Q1}, v_{D2}, v_{Q2}, i_{lD}, i_{lQ}$.
Re-write the equations in the form $Ax = b$. Linear equation solver is needed to obtain the algebriac variables.

3 Calculation of Initial Conditions

Step 1: Obtain the steady state network phasor solution (power flow solution). This ensures the generator power injection, terminal voltage phasor, current phasor are known.
Step 2: Back-calculate the steady state values of the generator states. Following is the problem at hand:

- **Given:** P_g, $\bar{V}$, $\bar{I}$, $\omega_o = \omega_B$ as well as the parameters of the synchronous machine and the exciter.
- **To find:** Equilibrium values of states (i.e. ψ_{do}, ψ_{qo}, ψ_{Fo}, ψ_{Go}, ψ_{Ko}, δ_o, X_{Eo}, X_{Go}, E_{fdo}).

NOTE:
For $\bar{V} = V\angle\theta$ and $\bar{I} = |I|\angle\phi$, the d-q variables are:

$$\begin{aligned} v_d &= V \sin(\theta - \delta) \\ v_q &= V \cos(\theta - \delta) \\ i_d &= |I| \sin(\phi - \delta) \\ i_q &= |I| \cos(\phi - \delta) \end{aligned}$$

3.1 Synchronous Machine

In steady-state, we have to set each derivative to zero. From (1) and (2),

$$\psi_{H0} = \psi_{d0} \tag{31}$$

$$\psi_{F0} = \psi_{d0} + \frac{x'_d}{x_d - x'_d} E_{fd0} \tag{32}$$

Substitute in (5),

$$\cancel{\psi_{d0}} = x''_d i_{d0} + \cancel{\psi_{d0}} - \cancel{\frac{x''_d}{x'_d}\psi_{d0}} + \cancel{\frac{\cancel{x_d} x''_d}{\cancel{x_d} x'_d}\psi_{d0}} - \frac{x''_d \cancel{x'_d}}{x_d \cancel{x'_d}}\psi_{d0} + \frac{\cancel{(x_d - x'_d)}}{x_d}\frac{\cancel{x'_d}}{\cancel{(x_d - x'_d)}}\frac{x''_d}{\cancel{x'_d}}E_{fd0} \tag{33}$$

This equation simplifies to:

$$\psi_{d0} = x_d i_{d0} + E_{fd0} \tag{34}$$

Similarly, from (3) and (4),

$$\psi_{K0} = \psi_{q0} \tag{35}$$
$$\psi_{G0} = \psi_{q0} \tag{36}$$

Substitute in (6),

$$\cancel{\psi_{q0}} = x''_q i_{q0} + \cancel{\psi_{q0}} - \cancel{\frac{x''_q}{x'_q}\psi_{q0}} + \cancel{\frac{x''_q}{x'_q}\psi_{q0}} - \frac{x''_q}{x_q}\psi_{q0} \tag{37}$$

This equation simplifies to:

$$\psi_{q0} = x_q i_{q0} \tag{38}$$

If $R_a \longrightarrow 0$, then from (7) and (8), we get:

$$\psi_{d0} = v_{q0} \tag{39}$$
$$\psi_{q0} = -v_{d0} \tag{40}$$

Substitute (34) and (38) in the above equations, we get:

$$v_{q0} = x_d i_{d0} + E_{fd0} \tag{41}$$
$$v_{d0} = -x_q i_{q0} \tag{42}$$

Add 'j' times (42) with (41), we get:

$$(v_{q0} + jv_{d0}) + jx_q i_{q0} = E_{fd0} + x_d i_{d0} \tag{43}$$

Subtract $jx_q i_{d0}$ from both sides, we get:

$$(v_{q0} + jv_{d0}) + jx_q (i_{q0} + ji_{d0}) = E_{fd0} + (x_d - x_q)\, i_{d0} \tag{44}$$

Multiply the equation by $e^{j\delta_0}$,

$$(v_{q0} + jv_{d0})\, e^{j\delta_0} + jx_q (i_{q0} + ji_{d0})\, e^{j\delta_0} = [E_{fd0} + (x_d - x_q)\, i_{d0}]\, e^{j\delta_0} \tag{45}$$

Now,

$$\begin{aligned}(v_{q0} + jv_{d0})\, e^{j\delta_0} &= V[\cos(\theta - \delta_0) + j\sin(\theta - \delta_0)]\, e^{j\delta_0} \\ &= Ve^{j(\theta-\delta_0)}.e^{j\delta_0} \\ &= Ve^{j\theta} \\ &= V\angle\theta\end{aligned}$$

Similarly,

$$\begin{aligned}(i_{q0} + j i_{d0})\, e^{j\delta_0} &= |I|\left[\cos(\phi - \delta_0) + j\sin(\phi - \delta_0)\right] e^{j\delta_0} \\ &= |I| e^{j(\phi-\delta_0)}.e^{j\delta_0} \\ &= |I| e^{j\phi} \\ &= |I|\angle\phi\end{aligned}$$

Now, (45) becomes:

$$V\angle\theta + jx_q|I|\angle\phi = \left[E_{fd0} + (x_d - x_q)\, i_{d0}\right] e^{j\delta_0} \tag{46}$$

Now, $[E_{fd0} + (x_d - x_q)\, i_{d0}]$ is a real number,

$$\therefore \delta_0 = \angle\left[V\angle\theta + jx_q|I|\angle\phi\right] \tag{47}$$

Once, δ_0 is known, we can find out v_{d0}, v_{q0}, i_{d0} and i_{q0}. Using these, we can find E_{fd0} in the following way:

$$E_{fd0} = |V\angle\theta + jx_q|I|\angle\phi| - (x_d - x_q)\, i_{d0} \tag{48}$$

Once these have been found, we can find the equilibrium values of all other states of the machines using (31), (32), (35), (36), (39) and (40). And,

$$\omega_0 = \omega_B \tag{49}$$

$$P_{m0} = \psi_{d0} i_{q0} - \psi_{q0} i_{d0} \tag{50}$$

3.2 Exciter

We know that in steady-state,

$$X_{E0} = E_{fd0} \tag{51}$$

Once X_{E0} is known, we can find V_{ref0} using

$$V_{ref0} = V + \frac{E_{fd0}}{k_A} \tag{52}$$

4 Study of Dynamic Stability

The procedure for dynamic security assessment has been shown here.

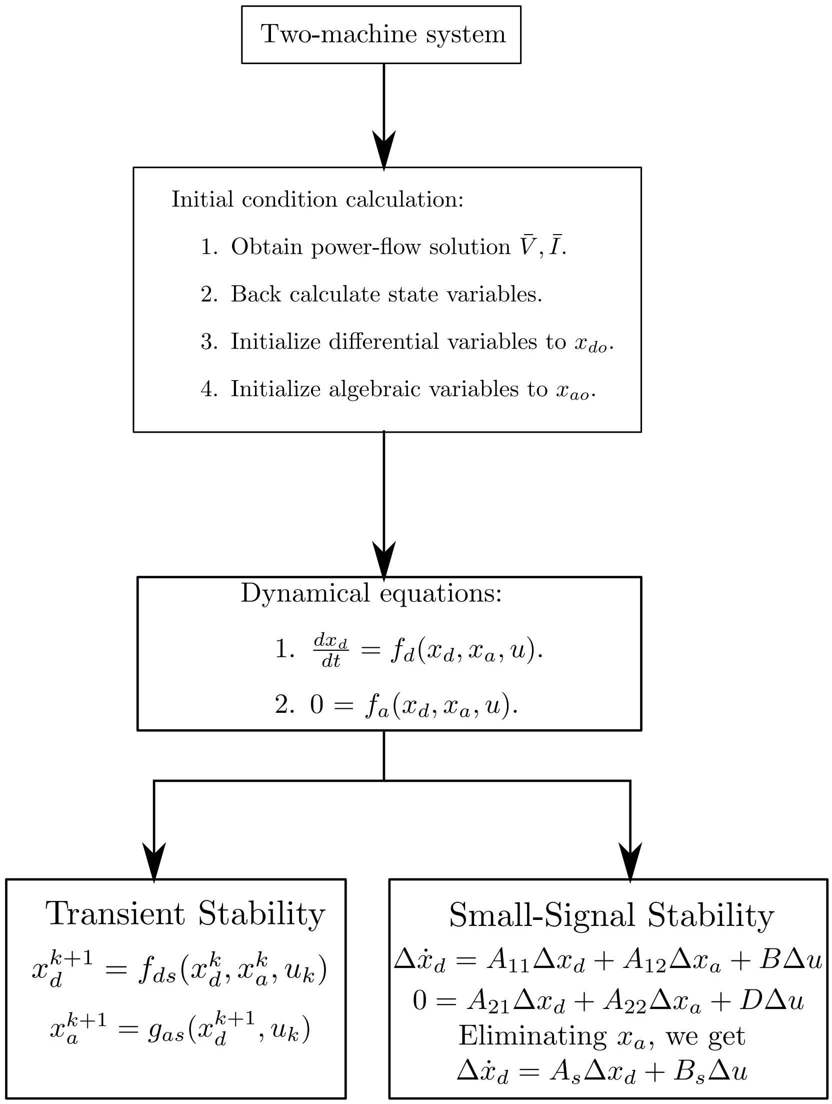

Figure 3: Assessment of Dynamic Stability