DYNAMICS OF BIOREACTOR SYSTEM

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Linear vs non-linear and CONTROLLERS

INTRODUCTION

System Equations

$$\frac{dX}{dt} = -DX + \mu(S,P)X$$

$$\dot{S} = -\mu(S)X/Y_{X/S} + D(S_f - S)$$

$$\frac{dP}{dt} = -DP + [\alpha\mu(S,\ P) + \beta]X$$

$$\mu(S,P) = \frac{\mu_m\left(1 - \frac{P}{P_m}\right)S}{K_m + S + \frac{S_2}{K_i}}$$

Parameter Values

Parameter	Values
$Y_{X,S}$	0.4 g/g
β	0.2 h^{-1}
P_m	50 g/L
K_i	22 g/L
α	2.2 g/g
μ_m	0.48 h^{-1}
k_m	1.2 g/L
S_f	20 g/L

Objective:-

- To study dynamics of Bioreactor system
- To compare linear vs non-linear model
- To discuss stability of zeros and poles

MODEL PARAMETERS AND DETAILS

– Step Change – 2% of D0 (steady state) at instants 100, 200, 300.
– Total Step Change – 8% of D0 (Steady State) between time instant 100 and 300.

– Transfer Functions between input and outputs :–

```
From input to output...
         -5.996 s^2 - 2.189 s - 0.1976
 1:  ---------------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303

          14.99 s^2 + 5.473 s + 0.494
 2:  ---------------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303

         -19.13 s^2 - 7.756 s - 0.7862
 3:  ---------------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303
```

1. X(s) [Biomass]
2. S(s) [Substrate]
3. P(s) [Product]

After using minreal function

```
From input to output...
        -5.996 s - 0.9782
 1:  ------------------------
     s^2 + 0.2931 s + 0.02625

         14.99 s + 2.446
 2:  ------------------------
     s^2 + 0.2931 s + 0.02625

         -19.13 s - 3.892
 3:  ------------------------
     s^2 + 0.2931 s + 0.02625
```

STEADY STATE VALUES AND ANALYSIS

– Used Fsolve with different initial guesses and found majorly 2 steady states with values:-

Variable	Value at SS
X_ss	0
S_ss	20
P_ss	0

Variable	Value at SS
X_ss	5.99564
S_ss	5.01089
P_ss	19.12670

WHY?

Let's do stability analysis and understand why one steady state is stable and other is unstable?

JACOBIAN MATRIX AND ITS EIGENVALUES

Unstable State Jacobian Matrix Eigenvalues

```
Eigenvalues of A:
   -0.2020
   -0.2020
    0.0418
```

Stable State Jacobian Matrix Eigenvalues

```
Eigenvalues of A:
  -0.1466 + 0.0691i
  -0.1466 - 0.0691i
  -0.2020 + 0.0000i
```

The complex exponential in turn can be written

$$e^{i\nu_j t} = \cos(\nu_j t) + i\sin(\nu_j t).$$

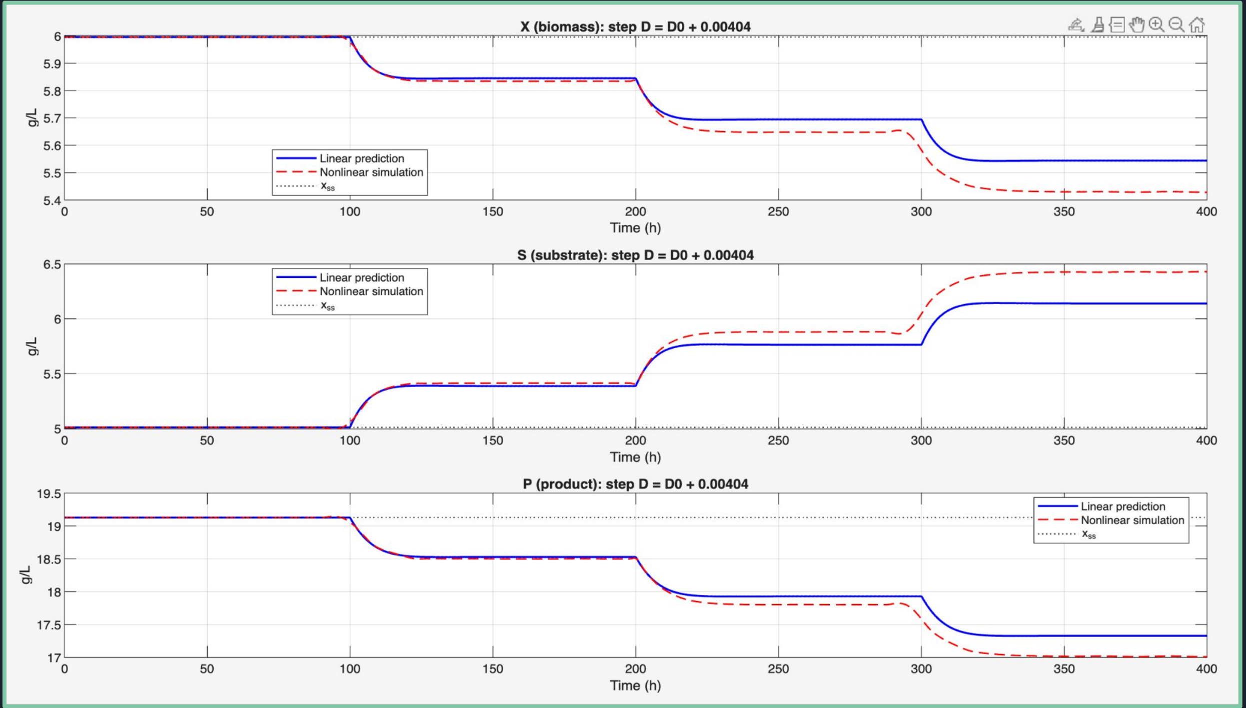
The complex part of the eigenvalue therefore only contributes an oscillatory component to the solution. It's the real part that matters: If $\mu_j > 0$ for any j, $e^{\mu_j t}$ grows with time, which means that trajectories will tend to move away from the equilibrium point. This leads us to a very important theorem:

Theorem 1 *An equilibrium point* $\mathbf{x}^*$ *of the differential equation 1 is stable if all the eigenvalues of* $\mathbf{J}^*$*, the Jacobian evaluated at* $\mathbf{x}^*$*, have negative real parts. The equilibrium point is unstable if at least one of the eigenvalues has a positive real part.*

Fun Fact

Poles of Transfer Function are in fact Eigenvalues of the Jacobian Matrix itself that is why we look for negative real values for a stable system.

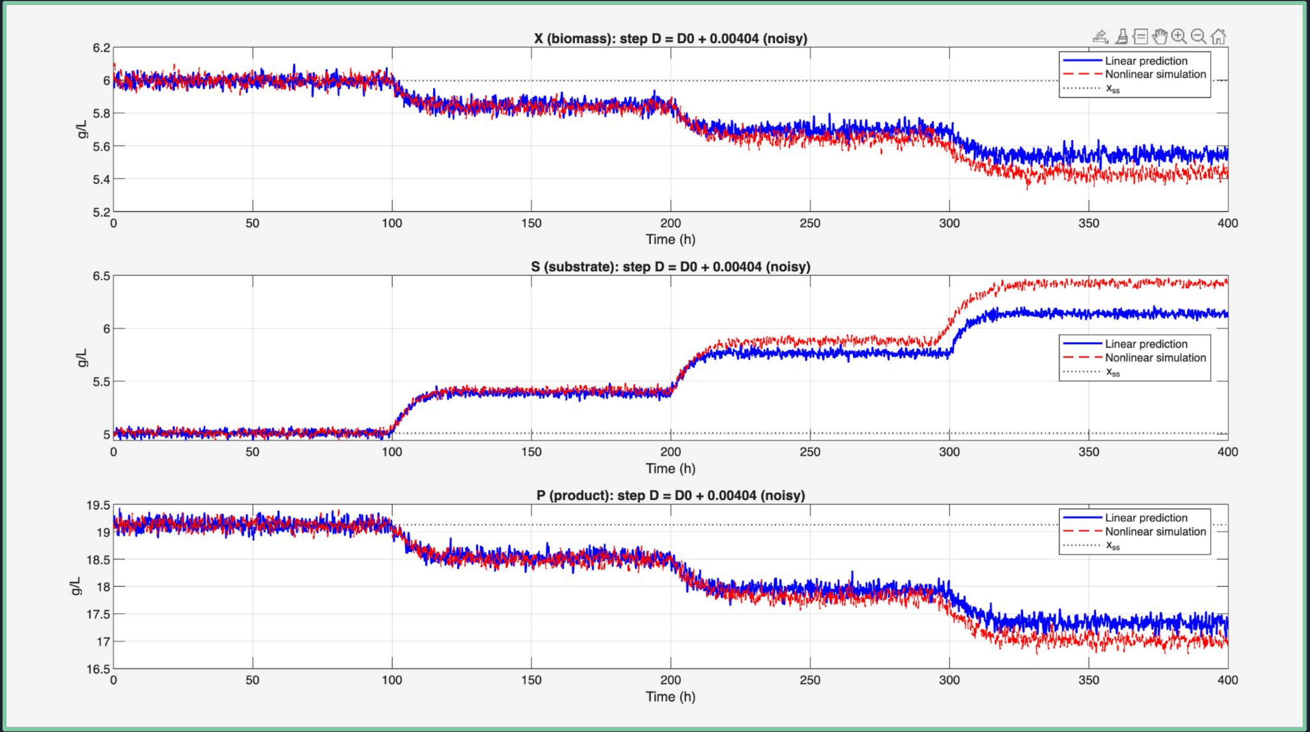
MODEL RESULTS



– Step Change – 2% of D0 (steady state) at instants 100, 200, 300.

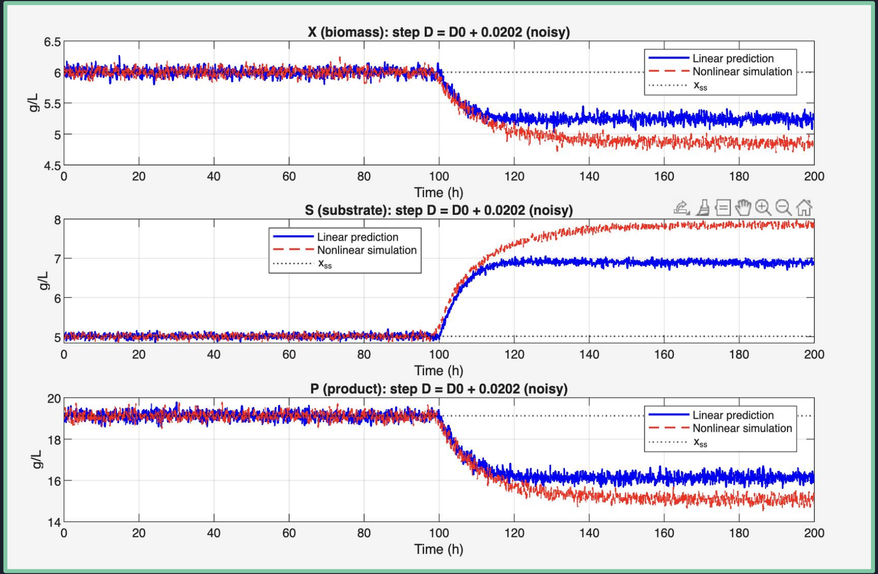
– Total Step Change – 8% of D0 (Steady State) between time instant 100 and 300.

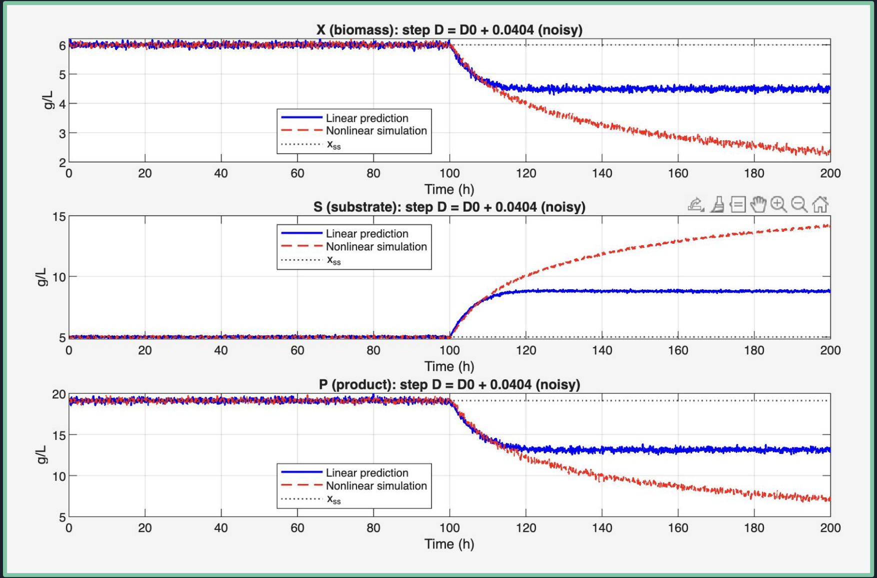
MODEL RESULTS [with gaussian noise]



– Step Change – 2% of D0 (steady state) at instants 100, 200, 300.
– Total Step Change – 8% of D0 (Steady State) between time instant 100 and 300.

Step Change of 10% Vs 20% of D0





As we increase deltaD (step change), the model drifts from the nonlinear solution since linearization holds only near steady state.

K_p And $\boldsymbol{\tau}$ Calculations graphically (for 5% step Change)

To calculate K_p we will calculate the :- Output Change/Input Change

For Transfer Function 1 :-

$K_p = -37.26$
$\tau = 5.953$

For Transfer Function 3 :-

$K_p = -93.69$
$\tau = 6.853$

For Transfer Function 2 :-

$K_p = 93.155$
$\tau = 5.665$

Poles of the stable system

```
From input to output...
       -5.996 s^2 - 2.189 s - 0.1976
 1:  ----------------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303

        14.99 s^2 + 5.473 s + 0.494
 2:  ----------------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303

       -19.13 s^2 - 7.756 s - 0.7862
 3:  ----------------------------------------
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```

POLES

1) -0.1466 + 0.0691 *i*
2) -0.1466 - 0.0691 *i*
3) -0.2020

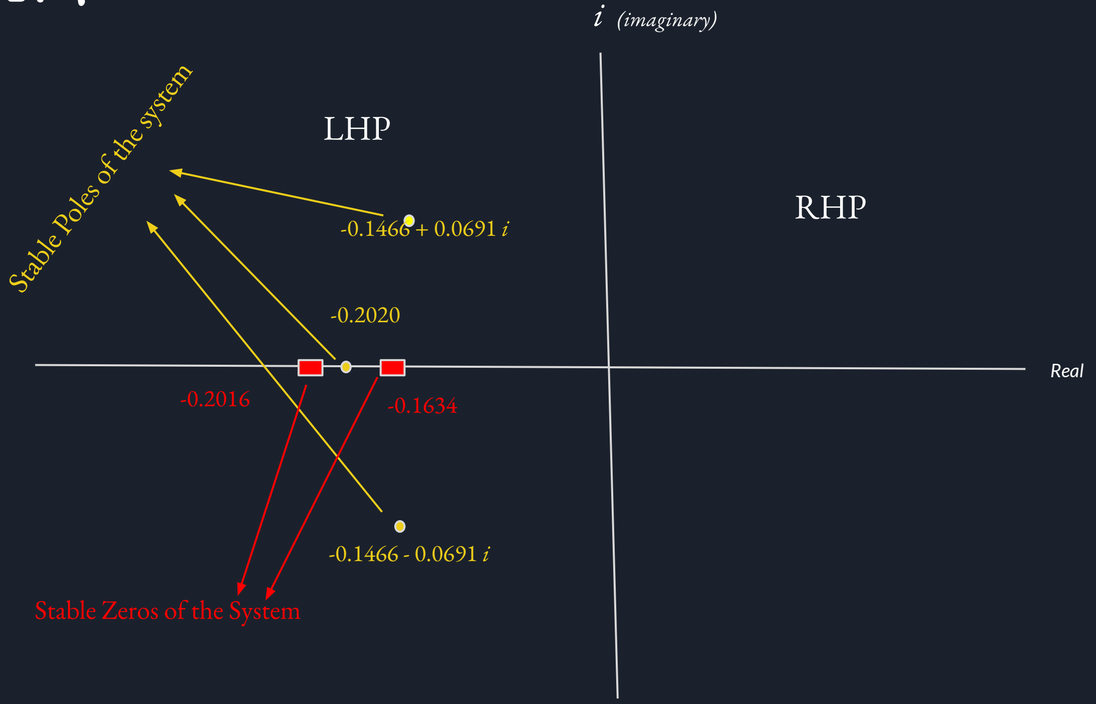
As first two roots lie on the left hand plane they show stable oscillatory behaviour as it is decaying. Whereas the pole lies purely on negative real axis and is stable.

Zeros of the system

Zeros

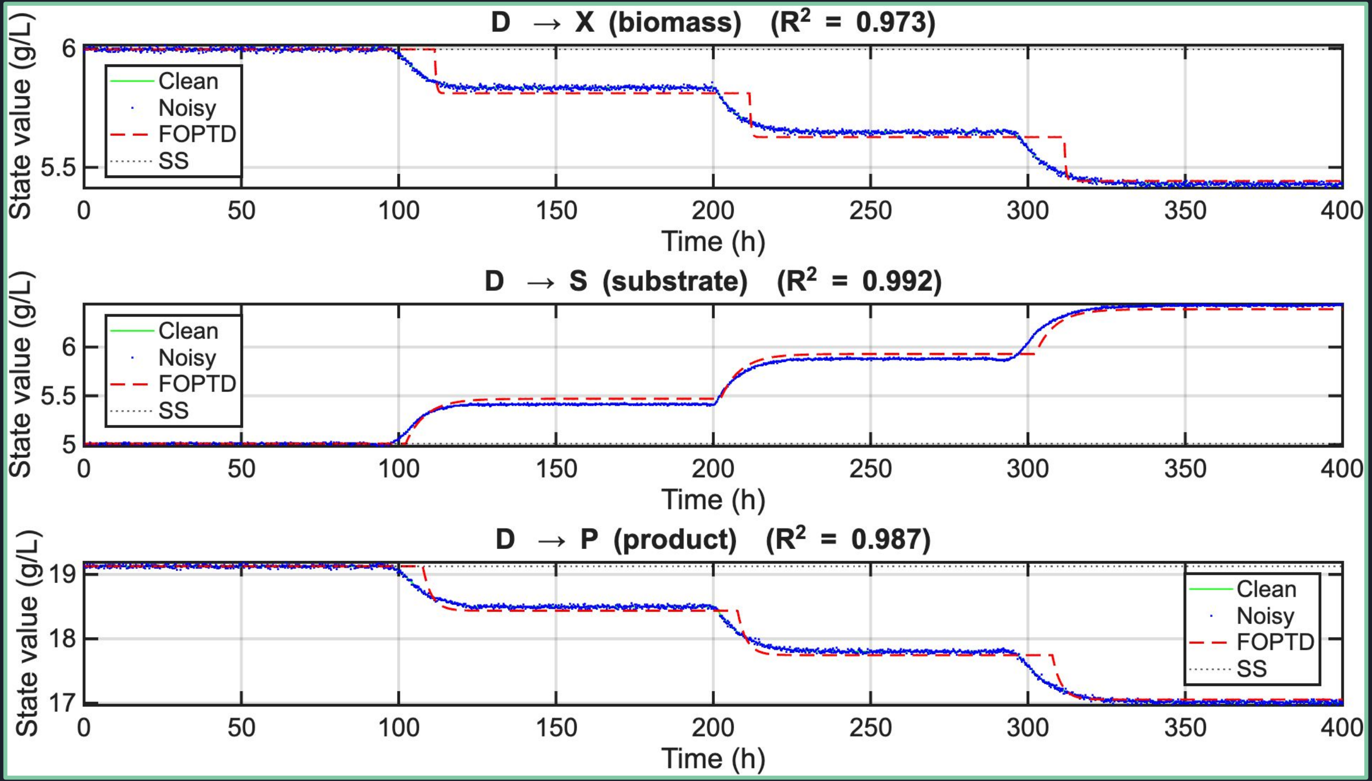
- Transfer function 1
  1) -0.201642
  2) -0.163435
- Transfer function 2
  1) -0.1632
  2) -0.2018
- Transfer function 3
  1) -0.2027 - 0.0017 *i*
  2) -0.2027 + 0.0017 *i*

As the zeros lie on the negative real axis, the system tends to be stable.



Other zeros have not been shown to prevent plot from being congested

FOPTD Model (Non-Linear Regression)



PROCESS PARAMETER VALUES USING NON-LINEAR REGRESSION

```
Kp_hat     = 45.6265
TauP_hat   = 0.543071
Theta_hat  = 11.3328
SSE        = 2.31616
```

X Vs D

- These are the predicted model parameters for each output.
- These have been calculated using fmincon function

```
Kp_hat     = -113.56
TauP_hat   = 6.3714
Theta_hat  = 2.30115
SSE        = 4.08225
```

S Vs D

```
Kp_hat     = 170.82
TauP_hat   = 2.84152
Theta_hat  = 7.59698
SSE        = 16.1207
```

P Vs D

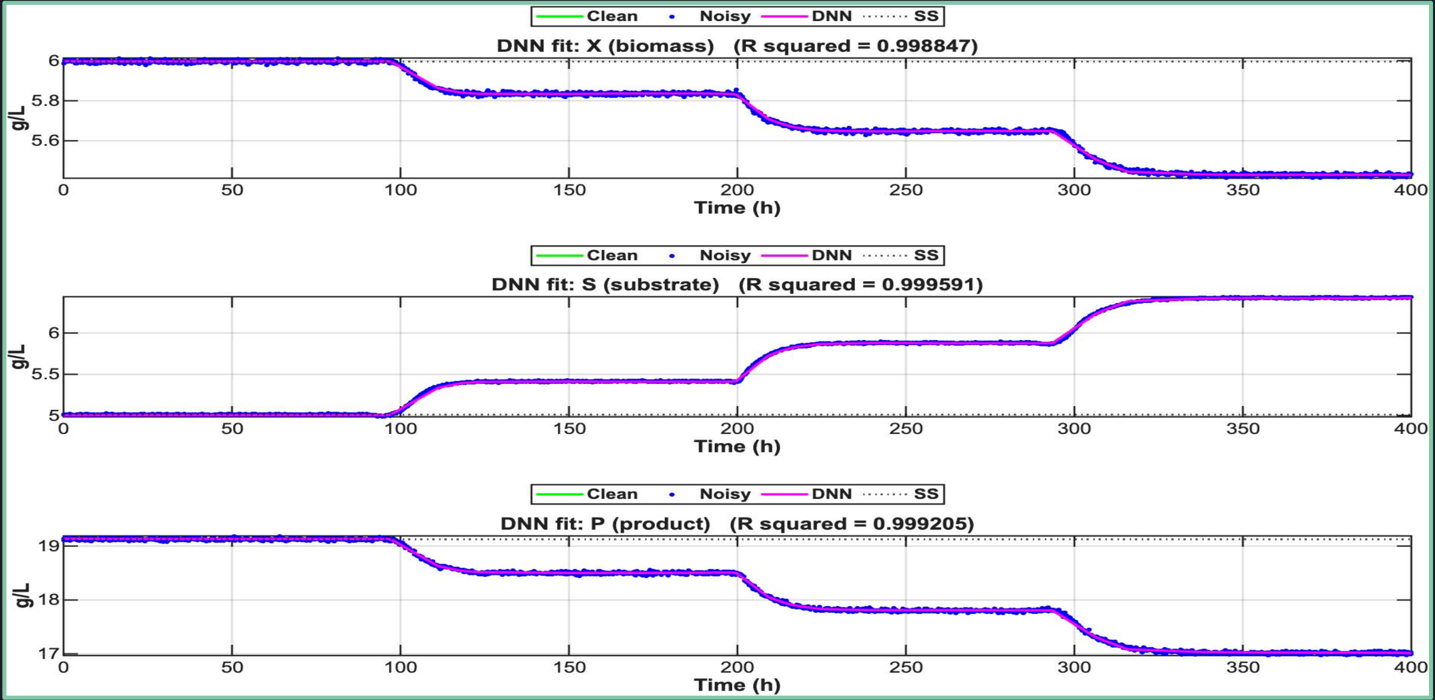
Reaction Curve Method (Intuitive)

- Apply a unit step change to input and record the output step response.
- Determine the process gain Kp from the final steady output change after the step.
- Normalize response by dividing by Kp, so it ideally goes from [0,1].
- Dead time Tau D is when the normalized response first reaches 1% of final value indicates delay before reaction starts.
- Time constant Tau P is the time to reach 63.2% of final value after Tau D indicates speed of system response.
- Fit the FOPTD model by comparing this analytical curve with the actual step response.

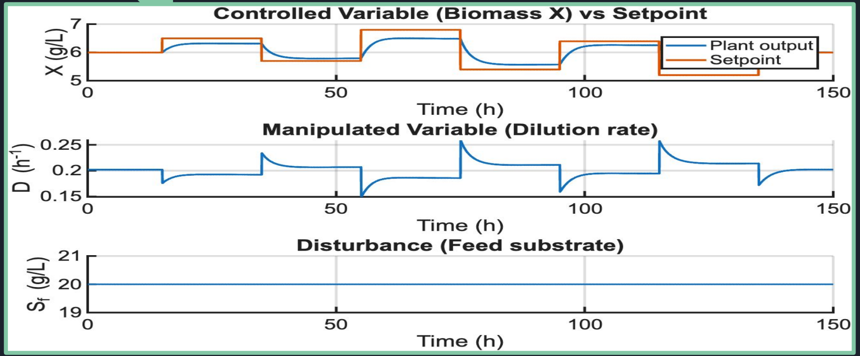
$$G(s) = \frac{K_P}{\tau_P s + 1} e^{-\tau_D s}$$

	Kp	Tau_P	Tau_D	R2
X	-37.262	5.6009	0.080013	0.99398
S	93.155	5.6009	0.080013	0.99398
P	-148.26	6.7811	0.080013	0.99516

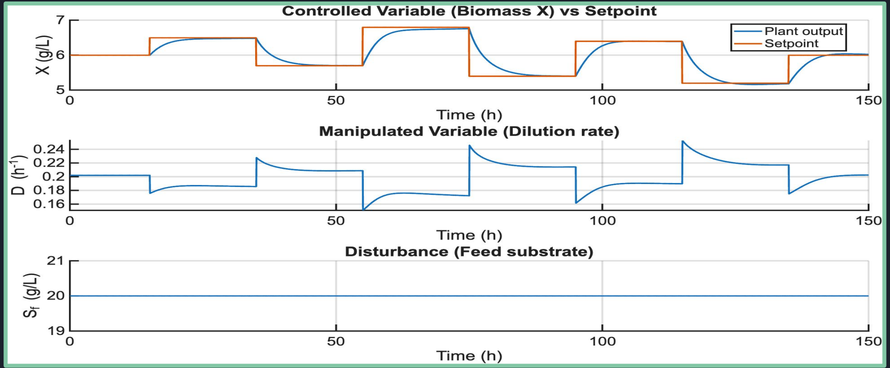
Deep Neural Network



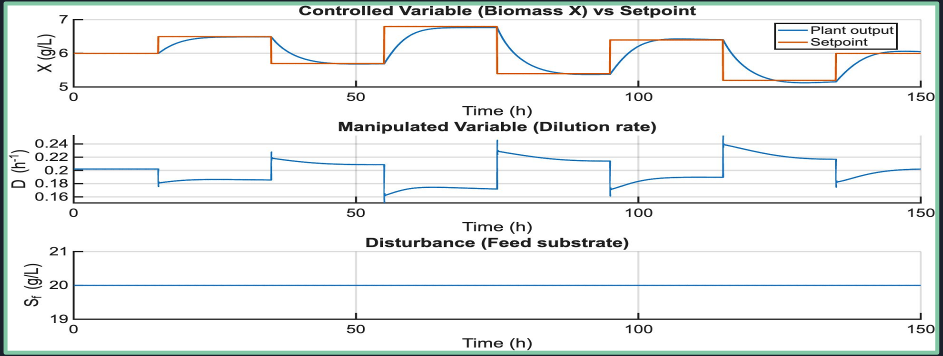
X (Biomass) - Servo Problem



P controller

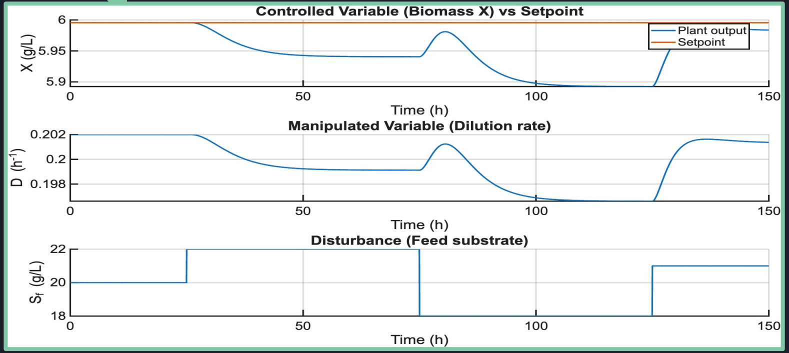


PI Controller

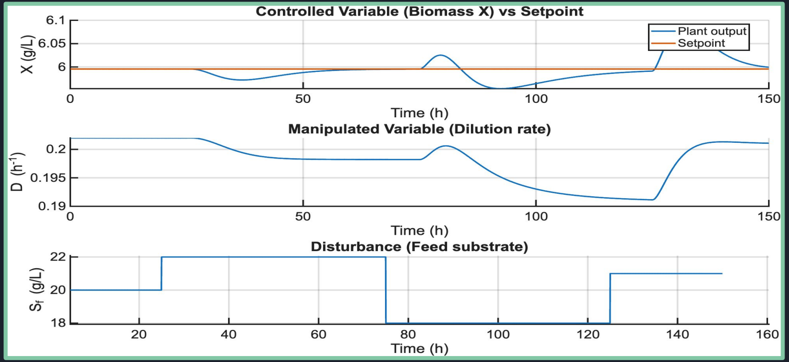


PID Controller

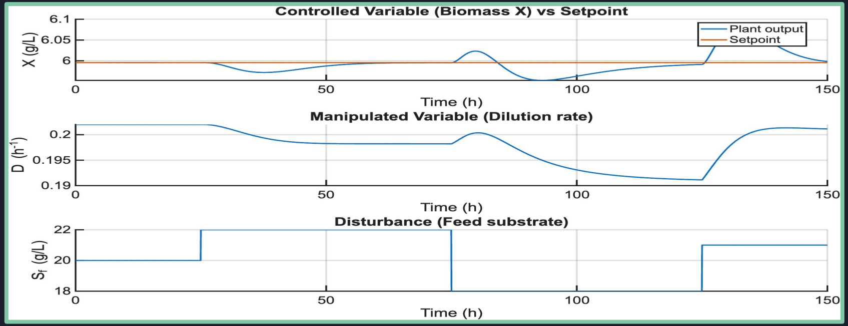
X (Biomass) - Regulatory Problem



P controller

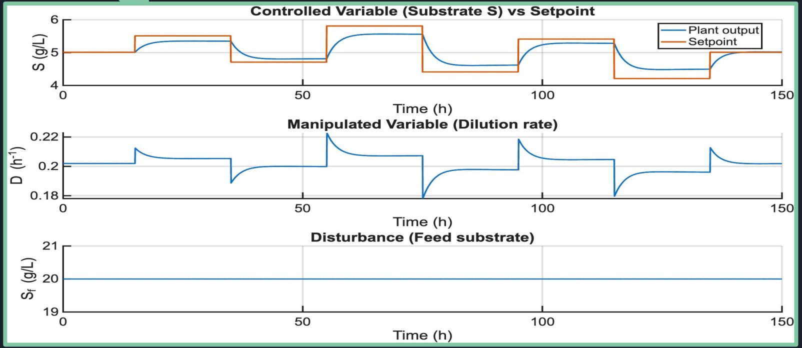


PI Controller



PID Controller

Substrate Concentration - Servo Problem



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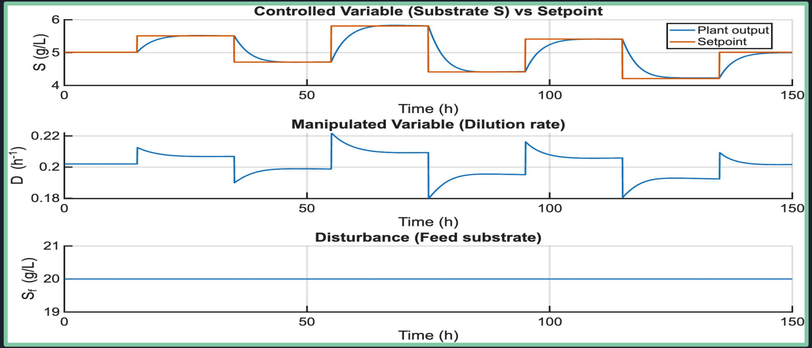
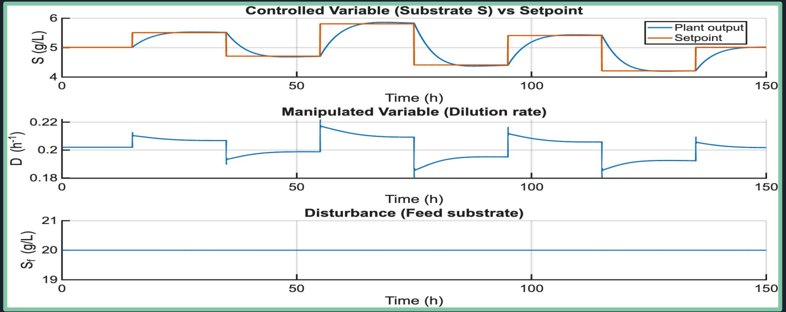
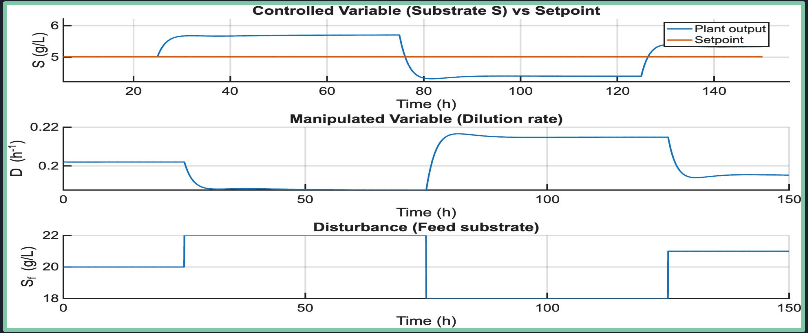


PI Controller



PID Controller

Substrate Concentration - Regulatory Problem



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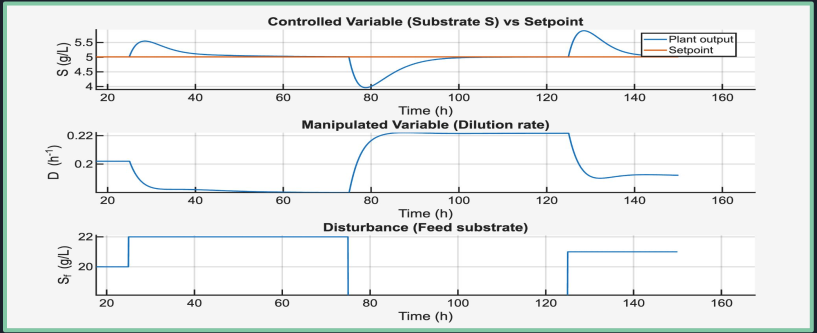
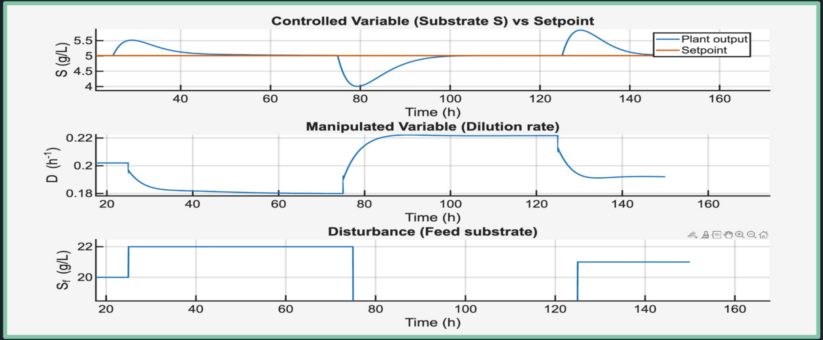
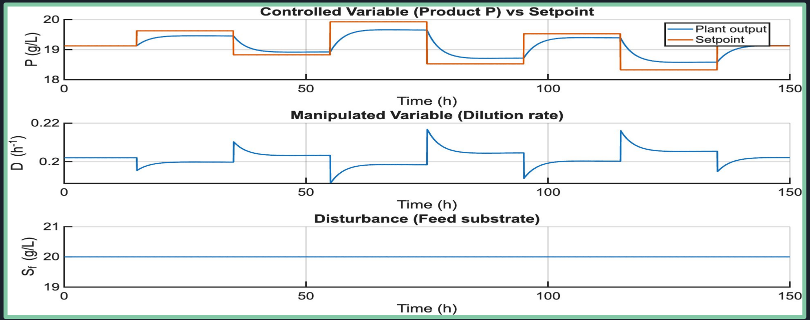


PI Controller

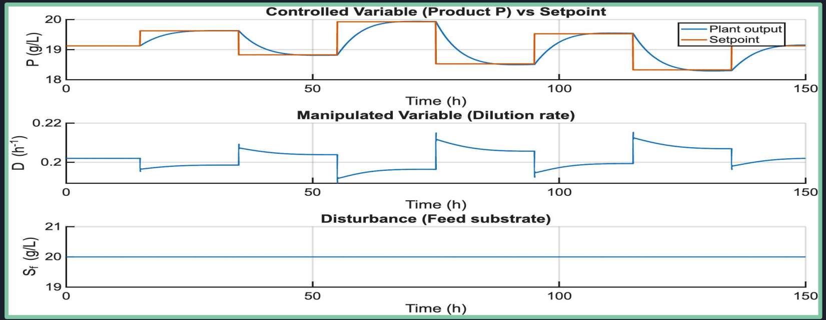


PID Controller

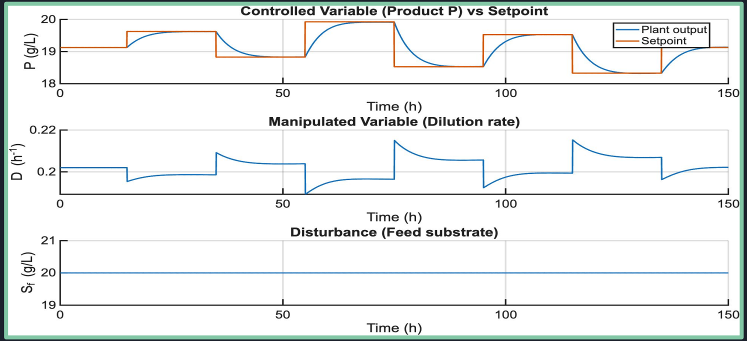
Product Concentration - Servo Problem



P controller

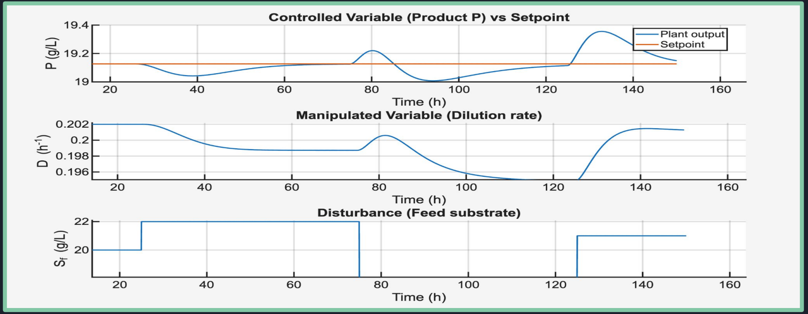


PI Controller

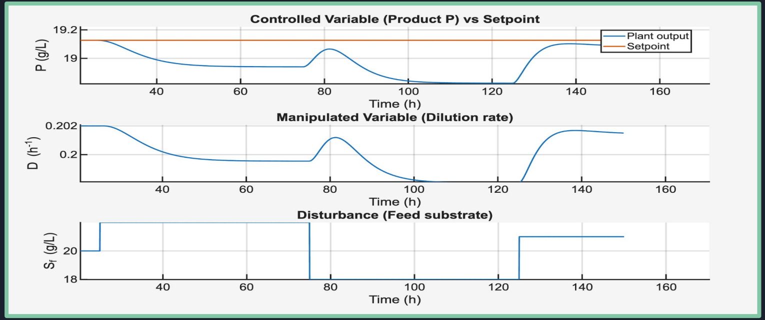


PID Controller

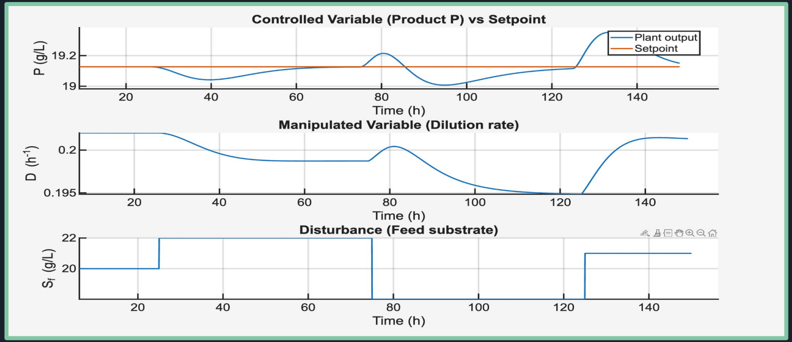
Product Concentration - Regulatory Problem



P controller



PI Controller



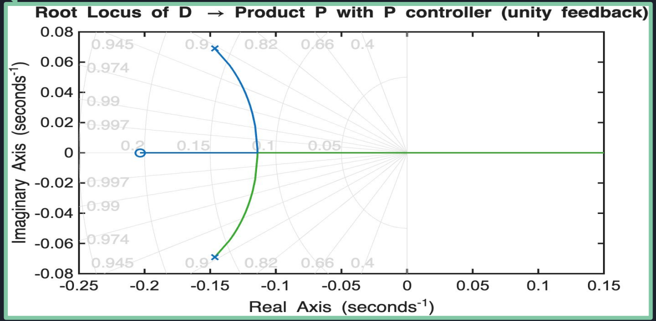
PID Controller

Kc values from Routh-Hurwitz Criterion

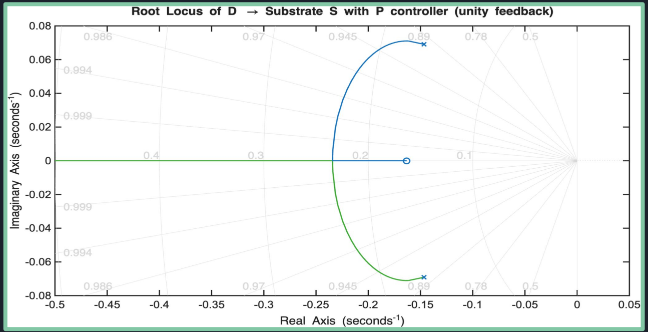
Kc range for finding actual range:- [-200,200]

- X (Biomass) Transfer Function:- [-200.0000, -0.0500]
- S (Substrate Conc.) Transfer Function:- [0.0500, 200.0000]
- P (Product Conc.) Transfer Function:- [-200.0000, -0.0500]

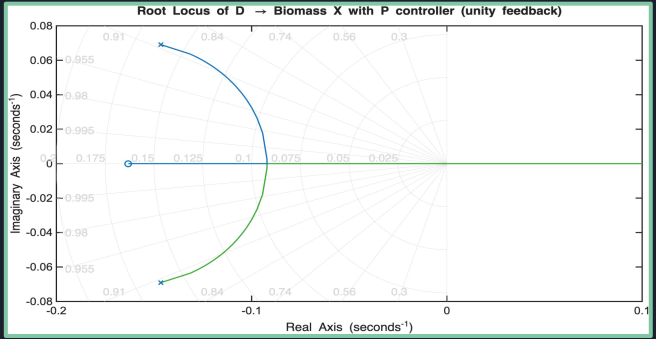
Root Locus Diagrams



X (Biomass)



S (Substrate Concentration)



P (Product Concentration)

THANK YOU !