DYNAMICS OF BIOREACTOR SYSTEM

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Linear vs non-linear

INTRODUCTION

System Equations

$$\frac{dX}{dt} = -DX + \mu(S,P)X$$

$$\dot{S} = -\mu(S)X/Y_{X/S} + D(S_f - S)$$

$$\frac{dP}{dt} = -DP + [\alpha\mu(S, P) + \beta]X$$

$$\mu(S,P) = \frac{\mu_m\left(1 - \frac{P}{P_m}\right)S}{K_m + S + \frac{S_2}{K_i}}$$

Parameter Values

Parameter	Values
$Y_{X,S}$	0.4 g/g
β	0.2 h^{-1}
P_m	50 g/L
K_i	22 g/L
α	2.2 g/g
μ_m	0.48 h^{-1}
k_m	1.2 g/L
S_f	20 g/L

Objective:-

- To study dynamics of Bioreactor system
- To compare linear vs non-linear model
- To discuss stability of zeros and poles

MODEL PARAMETERS AND DETAILS

– Step Change – 2% of D0 (steady state) at instants 100, 200, 300.
– Total Step Change – 8% of D0 (Steady State) between time instant 100 and 300.

– Transfer Functions between input and outputs :–

```
From input to output...
        -5.996 s^2 - 2.189 s - 0.1976
 1:  -----------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303

         14.99 s^2 + 5.473 s + 0.494
 2:  -----------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303

        -19.13 s^2 - 7.756 s - 0.7862
 3:  -----------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303
```

1. X(s) [Biomass]
2. S(s) [Substrate]
3. P(s) [Product]

After using minreal function

```
From input to output...
       -5.996 s - 0.9782
 1:  -----------------------
     s^2 + 0.2931 s + 0.02625

        14.99 s + 2.446
 2:  -----------------------
     s^2 + 0.2931 s + 0.02625

        -19.13 s - 3.892
 3:  -----------------------
     s^2 + 0.2931 s + 0.02625
```

STEADY STATE VALUES AND ANALYSIS

– Used Fsolve with different initial guesses and found majorly 2 steady states with values:-

Variable	Value at SS
X_ss	0
S_ss	20
P_ss	0

Variable	Value at SS
X_ss	5.99564
S_ss	5.01089
P_ss	19.12670

WHY?

Let’s do stability analysis and understand why one steady state is stable and other is unstable?

JACOBIAN MATRIX AND ITS EIGENVALUES

Unstable State Jacobian Matrix Eigenvalues

```
Eigenvalues of A:
   -0.2020
   -0.2020
    0.0418
```

Stable State Jacobian Matrix Eigenvalues

```
Eigenvalues of A:
  -0.1466 + 0.0691i
  -0.1466 - 0.0691i
  -0.2020 + 0.0000i
```

The complex exponential in turn can be written

$$e^{i\nu_j t} = \cos(\nu_j t) + i\sin(\nu_j t).$$

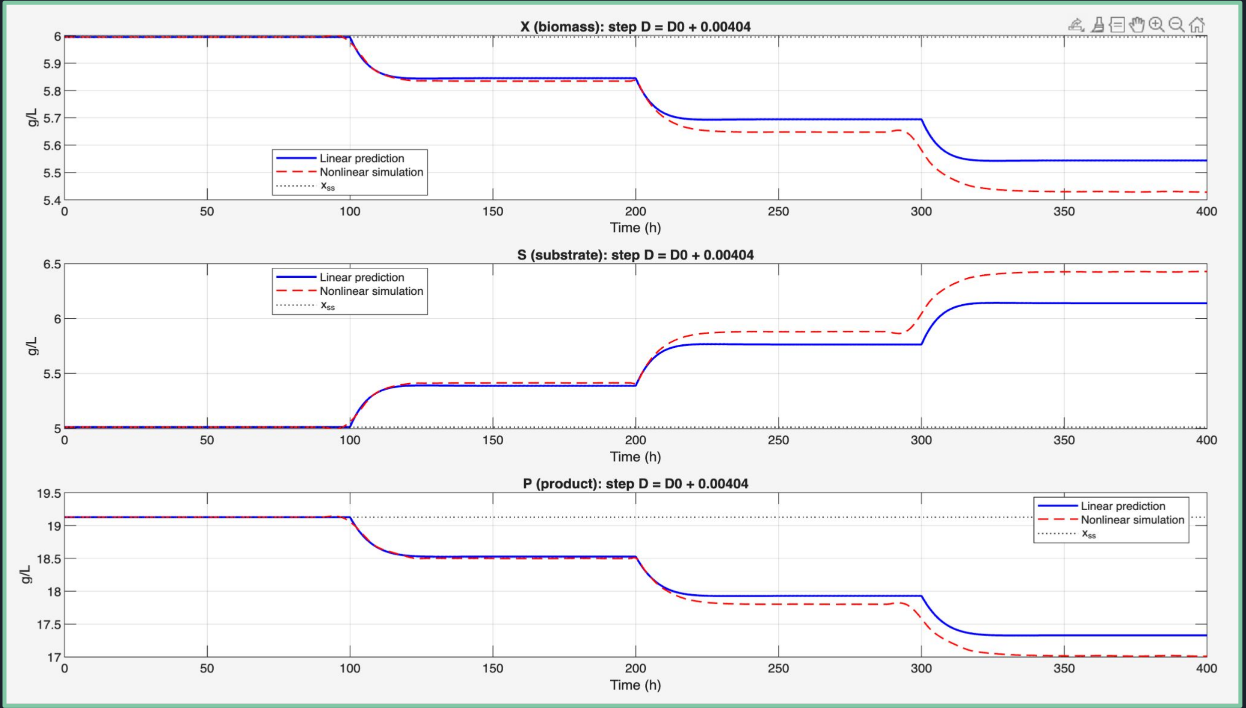
The complex part of the eigenvalue therefore only contributes an oscillatory component to the solution. It's the real part that matters: If $\mu_j > 0$ for any j, $e^{\mu_j t}$ grows with time, which means that trajectories will tend to move away from the equilibrium point. This leads us to a very important theorem:

Theorem 1 *An equilibrium point $\mathbf{x}^*$ of the differential equation 1 is stable if all the eigenvalues of $\mathbf{J}^*$, the Jacobian evaluated at $\mathbf{x}^*$, have negative real parts. The equilibrium point is unstable if at least one of the eigenvalues has a positive real part.*

Fun Fact

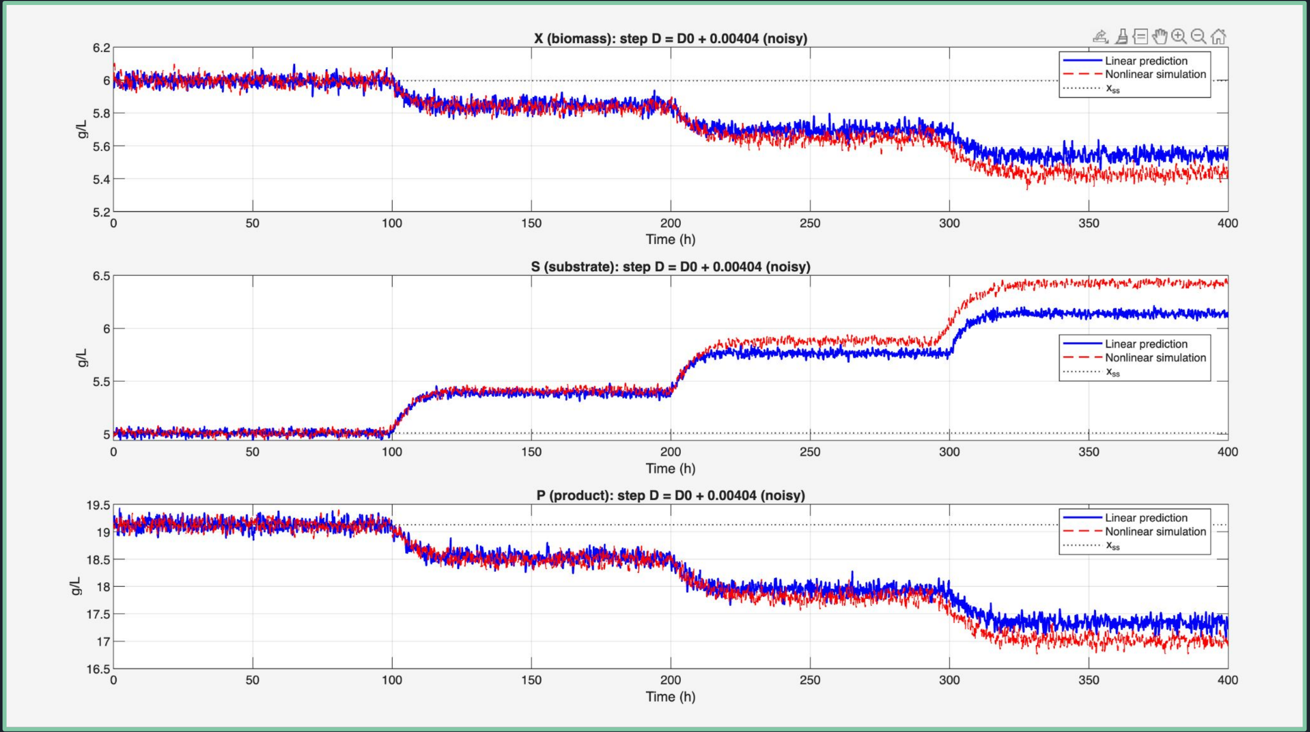
Poles of Transfer Function are in fact Eigenvalues of the Jacobian Matrix itself that is why we look for negative real values for a stable system.

MODEL RESULTS



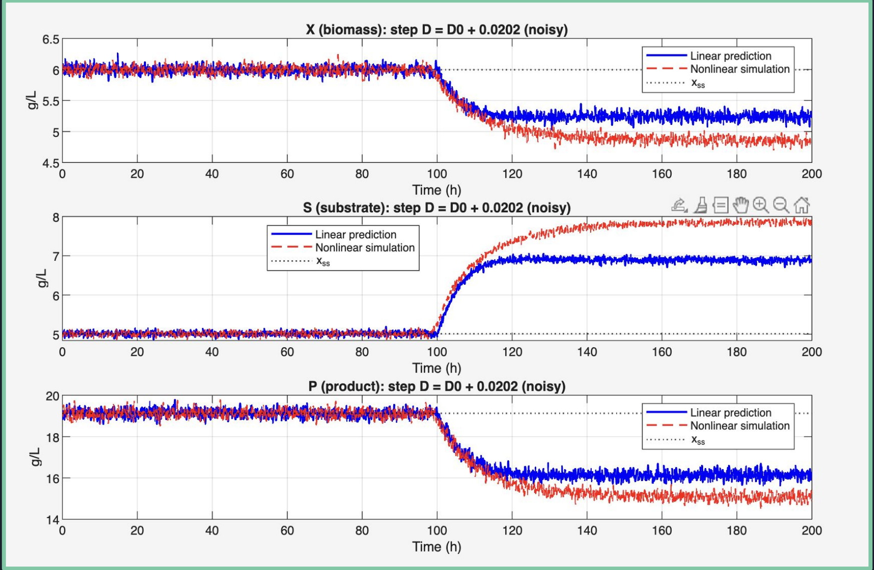
– Step Change – 2% of D0 (steady state) at instants 100, 200, 300.
– Total Step Change – 8% of D0 (Steady State) between time instant 100 and 300.

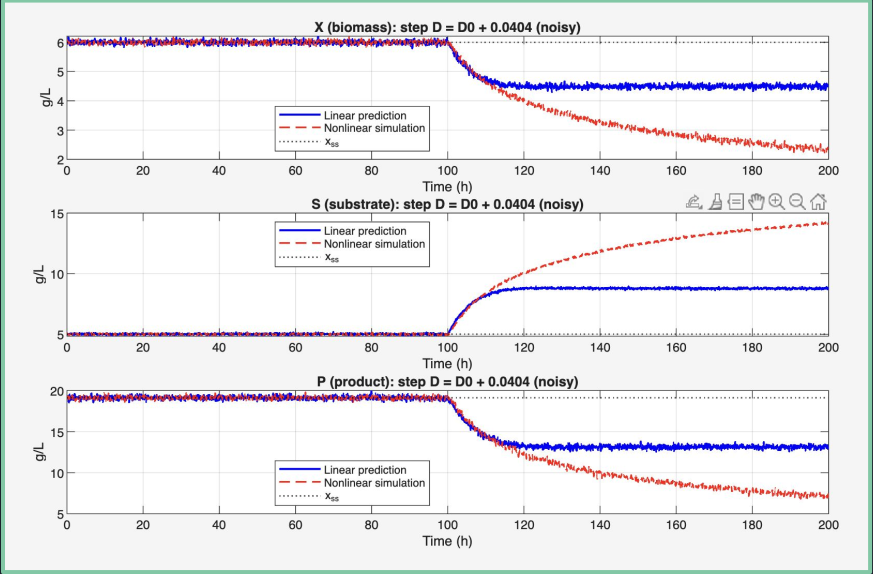
MODEL RESULTS [with gaussian noise]



– Step Change – 2% of D0 (steady state) at instants 100, 200, 300.
– Total Step Change – 8% of D0 (Steady State) between time instant 100 and 300.

Step Change of 10% Vs 20% of D0





As we increase deltaD (step change), the model drifts from the nonlinear solution since linearization holds only near steady state.

K_p And τ Calculations graphically (for 5% step Change)

To calculate K_p we will calculate the :- Output Change/Input Change

For Transfer Function 1 :-

$K_p = -37.26$
$\tau = 5.953$

For Transfer Function 3 :-

$K_p = -93.69$
$\tau = 6.853$

For Transfer Function 2 :-

$K_p = 93.155$
$\tau = 5.665$

Poles of the stable system

```
From input to output...
         -5.996 s^2 - 2.189 s - 0.1976
 1:  ---------------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303

          14.99 s^2 + 5.473 s + 0.494
 2:  ---------------------------------------
     s^3 + 0.4951 s^2 + 0.08546 s + 0.005303

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```

POLES

1) -0.1466 + 0.0691 *i*
2) -0.1466 - 0.0691 *i*
3) -0.2020

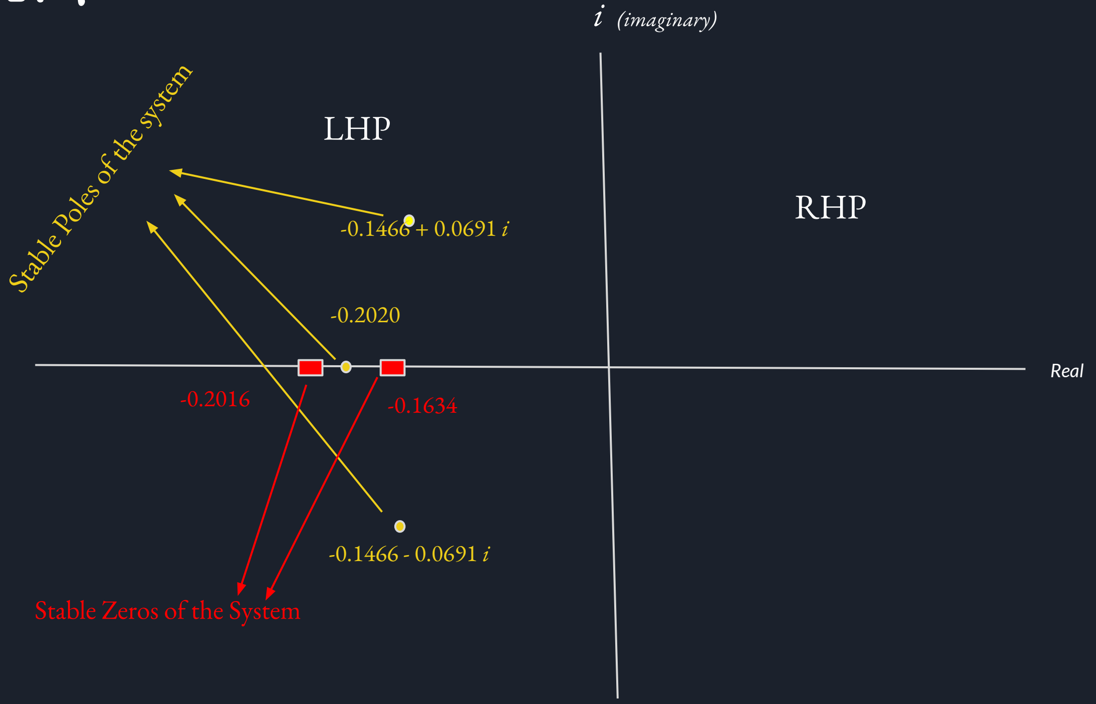
As first two roots lie on the left hand plane they show stable oscillatory behaviour as it is decaying. Whereas the pole lies purely on negative real axis and is stable.

Zeros of the System

Zeros

- Transfer function 1
1) -0.201642
2) -0.163435
- Transfer function 2
1) -0.1632
2) -0.2018
- Transfer function 3
1) -0.2027 - 0.0017 i
2) -0.2027 + 0.0017 i

As the zeros lie on the negative real axis, the system tends to be stable.



Other zeros have not been shown to prevent plot from being congested