

## Design PDA for the followings:

- a)  $\Sigma = \{x, y\}$ ,  $L = \{x^n y^n \mid n \geq 1\}$  Hints: Push x's, Then pop x's for y's in input string
- b)  $\Sigma = \{x, y\}$ ,  $L = \{x^n y^{2n} \mid n \geq 1\}$  Hints: for each x in input, push two x's for having the same no of x's & y's. Then pop x's for y's in input string
- c)  $\Sigma = \{x, y\}$ ,  $L = \{x^{2n} y^n \mid n \geq 1\}$  Hints: for every 2 x's, push only 1 x
- d)  $\Sigma = \{x, y\}$ ,  $L = \{x^n y^{n+1} \mid n \geq 1\}$  Hints:  $x^n y^n y$
- e)  $\Sigma = \{x, y\}$ ,  $L = \{x^m y^n \mid n > m \geq 0\}$  Hints:  $m \geq 0$ ,  $n > m$ ,  $n \geq m+1$ . So,  $x^m y^n = x^m y^{m+1} y \dots$  (at least one extra y at end of input string)
- f)  $\Sigma = \{a, b, c, d\}$ ,  $L = \{a^m b^n c^n d^m \mid m, n \geq 1\}$  Hints: a,d pair, b,c pair Push a's. Push b's. Pop b's for c's in input string. Pop a's for d's in input string
- g)  $\Sigma = \{a, b\}$ ,  $L = \{a^n b a^m b a^{m+n} \mid m, n \geq 1\}$  Hints:  $a^n b a^m b a^n$ . Push all a's, ignore b. Push all a's, ignore b. pop a's for a's in input
- h)  $\Sigma = \{x, y\}$ ,  $L = \{x^n y^m x^n \mid m, n \geq 1\}$  Hints: push all x's. Then transition for y. Ignore y's. pop x's for x's in input string
- i)  $\Sigma = \{x, y, z\}$ ,  $L = \{x^n y^m z^{(n+m)} \mid n, m \geq 1\}$  Hints: push all x's & y's. Then pop x's & y's for z's OR  
it can be solved with this logic  $x^n x^m z^m z^n$
- j)  $\Sigma = \{x, y, z\}$ ,  $L = \{x^{(n+m)} y^m z^n \mid n, m \geq 1\}$  Hints: push all x's. Pop x's for y's & pop x's for z's **sequentially**.  
OR it can be solved with this logic  $x^n x^m y^m z^n$
- k)  $\Sigma = \{x, y, z\}$ ,  $L = \{x^n y^{(n+m)} z^m \mid n, m \geq 1\}$  Hints:  $x^n y^n y^m z^m$ . push all x's, then pop x's for y's. when stack empty, push all y's, then pop y's for z's.
- l)  $\Sigma = \{x, y\}$ ,  $L = \{\text{no of x's are greater than the no of y's}\}$  Hints: push all x's and y's. Pop x's for y's (at least one x extra at end)
- m)  $\Sigma = \{x, y\}$ ,  $L = \{x^n y^m \mid m, n \geq 1 \text{ \& } m > n+2\}$   
Hints: minimum value of n is 1. So,  $m > n+2$ ,  $m > 1+2$ ,  $m > 3$ . minimum no of x is 1 & minimum no of y is 4.  
Again,  $m > n+2$ ,  $m \geq n+3$   
Push all x's in stack, then pop x's for y's. Then you should have at least 3 extra y's in the input String.
- n) Design a PDA to match parenthesis. Hints: Push all '('s. Pop '(' for ')' in the string.

## Conceptual Questions:

1. How can you relate a NFA and PDA?
2. How do we choose the next move in PDA?
3. What are the ways to accept strings in PDA?
4. How many arguments does a PDA take in its transition function?
5. How to convert an empty stack PDA into a final state PDA?
6. How to convert a final state PDA into an empty stack PDA?