

T A C D

- * symbol :- $\sigma, \Sigma, \emptyset, q_0$.
- * Alphabets :- $\{a, b\}; \{0, 1\}$
- * Strings :- a, ab, ba, b, aaa, \dots
- * Language :- No. of a's is even $\rightarrow aa, aaaa, \dots$

ϵ^0/E (epsilon) \rightarrow length of string is zero.
 $\epsilon^1 \rightarrow$ length of string is one.
 $\epsilon^* \rightarrow \epsilon^1 * \epsilon^2 * \epsilon^3 \dots$ // Kleen closure.

In eg { * Language \rightarrow Valid strings are part of language.
 * set of valid codes is string.

Finite Automata (FA)



* DFA :-

Defn :- $(Q, \Sigma, \delta, q_0, F)$

set of input alphabets $\{a, b\}$

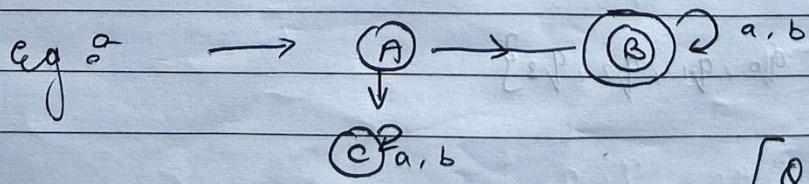
set of all states $\rightarrow \{A, B, C\}$

transition fn (δ)

starting state $q_0 \rightarrow a$

set of all final states $q_f \rightarrow \{B\}$

representation $\rightarrow Q \times \Sigma = Q$



In DFA with ~~one~~ one symbol it will go to only one state

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Question 1

Construct a DFA that accepts set of all strings over a, b of length 2.

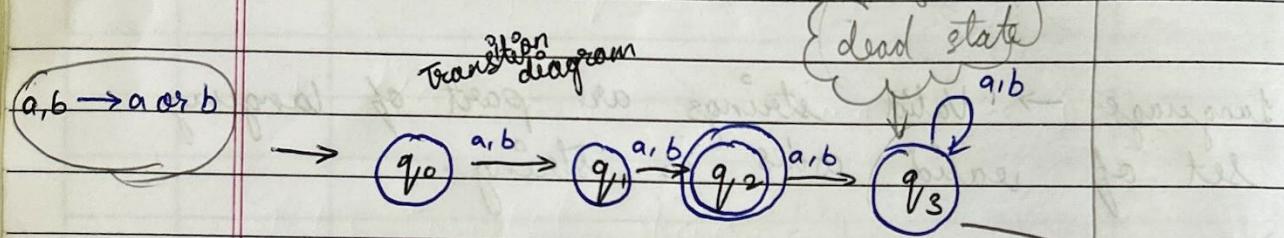
Solⁿ :-

$$\Sigma = \{a, b\}$$

$$\text{Language} = L = \{aa, ab, ba, bb\}$$

$$(Q, \Sigma, \delta, q_0, F)$$

$$\delta : Q \times \Sigma \rightarrow Q$$



length is given 2 as q_2 is final state.

If anything greater than 2 comes it will be transferred to death state

~~transition table~~

$Q \setminus \Sigma$	a	b
q_0	q_1	q_1
q_1	q_2	q_2
$* q_2$	q_3	q_3
q_3	q_3	q_3

$\rightarrow q_0 \text{ goes to } q_1$
 $\rightarrow q_1 \text{ goes to } q_2$
 $\rightarrow q_2 \rightarrow q_3$
 $\rightarrow q_3 \text{ goes to } q_3$

* indicates final state or \emptyset .

$$\text{Final} = F = \{q_2\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

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Simulation :-

i) Transition $\rightarrow \delta$

$$\delta(q_0, ab) = \delta(q_1, b)$$

(q_0 at $a \rightarrow q_1$)

$$= q_2$$

= Final state \rightarrow accepted

ii) $\delta(q_0, aab) = \delta(q_1, ab)$

$$= \delta(q_2, b)$$

$$= q_3$$

= dead state

= Rejected

Finite automata is said to accept a language if all the strings in that language are accepted and rejected by the string which are not in language.

Q 2

~~Design~~ Design DFA where length of word is greater than and equal to over a, b .

Solⁿ:

$$\Sigma = \{a, b\}$$

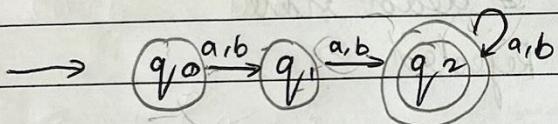
aa aag

$$|w| \geq 2$$

length of word or string.

$$L = \{aa, ab, ba, bb, aag, \dots\}$$

Take smallest string.



less than final is also reject

δ	a	b
q_0	q_1	q_1
q_1	q_2	q_2
* q_2	q_2	q_2

$$F = \{q_2\}$$

$$Q = \{q_0, q_1, q_2\}$$

Simulation :-

Transition $\rightarrow \delta(q_0, \underline{bbb}) \Rightarrow \delta(q_1, \underline{bb}) = \delta(q_2, \underline{b})$
 $= \cancel{\underline{q_2}}$

= Final state
= accepted

Q3

Design DFA where length of the word is ~~less than~~ less than 2 over a, b

Soln :-

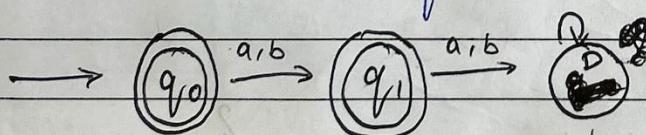
$$\Sigma = \{a, b\}$$

$$|w| < 2 \rightarrow 0, 1$$

$$L = \{a, b\}$$

$$E \rightarrow 0$$

→ Take smallest string.



Whenever there is E take first state as final state.

a Σ	a	b
q_0	q_1	q_1
q_1	D	D
D	D	D

$$F = \{q_0, q_1\}$$

$$Q = \{q_0, q_1, D\}$$

Transition $\rightarrow S(q_0, ab)$
 i) $= S(q_1, b)$
~~D~~
~~Dead state~~
~~Rejected~~

ii) $S(q_0, \epsilon)$
 $= q_0$
 $=$ Final state
 $=$ Accepted

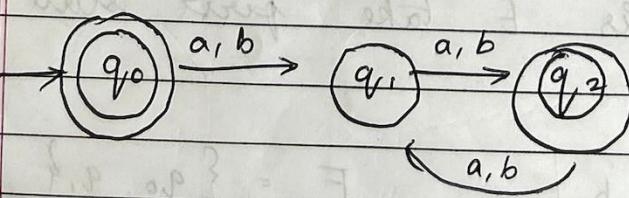
Design DFA where length of string is $\text{mod } 2 = 0$
 $\Sigma = \{a, b\}$

$$|w| \text{ mod } 2 = 0$$

[length of string is divisible by 2]

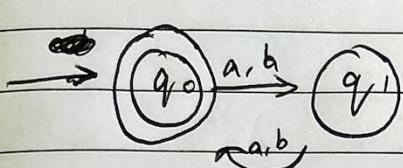
Sol:-

$$L = \{ \epsilon, aa, ab, ba, bb, aaa, \dots \}$$



Transition \rightarrow
 $S(q_0, ab)$
 $= S(q_1, b)$
 $= q_2$
 $=$ Final state
 $=$ accepted

minimum DFA :-



$\alpha \setminus \Sigma$	a	b	
*	q_0	q_1	q_1
*	q_1	q_0	q_0

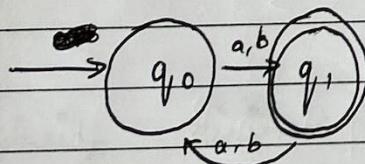
design DFA where remainder is one.

$$\Sigma = \{a, b\}$$

$$1 \% 2 = 1, 3, 5,$$

$$|w| \bmod 2 = 1$$

$$L = \{a, b, aaa, \dots\}$$



$\alpha \setminus \Sigma$	a	b	
*	q_0	q_1	q_1
*	q_1	q_0	q_0

$$F = \{q_1\}$$

$$\Delta = \{q_0, q_1\}$$

Transition :- $\rightarrow S(q_0, abb)$

$\rightarrow S(q_1, bb)$

$\rightarrow S(q_0, b)$

$\rightarrow S(q_1, q_1)$

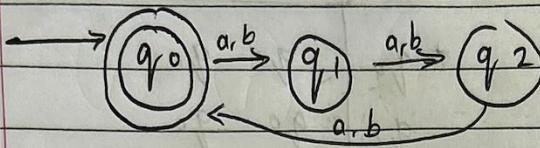
\rightarrow final state

\rightarrow accepted

#

$$|w| \bmod 3 = 0$$

\rightarrow AFD minimum



$$F = \{q_0\}$$

$$Q = \{q_0, q_1, q_2\}$$

<u>a</u>	<u>ϵ</u>	a	b
*	q_0	q_1	q_1
	q_1	q_2	q_2
	q_2	q_0	q_0

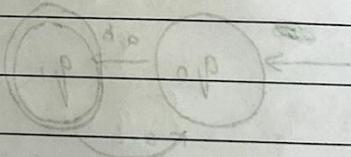
$$\text{Transition :- } \delta(q_0, aab) \rightarrow \delta(q_1, ab) \rightarrow \delta(q_2, b) \rightarrow \delta(q_0, a)$$

$= q_0 \rightarrow \text{Final state}$

$= \text{Accepted}$

#

$$|w| \bmod 3 = 1$$



$$F = \{q_1\}$$

$$Q = \{q_0, q_1, q_2\}$$

0 1 2

3 4 5

$$\text{Transition :- } \delta(q_0, aaba) \rightarrow \delta(q_1, aba)$$

$$\rightarrow \delta(q_2, ba)$$

$$\rightarrow \delta(q_0, a)$$

$\rightarrow q_1$

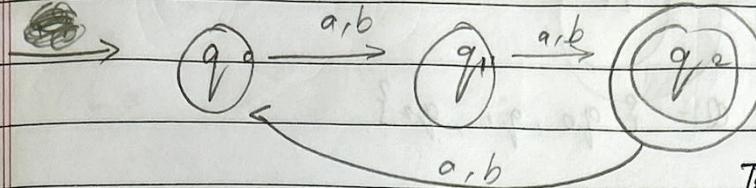
$= \text{Final state}$

$= \text{Accepted}$

<u>α</u>	a	b
q_0	q_1	q_1
*	q_1	q_2
q_2	q_0	q_0

$|w| \bmod 3 = 2$

$\alpha \setminus \epsilon$	a	b
q_0	q_1	q_1
q_1	q_2	q_2

Transition: $s(q_0, ab)$

$F = \{q_2\}$

$\delta = \{q_0, q_1, q_2\}$

$\rightarrow s(q_1, b)$

$\rightarrow q_2$

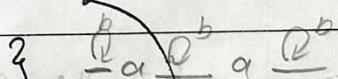
→ Accepted

Important Note :-

- If number ~~w~~ is divisible by 2 ; two states are required.
- If divisible by 3 then three states are required.

Design minimum DFA over input alphabet a, b where $n_a(w) = 2 \rightarrow \text{number of } a = 2$.

$L = \{aa, aab, baa, aba, \dots\}$



$F = q_2$

$Q = \{q_0, q_1, q_2, D\}$

$\alpha \setminus \epsilon$	a	b
q_0	q_1	q_0
q_1	q_2	q_1
$*q_2$	D	q^2

Transition s -

$= s(q_0, aaaaa)$

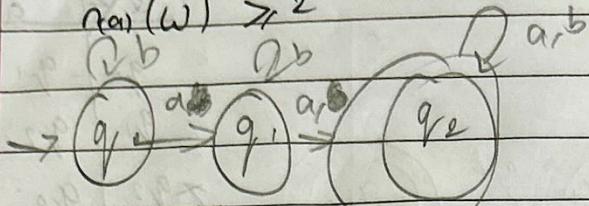
$= s(q_1, bbbbab)$

$= s(q_1, ab)$

$= s(q_2, b)$

$\therefore q_2 = \text{Final state}$

$$2) n_{\alpha}(\omega) \geq 2$$



$$F = \{q_2\}$$

$$\Omega = \{q_0, q_1, q_2\}$$

$$S = \epsilon \text{ hom } (\omega)$$

α^2	a	b
q_0	q_1	q_0
q_1	q_2	q_1
*	q_2	q_2

simulation $S(q_0, a)$

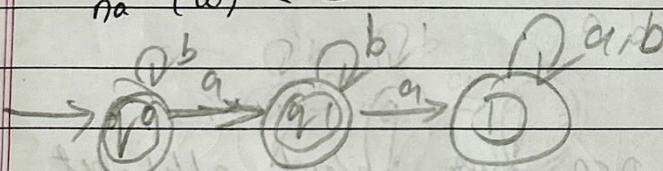
$$\rightarrow S(q_1, ab)$$

$$\rightarrow S(q_2, b)$$

$$\rightarrow q_2$$

Accepted.

$$3) n_{\alpha}(\omega) < 2$$



$$L = \{b, ab, abb, \dots\}$$

$$F = \{q_0, q_1\}$$

$$\Omega = \{q_0, q_1, D\}$$

$$S(q_0, b)$$

$$\rightarrow q_0$$

→ accepted

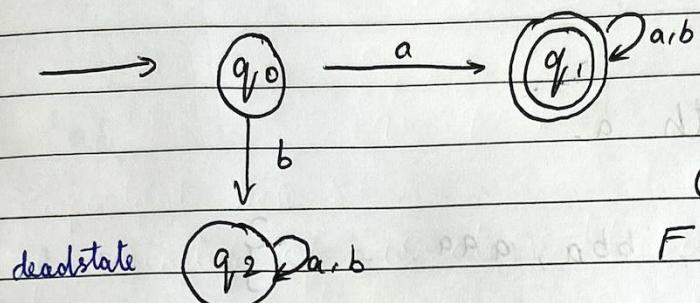
α^2	a	b
q_0	q_1	q_0
q_1	D	q_1

D	D	D

Design a DFA that accepts a set of all strings starting with a over input alphabet a, b.

Solⁿ:

$$L = \{a, aa, ab, abbb, \dots\}$$



$$\Omega = \{q_0, q_1, Dq_2\}$$

$$F = \{q_1\}$$

Transition:

$$\delta(q_0, a)$$

$$\delta(q_1, a)$$

$$q_1 = q_1$$

= Final state

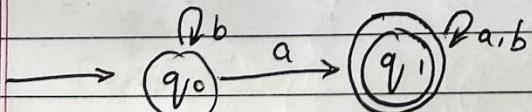
= accepted

$\alpha \setminus \Sigma$	a	b
q_0	q_1	q_2
q_1	q_1	q_1
q_2	q_2	q_2

Set of all strings containing a.

Solⁿ:

$$L = \{a, aa, ab, abbb, ba, \dots\}$$



$$\Omega = \{q_0, q_1\}$$

$$F = \{q_1\}$$

$$\delta(q_0, a)$$

$$\delta(q_1, a)$$

$$q_1 = q_1$$

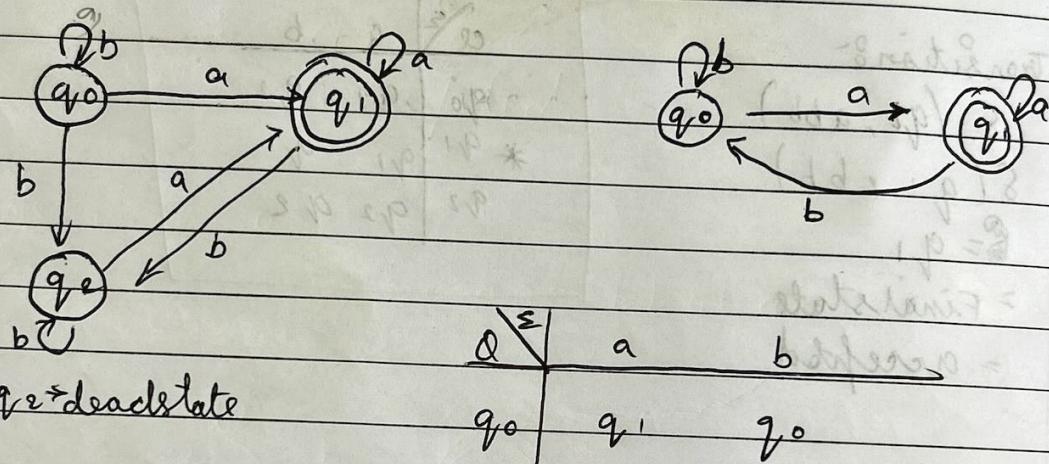
= Final state

= accepted

$\alpha \setminus \Sigma$	a	b
q_0	q_0	q_1
q_1	q_1	q_1

Ending with a.

$$L = \{ a, ba, bba, aaaa, \dots \}$$



Transition :-

$$\delta(q_0, abbaa)$$

$$\delta(q_1, bbbaa)$$

$$\delta(q_0, aa)$$

$$\delta(q_1, a)$$

$$= q_1$$

= Final state

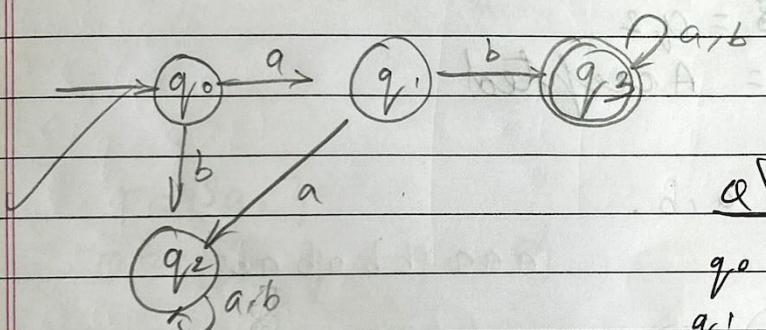
= accepted

construct a DFA which accepts set of all strings over input a, b which.

- i) $'ab' \rightarrow$ starts
- ii) contains ' ab '
- iii) ends with ' ab '

j) ' ab '

$$L = \{ab, abaa, abab, abbb, \dots\}$$



$\alpha \in$	a	b
q_0	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_3
$*q_3$	q_3	q_3

Transition :-
 $\delta(q_0, aba)$

$$= \delta(q_1, ba)$$

$$= \delta(q_2, a)$$

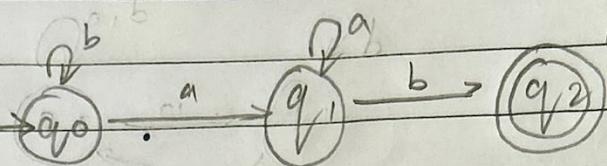
$$= q_3$$

Final state
accepted

$$F = \{q_3\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

2)



$$Q = \{q_0, q_1, q_2\}$$

$$F = \{q_2\}$$

α	ϵ	a	b
q_0		q_1	q_0
q_1		q_1	q_2
q_2	*	q_2	q_2

Transition :- $\delta(q_0, aab)$

$\delta(q_1, ab)$

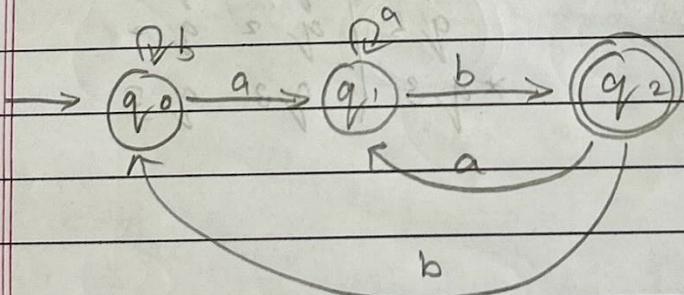
$\delta(q_1, b)$

$\delta = q_2$

= Accepted

3) ends with a.b.

aaa bb ab ab



$$Q = \{q_0, q_1, q_2\}$$

$$F = \{q_2\}$$

Transition :- $\delta(q_0, ab)$

$\delta(q_1, b)$

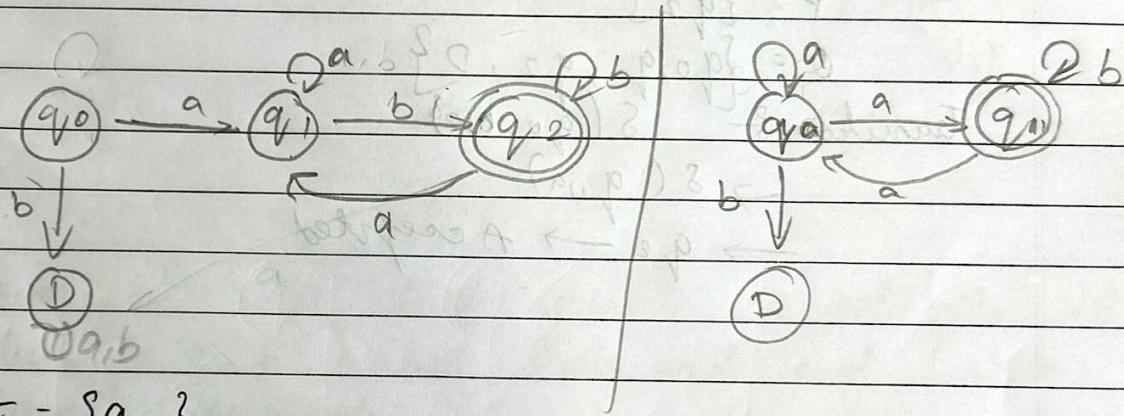
~~$\delta(q_2)$~~

~~Accepted~~

Construct a DFA which accepts set of all strings over input alphabet a,b.

- i) starts with a and ends with b.
- ii) starts with b and end with a.
- iii) starts with a and ends with a.
- iv) starts with b and ends with b.

i)



$$F = \{q_2\}$$

$$\Omega = \{q_0, q_1, q_2, q_3\}$$

Transition :-

$$\rightarrow q_0 (q_0, ab)$$

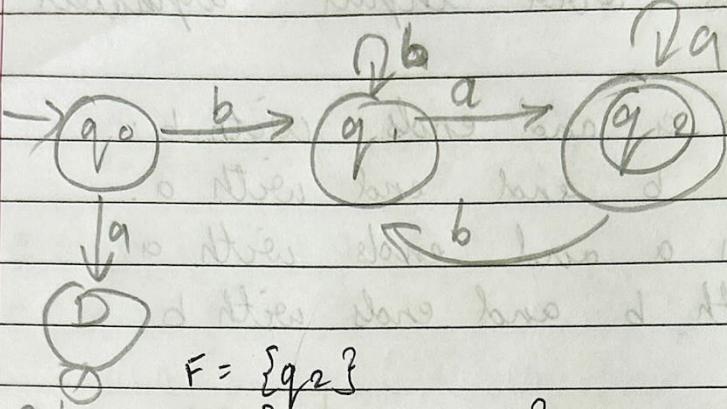
$$\rightarrow q_1 (q_1, b)$$

$$\rightarrow q_2$$

Accepted.

ii)

start b → end a



$$F = \{q_2\}$$

$$\Omega = \{q_0, q_1, q_2, D\}$$

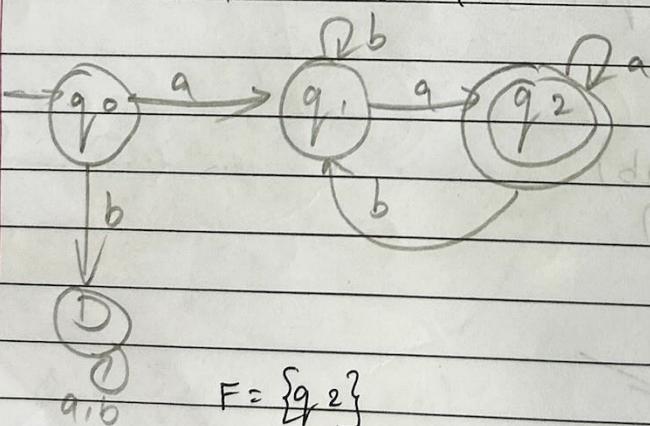
Transition $\stackrel{\delta}{\rightarrow} S(q_0, ba)$

$$\rightarrow \delta(q_1, a)$$

$\rightarrow q_2 \rightarrow \text{Accepted}$

iii)

starts a ends a :-



$$F = \{q_2\}$$

$$\Omega = \{q_0, q_1, q_2, D\}$$

Transition $\stackrel{\delta}{\rightarrow}$

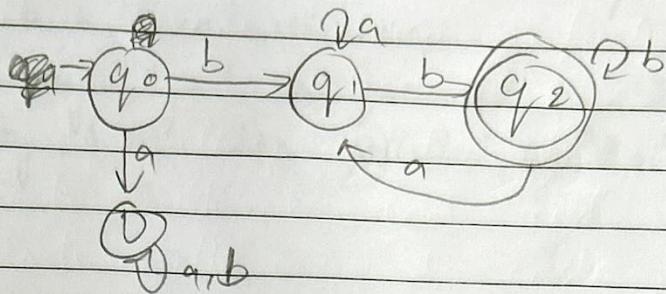
$$\rightarrow \delta(q_0, aa)$$

$$\rightarrow \delta(q_1, a)$$

$\rightarrow q_2$

$\rightarrow \text{Accepted}$

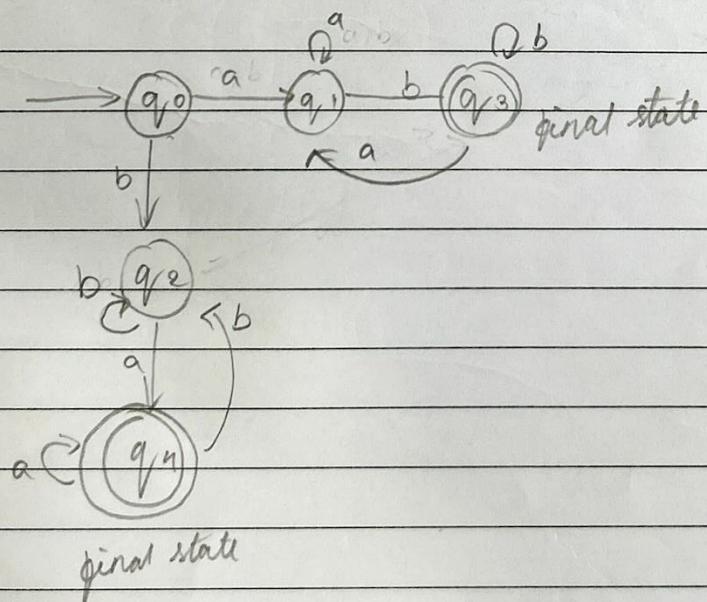
iv starts b ends with b :-



Construct a DFA which accepts a set of all strings including a, b and starts and end with different symbols.

A 18

$$L = \{ abbab, aab, abaab, baaba, \dots \}$$



		abaaaab		
		a	b	
		q0	q1	q2
	*	q0	q1	q2
	*	q1	q1	q3
	*	q2	q4	q2
*	q3	q1	q3	
*	q4	q4	q2	

$$F = \{q_3, q_4\}$$

$$\delta = \{q_0, q_1, q_2, q_3, q_4\}$$

#

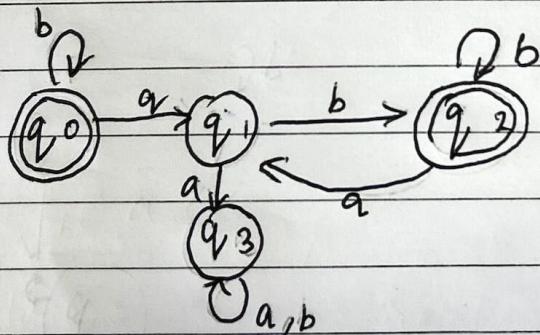
starts and ends with same ~~the~~ symbol

Q) Construct a DFA which accepts set of all strings of a, b in which

- i) Every ' a ' is followed by a ' b '
- ii) Every ' b ' is followed by a ' a '

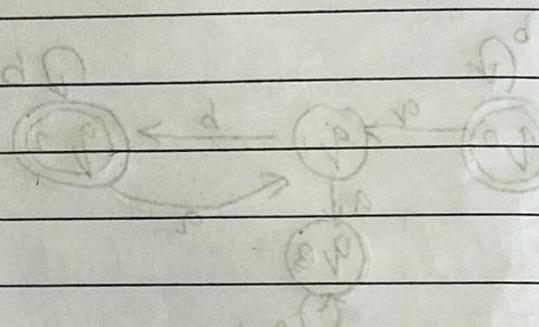
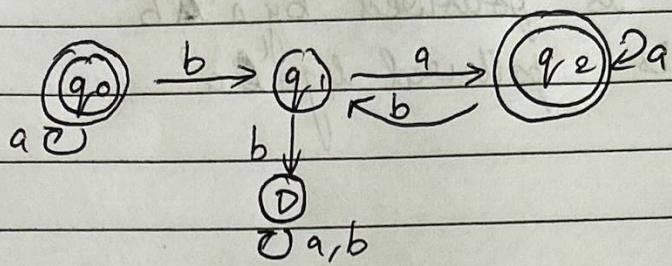
i)

$$L = \{b, ab, abbbab, \dots\}$$

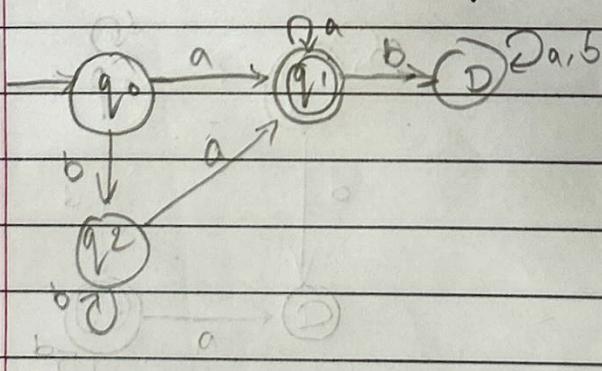


ii)

$$L = \{ba, a, baaaba, \dots\}$$



#) construct a MDFA which accepts all strings over 'a,b' in which every 'a' should not be followed by 'b'



ii) 'b' shouldn't be followed by 'a'

In which every a should be followed by "bb"

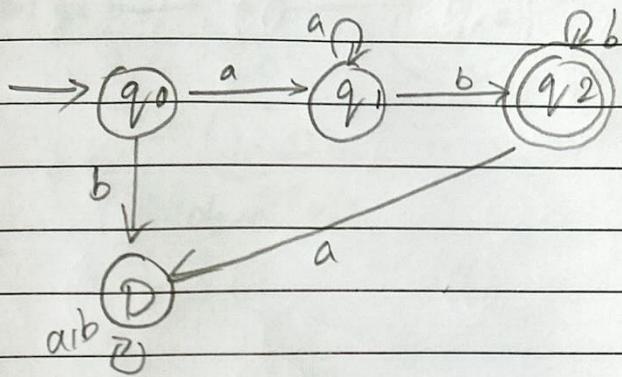
"p" and "n" both go together and "t" also goes with "d".

Every a should not be followed by "bb."

M DFA

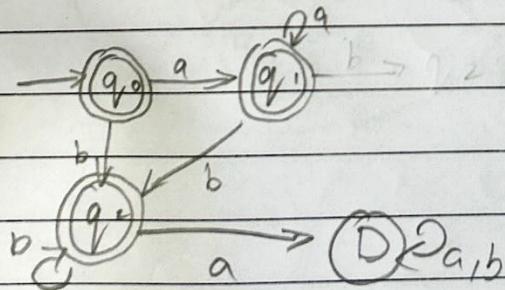
i) $L = \{a^n b^m \mid n, m \geq 1\}$

$$L = \{ab, aab, abb, \dots\}$$



ii) Q) $L = \{a^n b^m \mid n, m \geq 0\}$

$$L = \{ \epsilon, a, ab, aabb, abbb, \dots \}$$



(iii))

$$L = \{ a^i b^j c^k \mid i, j, k \geq 1 \}$$

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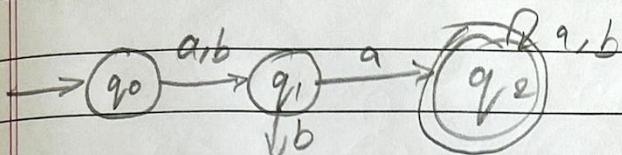
(iv))

$$L = \{ a^l b^m c^n \mid l, m, n \geq 0 \}$$

construct minimum DFA which accepts set of all strings over input {a, b} such that second symbol from RHS is a.

$$\rightarrow L = \{ \underline{a}, \underline{aa}, \underline{ba}, \underline{baa}, \underline{bab}, \dots \}$$

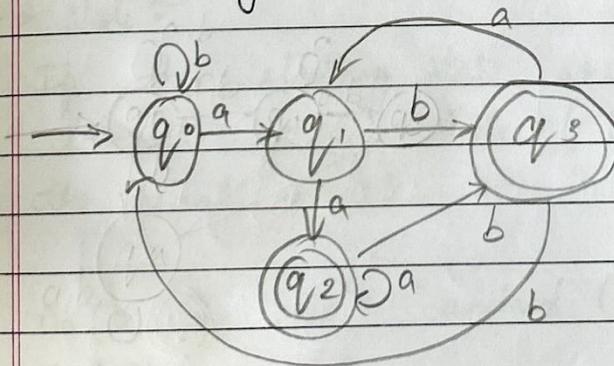
aabbab



D_{2a,b}
dead state

second symbol from RHS is a.

abbaaabab



3rd from LHS is a ~~thin~~ broad

Third from RHS is a ~~broad~~ broad

Construct a DFA which accepts all strings input of $\{a, b\}$ which accept strings of form $a^3b^w a^2$

$2/2 = \textcircled{C}$

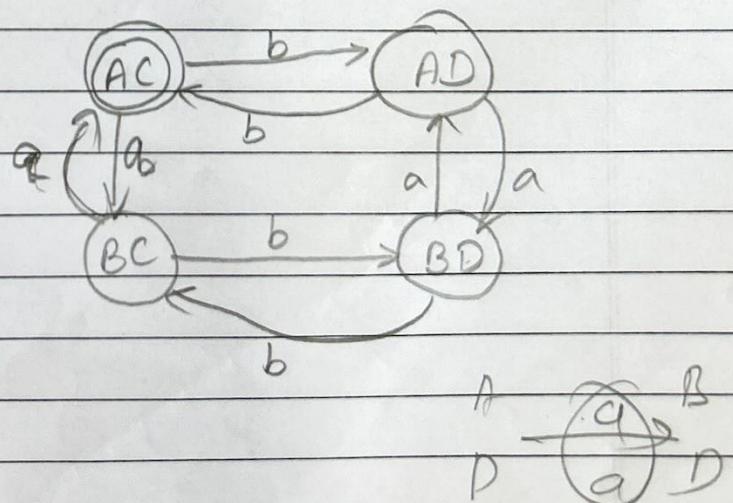
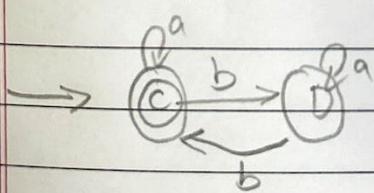
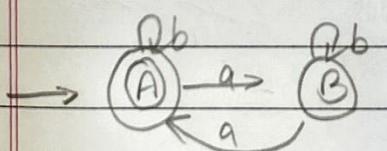
Cross Product Method of DFA :-

→ Note :-

If the starting state are coming together then in the product state (They both together ~~are~~ become the final state).

$$n_a(w) \equiv_0 \text{mod } 2 \text{ and and } (f f)$$

$$n_b(w) \equiv \text{mod } 2$$



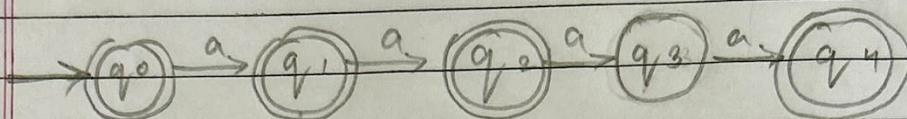
A \xrightarrow{a} B
D \xrightarrow{a} D

#

$$\{ a^n \mid n \geq 0, n_b = 3 \}$$

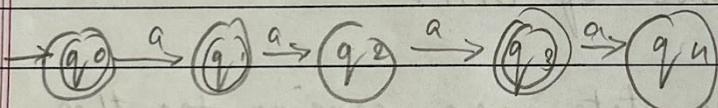
$$L = \{ \epsilon, a, aa, aaaa \}$$

Make for $n_b = 3$; then apply reversal rule and make initial state final and vice versa.



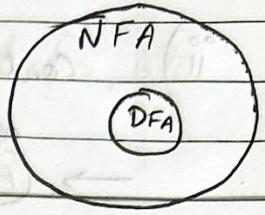
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$$\{ a^n \mid n \geq 0, n_b = 2, n_a = 4 \}$$



NFA

(Q, Σ, S, q_0, F)



$\delta: Q \times \Sigma \rightarrow 2^Q$

Transformation

DFA Q

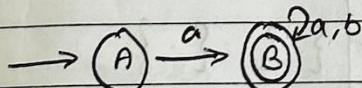
You can solve DFA for
all NFA ~~eg~~ questions.

Construct NFA for following language.

- i) starts with 'a'
- ii) contains 'a'
- iii) ends with 'a'

i)
starts with a

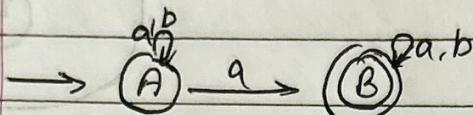
$$L = \{ a, aa, abb, \dots \}$$



	a	b
A	{B}	{}
B	{}	{B}

ii)

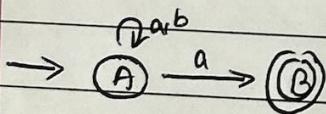
Contains 'a'.



	a	b
A	{A,B}	{A}
B	{B}	{B}

iii)

Ends with 'a' :-

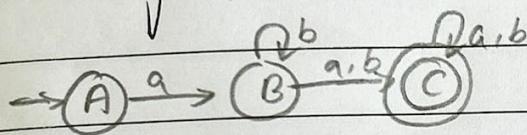


	a	b
A	{A,B}	{A}
B	{ }	{ }

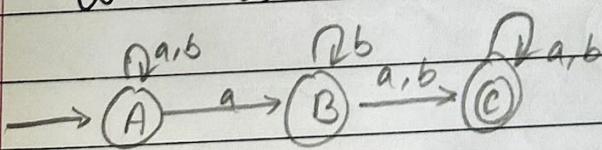
Design NFA over input alphabet {a, b} on following language.

- i) string starts with ab
- ii) contains ab
- iii) ends with ab

- i) string starts with ab

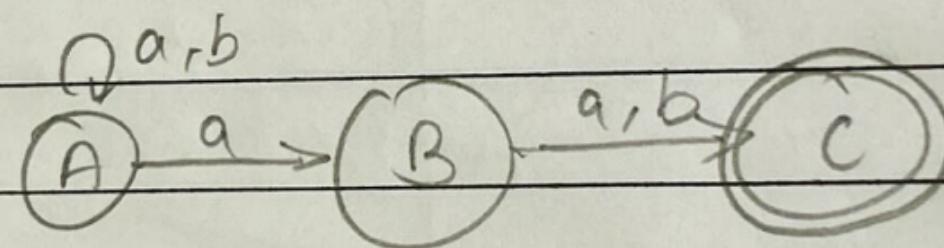


- ii) contains ab



second from RHS is a.

$$L = \{ aa, ab, aab, \cancel{baa}, \dots \}$$



DFA ($M \cdot W$)

① $n_a(\omega) \cong 0 \pmod{3} \rightarrow$ divide by 3 $\rightarrow 0$
 $n_b(\omega) \cong 0 \pmod{2} \rightarrow$ all b's $\rightarrow 0$

② $n_a(\omega) \pmod{3} = 1 \rightarrow$ by 3 \rightarrow rem=1
 $n_b(\omega) \pmod{3} = 2 \rightarrow$ by 3 \rightarrow rem=2

③ $n_a(\omega) \pmod{3} > n_b(\omega) \pmod{3}$

$n_a \text{ rem} > n_b \text{ rem}$

①

(0, 1) 0000

Construct a minimal DFA $\{0, 1\}$ when interpreted by binary number is divisible by 2.

Divisible by 3

divisible by 4.

Accepts set of all strings for input alphabet {0, 1, 2} when interpreted as ternary number when divisible by 4.

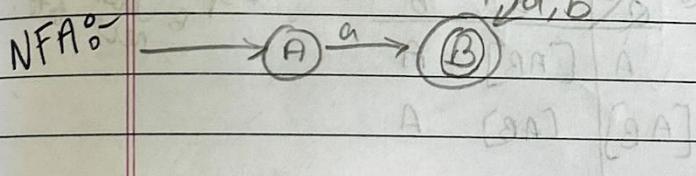
9) can have many end states.

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NFA to DFA :-

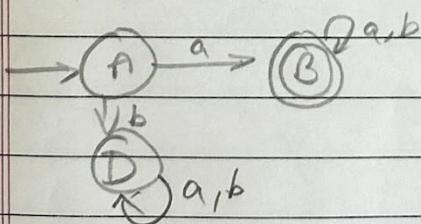
Rules :-

- 1) Starts with initial state
- 2) Find all reachable states from initial state.
- 3) If there is no state for any specific symbol, ~~consider~~ create dead state.
- 4) The states we get in NFA table, we should consider only that states in DFA table.
- 5) In NFA it goes ~~to~~ $\{A, B\}$, write it $[AB]$
- 6) Take union in NFA of A and B for new state $[AB]$
- 7) If it contains B (dead state) in comb take comb as end state
- 8) Construct NFA for given language and convert into DFA over input alphabet $\{a, b\}$ where all strings starts with a.



a, b	a	b
A	B	∅
*B	B	make dead state

→ DFA



a, b	a	b
A	B	D
*B	B	B
D	D	D

Take B and new state

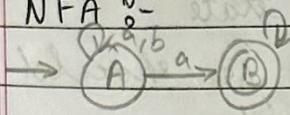
construct NFA for input $\{a, b\}^*$

NFA to DFA. \rightarrow Ending with a.

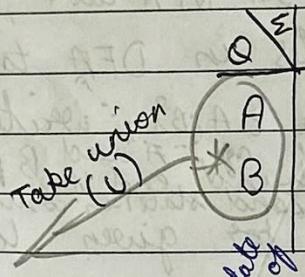
For given language

$$L = \{a, aa, baa, aba, \dots\}$$

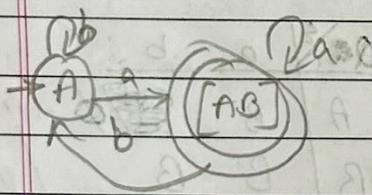
NFA :-



NFA



DFA :-



DFA

Take new state version of

Q	a	b
A	[AB]	A
[AB]	[AB]	A

* If we don't get
B in RHS table
don't take it.

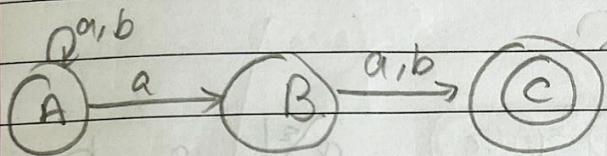
* If we get
B then create
new state

2nd

From RHS is a .

$$L = \{ ab, aab, \dots \}$$

NFA :-

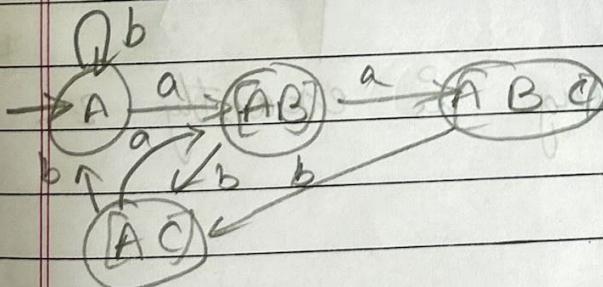


NFA

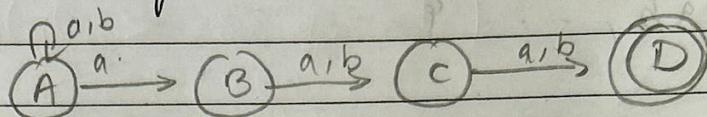
α/ϵ	a	b
A	{A, B}	A
B	C	C
C	\emptyset	\emptyset

union

DFA :-

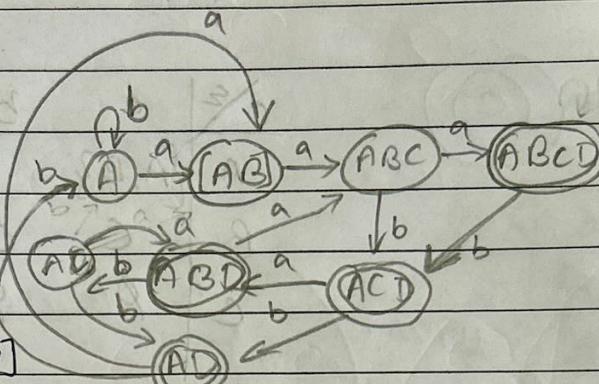


α/ϵ	a	b
A	[AB]	A
[AB]	[ABC]	[AC]
* [AG]	[ABq]	[AC]
* [AC]	[AB]	A

i)Design NFA \rightarrow DFA3rd from RHS is q.

Q/Σ	a	b
A	[A, B]	A
B	C	C
C	E	F
D	X	X

Q/Σ	a	b
A	[AB]	A
[AB]	[ABC]	[AC]
[ABC]	[ABCD]	[ACD]
[AC]	[PBD]	[AD]
[ACD]	[ABCD]	[ACD]
[ACD]	(ABC)	[AO]
[ABD]	[ABC]	[AC]
[AD]	[AB]	[A]



Q/Σ	a	b
A	[A, B]	A
[A, B]	[ABC]	[AC]
[ABC]	[ABCD]	[ACD]
[AC]	[ABD]	[AD]
[ABD]	[ABC]	[ACD]
[AD]	[AB]	[AO]

Length of the string is exactly 2.

O
H H
G - O

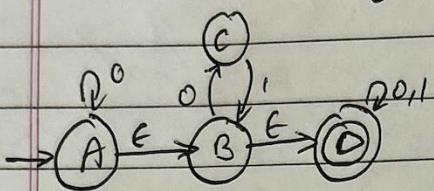
length of the string is almost 2.

at least 2.

#

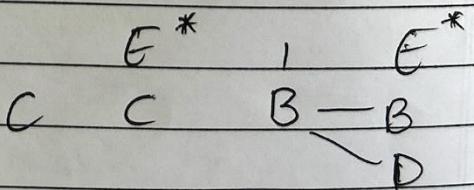
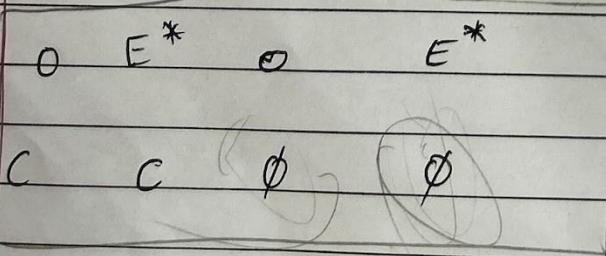
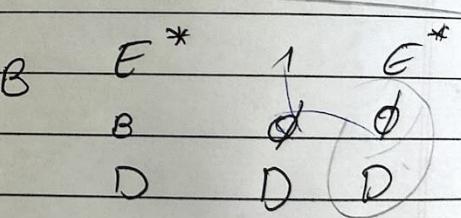
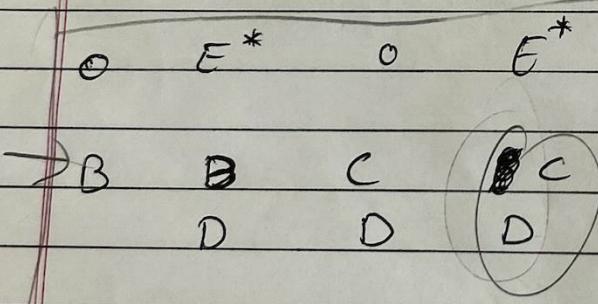
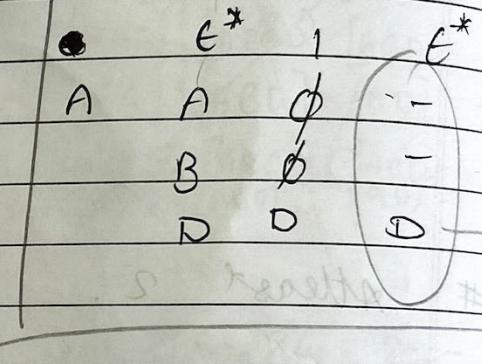
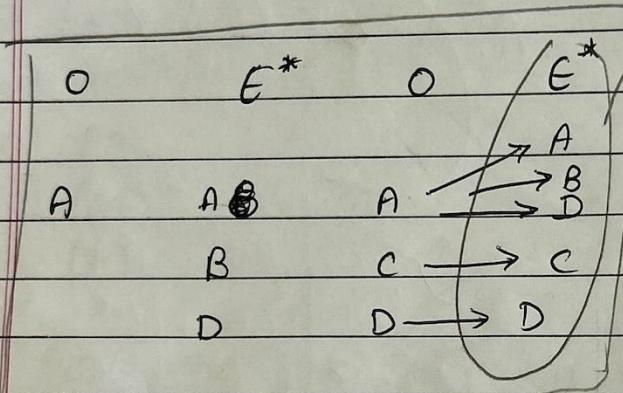
E NFA :-

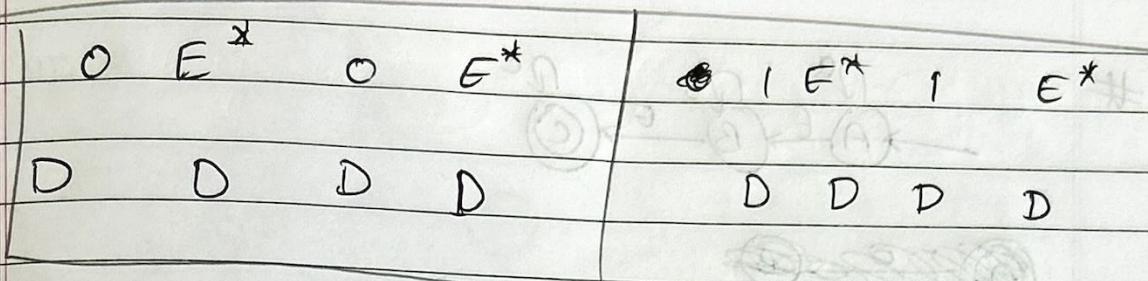
ENFA

 $(Q, \Sigma, \delta, q_0, F)$ $\delta: Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$ 

C	ϵ	0	1
A	$\{A, B, D\}$	A	\emptyset
B	$\{B, D\}$	C	\emptyset
C	$\{C\}$	\emptyset	B
D	D	D	D

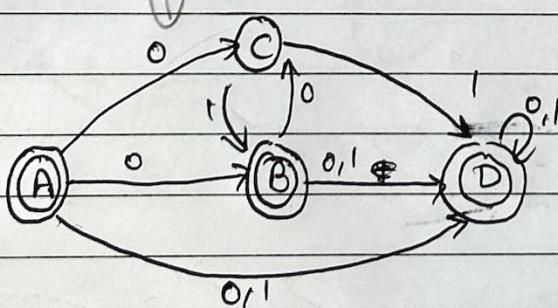
Set



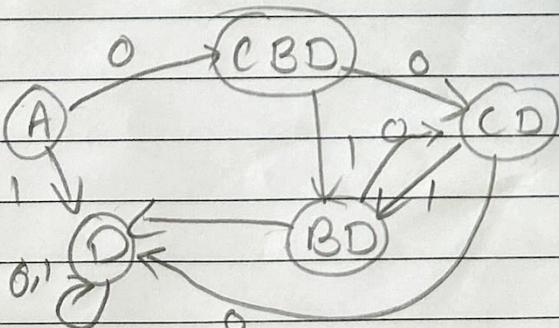


O/E	0	1
A	{A, B, C, D}	{D}
B	{C, D}	{D}
C	{ }	{B, D}
D	{D}	{D}

Whatever state that can reach the final state from some state by E. Those all states are final states.

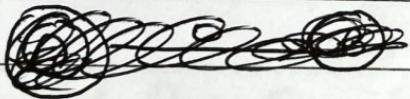
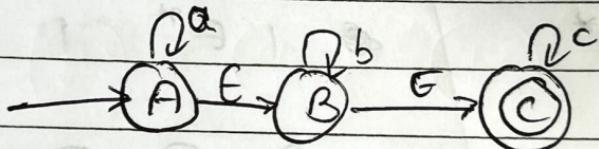


	0	1
*A	{C, B, D}	D
*B	{C, D}	D
C	{ }	{B, D}
*D	D	D

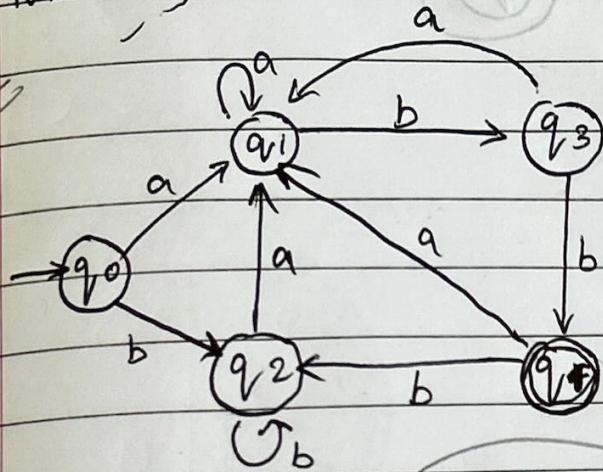


	0	1
*A	[CBD]	D
*CBD	[CD]	[BD]
*CD	D	[BD]
*BD	[CD]	[D]
D	D	D

#



~~DFA \rightarrow NDFA~~



a \ ϵ	a	b
q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
q_4	q_1	q_2

equivalent

equivalence :-

$$\Pi_0 \Rightarrow \{ q_0, \overbrace{q_1, q_2}, \{ q_3 \} \{ q_4 \} \}$$

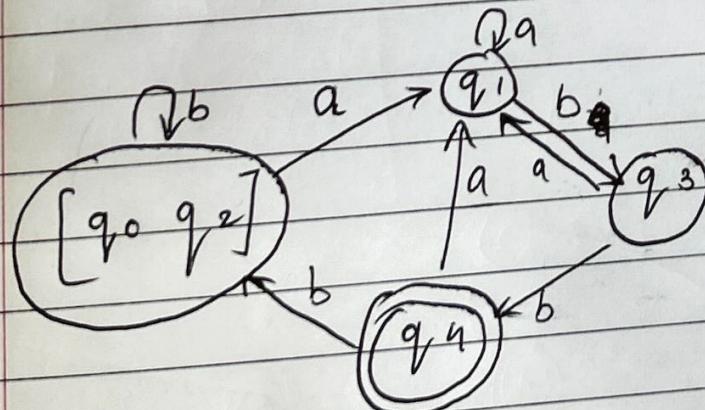
Check equivalent

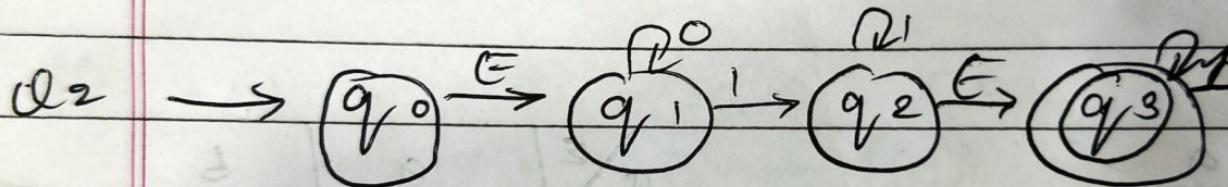
$$\Pi_1 \rightarrow \{ q_0; q_1; q_2 \} \{ q_3 \} \{ q_4 \}$$

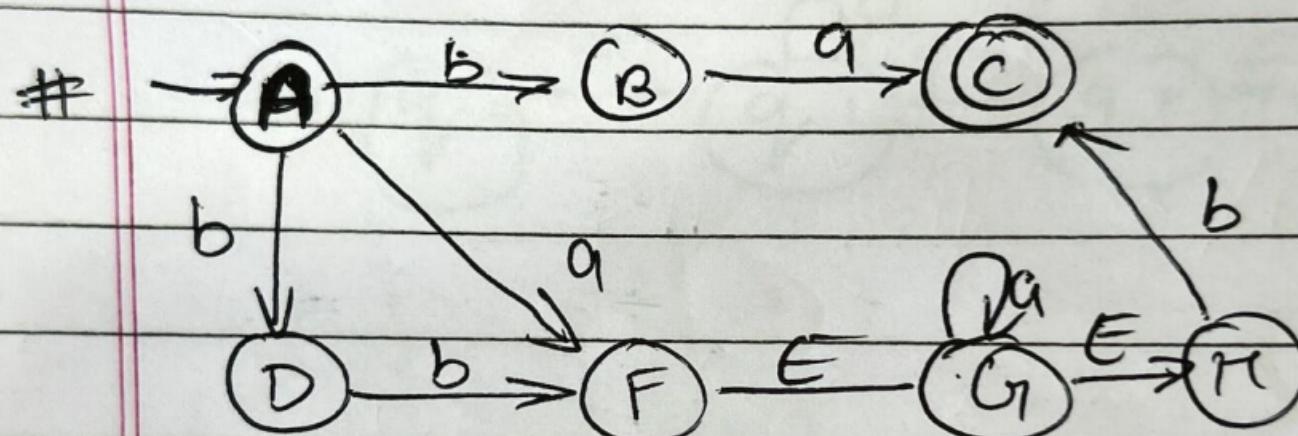
$$\Pi_2 \rightarrow \{ q_0, q_2 \} \{ q_1 \} \{ q_3 \} \{ q_4 \}$$

If in same
group remain

If 'a', 'b' of
them in
diff grp.
then split

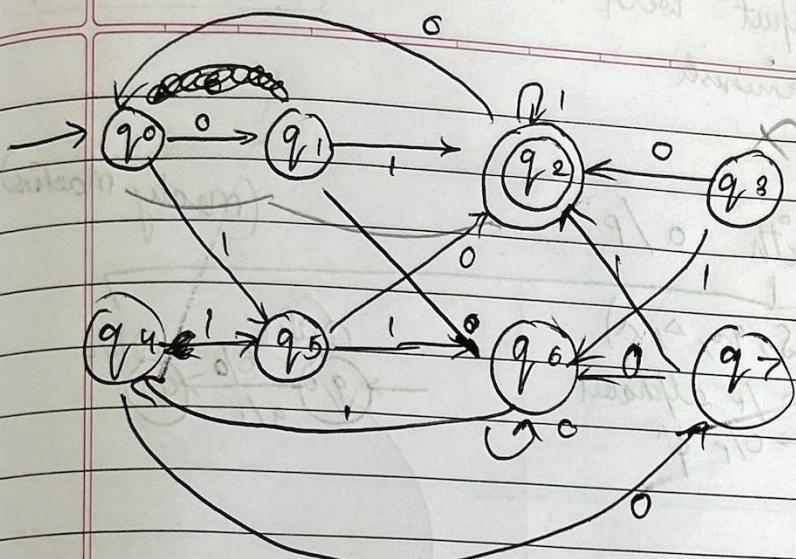






* Delete all nodes where we cannot reach from initial state.

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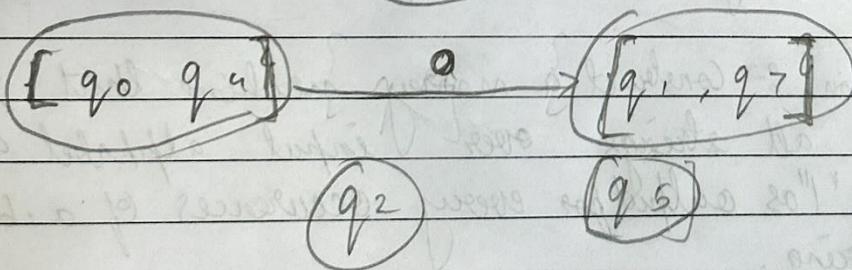


$$\Pi_0 \rightarrow \{q_0, q_1, q_4, q_5, q_6, q_7\}$$

\emptyset	0	1
q_0	q_1	q_5
q_1	q_6	q_2
q_2	q_0	q_2
q_3	q_2	q_6 delete
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

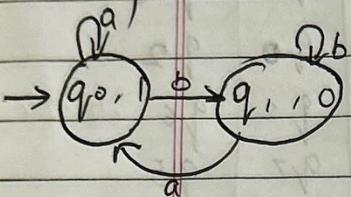
$$\Pi_1 \rightarrow \{q_0, q_4, q_6\} \quad \{q_1, q_7\} \quad \{q_5\} \quad \{q_2\}$$

$$\Pi_2 \rightarrow \{q_0, q_4\} \quad \{q_6\} \quad \{q_1, q_7\} \quad \{q_5\} \quad \{q_2\}$$



- * Moorey Machine has output with q .
 → By default without any condition write output q .
- * Mealy has output with state $(a \text{ and } b)$.
- * Both are deterministic.

(Moorey Machine)

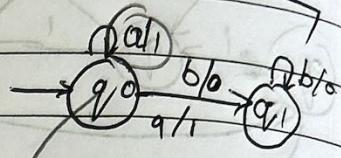


FA with o/p

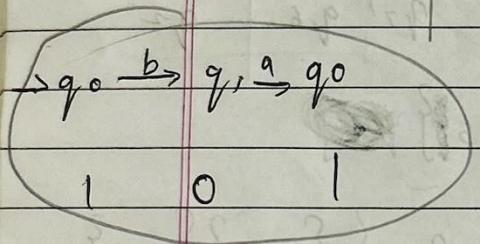
$$(Q, \Sigma, S, q_0, \delta, \lambda)$$

$\delta \rightarrow o/p$ alphabet
 $\lambda \rightarrow o/p \neq \lambda$

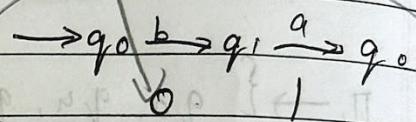
(Mealy machine)



$$\lambda: Q \rightarrow \Delta$$



$$\lambda: Q \times \Sigma \rightarrow \Delta$$

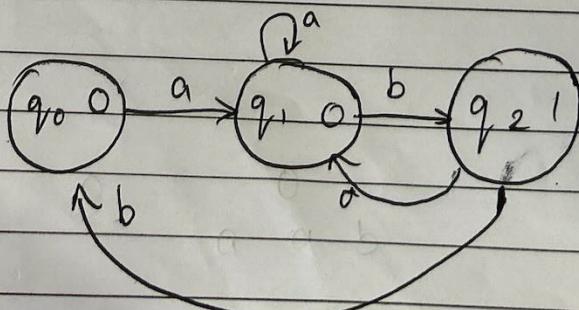


Design Moorey machine to count the occurrences of substring ab and prints 1 as output.

Description :- Construct a moorey machine that takes set of all strings over input alphabet a, b and prints "1" as output for every occurrences of a, b as a substring.

$$\Sigma = \{a, b\}$$

$$\Delta = \{0, 1\}$$



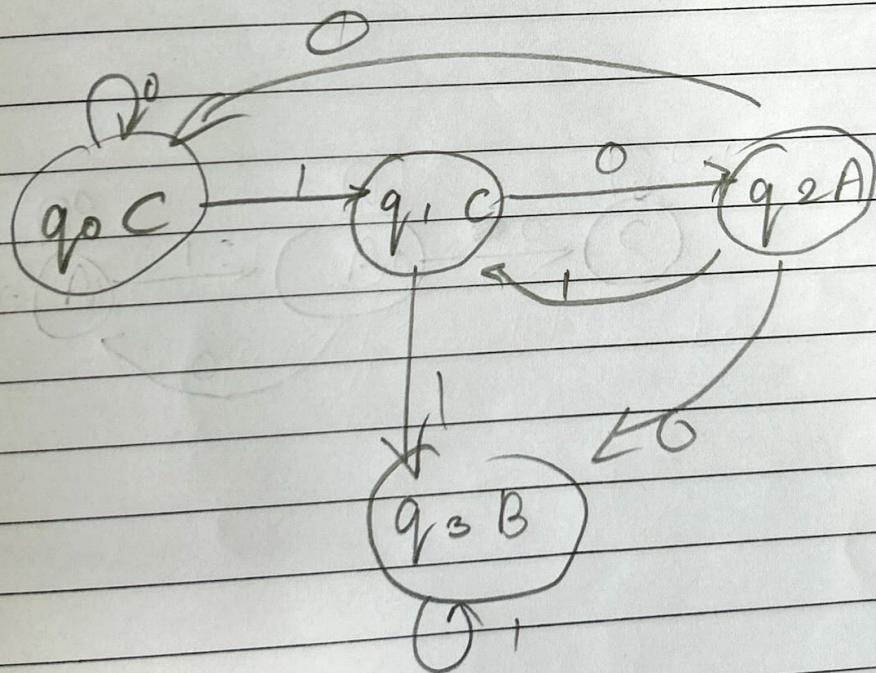
	a	b	0
q_0	q_1	q_2	0
q_1	q_1	q_2	0
q_2	q_1	q_0	1

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Simulation :- aq babab

Construct a Moorey machine for counting occurrence of substring baq and baba ✓

d) Construct Mealey machine that takes set of all strings over 0, 1 and produces a as output if input ends with 10 or produces b as output if input ends with 11 otherwise produces c.



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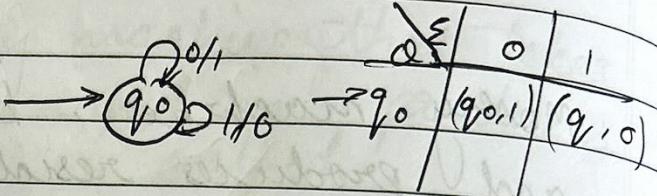
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Construct Moorey machine that takes binary numbers as input and produces residue module 3 as output.

Moseley machine, base 4 numbers as input
and produces residue modulo 5 as output.
(0, 1, 2, 3)

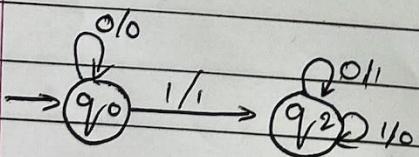
Mealey Machine :-

$$\begin{array}{l} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{array}$$

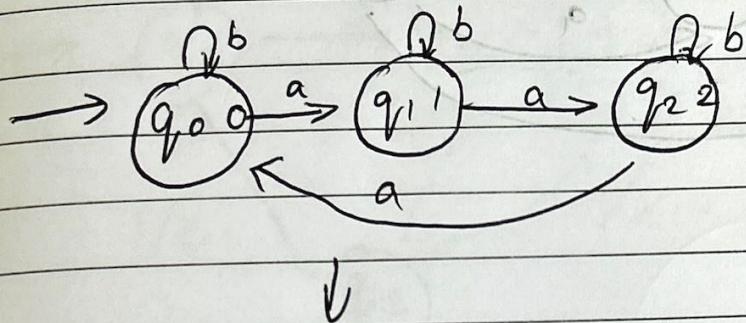


Construct mealey machine for 2's complement

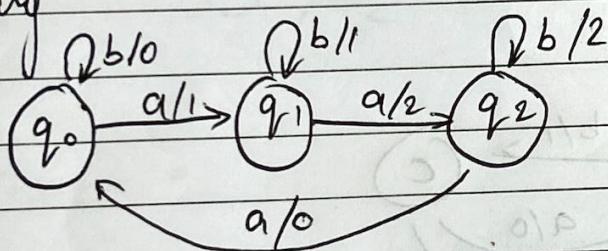
- * Binary No as an input and produces 2's complement as output assume the string is read from LSB to MSB and carry is discarded.



Moore \rightarrow Mealy M/c



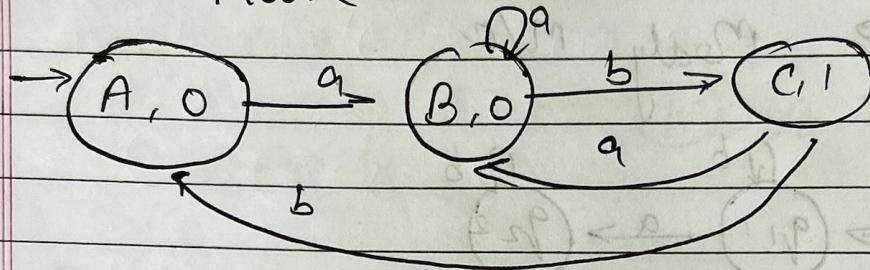
mealy



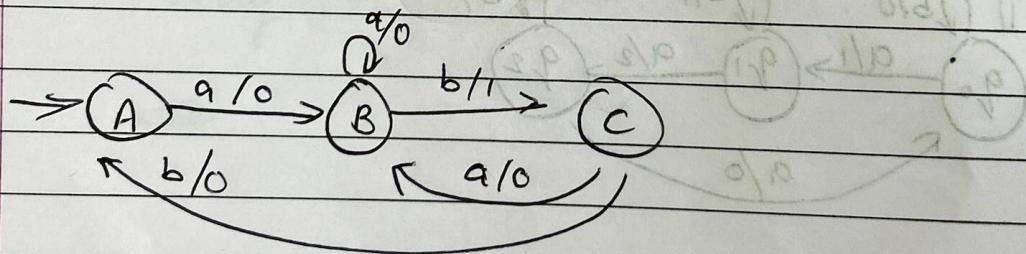
- 1) Check where the variable or elements ($a, b, 1, 0$) are going and write with the upcoming output of ~~state~~ state.

Q

MOORE

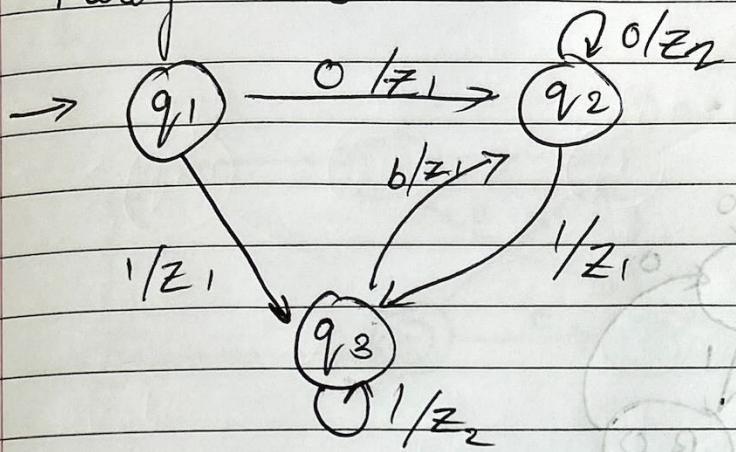


Mealy

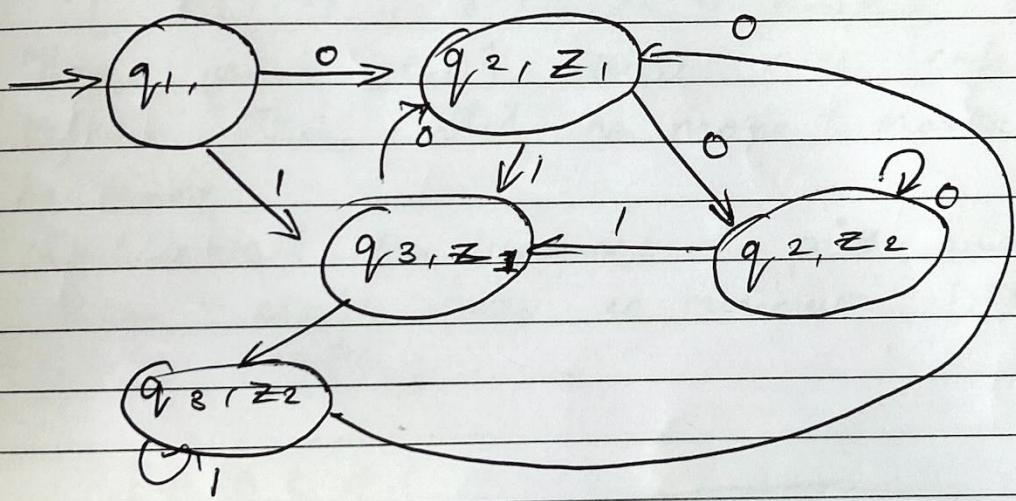


(S, Σ, Δ) of mealy to moore will consider state (1) value whenever all other states has miss zero state also 2

Mealy to Moore



Moore :-



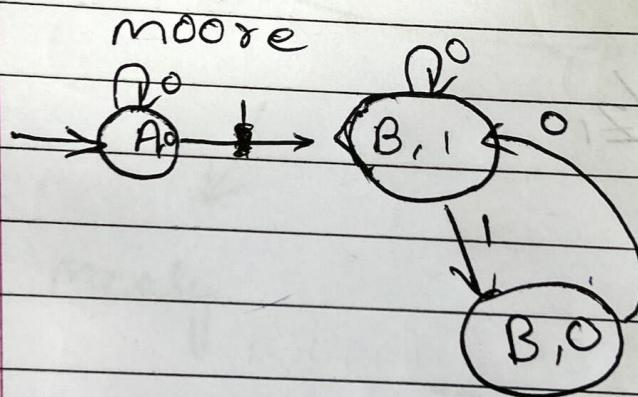
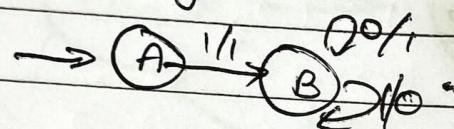
Step 1 :-

→ check start state for eg (q_1) and for 0 which output it has which here is z_1 so the (q_1) will go to (q_2, z_1) for input 0.

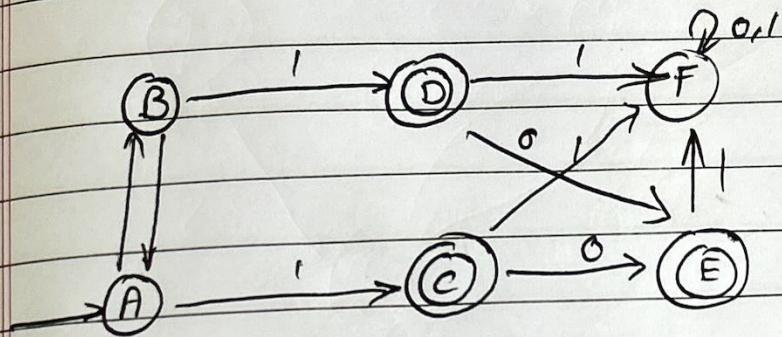
- For 0 input on q_2 it will go to (q_2, z_2) . But this state does not exist. So make a new state.
- Complete the whole diagram step by step to reduce confusion.

#

Mealy M/C to moore :-



Myhill Nersode

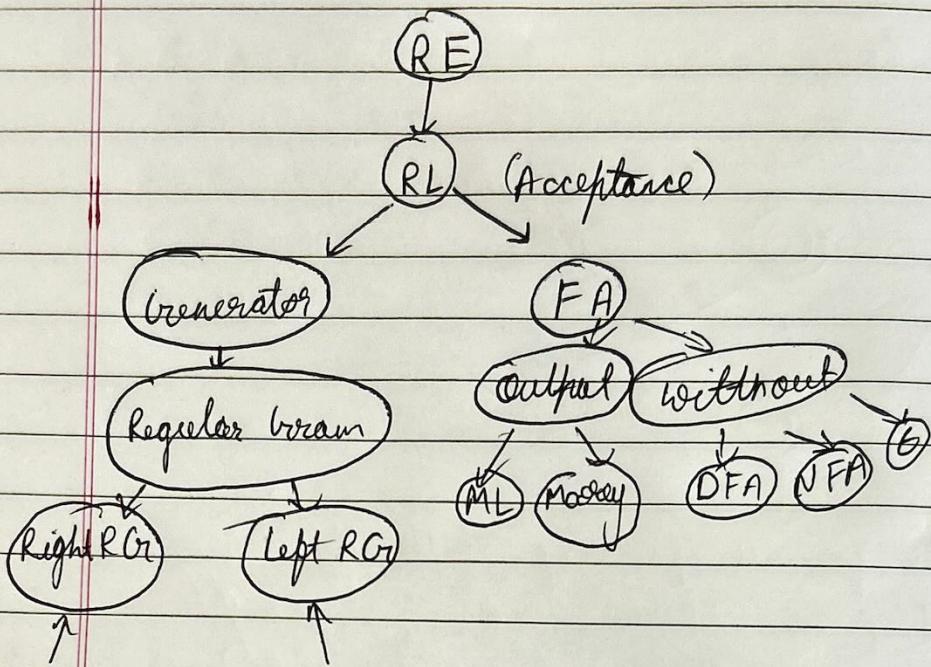


- i) draw table for all pairs of states (P, Q)
- ii) mark all pairs where $P \in F$ and $Q \notin F$
- iii) If there are any unmarked pairs (P, Q) such that $[\delta(P, x), \delta(Q, x)]$ is marked.
Then mark $[P, Q]$ where x is input symbol
- iv) Repeat this until no more markings can be made.
- v) combine all the unmarked pairs and make them single states in minimum DFA.

	A	B	C	D	E	F
A						
B		X	X			
C	X	X				
D	X	X				
E	X					
F				X	X	

cross when one of
2 states is an end
state

$$\delta(A, 0) \ni B$$
$$\delta(B, 0) \ni A$$
$$\delta(A, 1) \ni C$$
$$\delta(B, 1) \ni D$$



Provide regular expression for given language
 Set of all strings whose length is equal to
 2 over input alphabet a, b

$$L = \{aa, ab, ba, bb\}$$

$$\begin{aligned} &\rightarrow aa + ab + ba + bb \\ &\rightarrow a(a+b) + b(a+b) \\ &\rightarrow (a+b)^2 \end{aligned}$$

Exactly length equals to three :-

$$L = \{aaa, bbb, abb, baa, aba, bab\}$$

$$\begin{aligned} &\rightarrow aaa + bbb + abb + baa + aba + bab \\ &\rightarrow a(aa+bb+ba) + b(bb+aa+ab) \\ &\rightarrow (a+b)(aa+bb+ba)ab \\ &\rightarrow (a+b)(a+b)(a+b) \end{aligned}$$

length equals four

$$L = \{ \text{aaaa}, \text{bbbb}, \text{abbb}, \text{baaa}, \text{abba}, \text{baab}, \text{bbaa}, \text{aabb}, \text{abab}, \text{baba}, \text{babab}, \text{aaba}, \text{aaab}, \text{bbbba} \}$$

$$= (a+b)(a+b)(a+b)(a+b)$$

$$(a+b)(a+b)(a+b)^*$$

Almost 2^o-

$$L = \{ \epsilon, a, b, aa, ab, bb, ab \}$$

↓
↓
↓

$$\epsilon + \epsilon + a + b + aa + ab + bb + ab$$

$$\rightarrow (\epsilon + a + b)(\epsilon + a + b)$$

$$\rightarrow \epsilon \times \epsilon + \epsilon \times a + \epsilon \times b + a \times a + a \times b + b \times a + b \times b + a \times b + b \times a$$

$$\cancel{\rightarrow \epsilon + a + b + aa + ab + bb + ba}$$

Write a regular expression where length of string is even.

$$L = \{ aa, bb, \epsilon, ab, ba, \dots \}$$

Length of strings is odd :-

* We can get multiple answers to the questions.

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Set of all string divisible by 3.

$\{d, dd, ddd, dddd, \dots\} = \Sigma$

$\{d^k \mid k \in \mathbb{N}\}$

$\{d^3, dd^3, ddd^3, \dots\} = \Sigma$

$\{d^3, dd^3, ddd^3, \dots\} = \Sigma$

Set of all string whose length is congruent to
 $\equiv 2 \pmod{3}$.

$\{d^2, dd^2, ddd^2, \dots\} = \Sigma$

→ R.E for set of strings, where no. of 'a' is 2.

$$\Sigma = \{a, b\}$$

$$L = aa, baab, aab, baa, babab, \dots \}$$

$$b^* ab^* ab^*$$

→ R.E no. of 'a' be atleast 2.

$$\Sigma = \{a, b\}$$

$$L = \{aa, aaa, aab, baa, \dots \}$$

$$ab^* ab^* a^* b^*$$

→ R.E no. of 'a' atleast 2.

$$\Sigma = \{a, b\}$$

$$b^* (E + a) b^* (E + a) b^*$$

→ Number of a's even

$$\Sigma = \{a, b\}$$

$$L = \{E, b, aa, aab, aabb, \dots \}$$

$$(b^* a b^* a b^*)^* + b^*$$

(Or)

$$(b^* a b^* a b^*)^* \cdot b^*$$

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starting with a , end with a , contains a .

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Starting and ending with different symbol.
and same symbol.

→ 2 ds should not be together.

$$d = 0 \cdot 9 = 9 \cdot 0$$

$$d = 3 \cdot 9 = 9 \cdot 3$$

✓

$$*9 = 3 + 9 *9 = 199 + 3$$

$$*(*)_d + *_0 = *(d+0)$$

$$*(*)_d *_0 =$$

$$*(d + 0) =$$

$$*(*)_{d+0} =$$

$$*(*)_{dd} *_0 =$$

$$*(*)_{d0} *_1 =$$

set of string - where number of 2 as and
number of 2 b's together

Properties :-

$$*\quad \emptyset + R = R + \emptyset = R$$

$$\emptyset \cdot R = R \cdot \emptyset = \emptyset$$

$$ER = RE = R$$

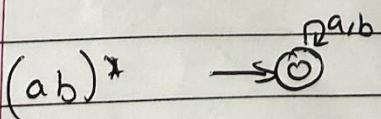
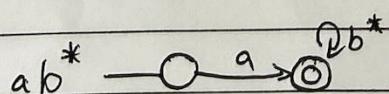
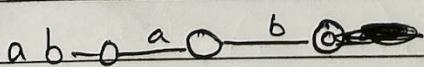
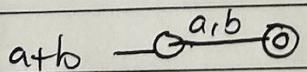
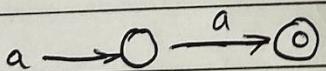
$$E^* = E$$

$$\emptyset^* = E$$

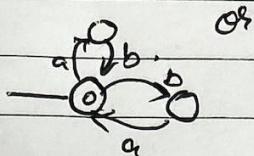
$$E + RR^* = R^*R + E = R^*$$

$$\begin{aligned} (a+b)^* &= (a^* + b^*)^* \\ &= (a^* b^*)^* \\ &= (a^* b)^* \\ &= (a + b^*)^* \\ &= a^* (ba^*)^* \\ &= b^* (ab^*)^* \end{aligned}$$

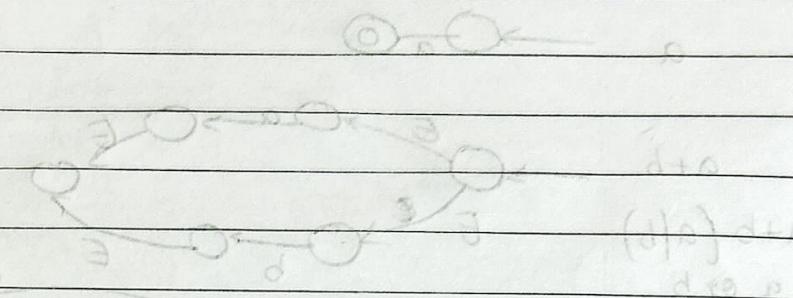
RE to Finite Automata :-



$$(ab + ba)^* \rightarrow \text{NFA}$$

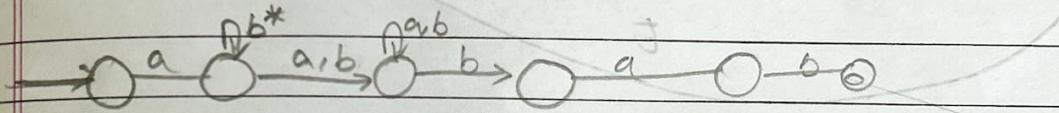


or

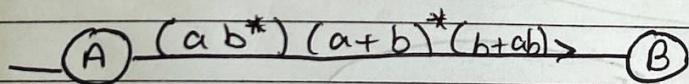


Design transition diagram for regular expression

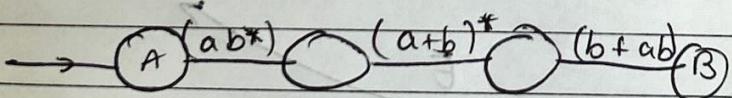
$$R = (ab^*)^* (a+b)^* (b+ab)$$



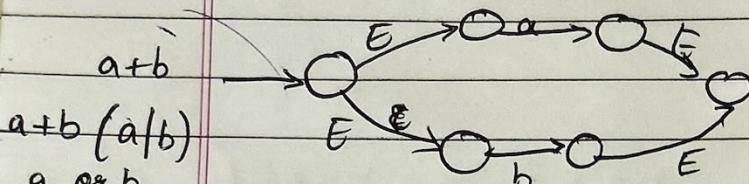
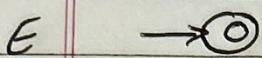
Step 1 :-



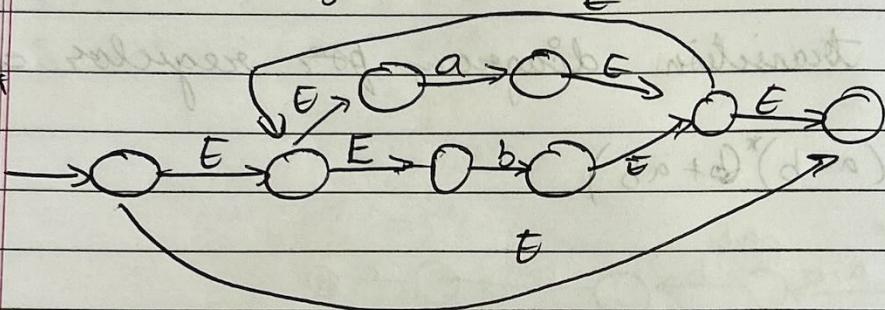
Step 2 :-



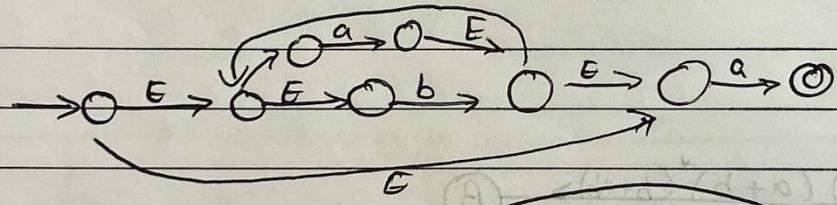
RE \rightarrow FA



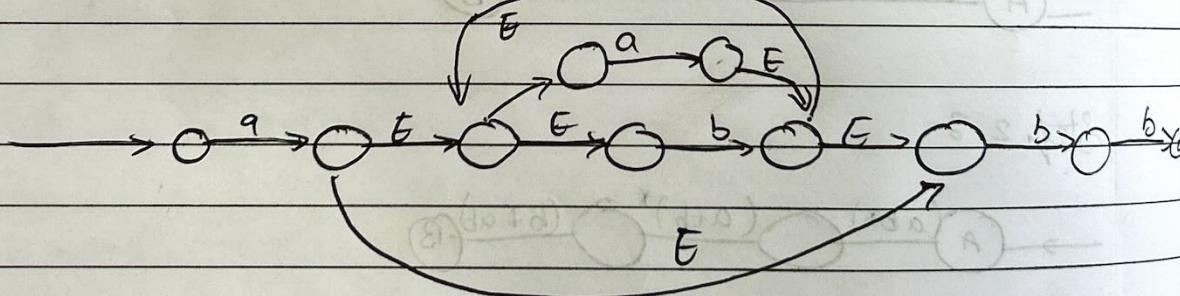
$(a+b)^*$



$(a+b)^*a$



$a(a+b)^*bb$



#

$10 + (0+11)_0 * 1$

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Q2

$$(0+1)^* (00+11)(0+1)^*$$

Q3

$$(00+1)^* (10)^*$$

Identities for regular expression :-

$$I_1 = \phi + R = R$$

$$I_2 = \phi R = R\phi = \phi$$

$$I_3 = \Lambda R = R\Lambda = R$$

$$I_4 = \Lambda^* = \Lambda = \phi^* = \Lambda \quad \text{Epsilon or } \epsilon$$

$$I_5 = R + R = R$$

$$I_6 = R^* R^* = R^*$$

$$I_7 = RR^* = R^*R$$

$$I_8 = (R^*)^* = R^*$$

$$I_9 = \Lambda + RR^* = R^*$$

$$I_{10} = (P\phi)^* P = P(\phi P)^*$$

$$I_{11} = (P + \phi)^* = (P^* Q^*)^* = (P^* + Q^*)^*$$

$$I_{12} = (P + \phi)R = PR + \phi R$$

$$= R(P + \phi)$$

$$= RP + R\phi$$

Theorem 2

P and ϕ are Regular expression
 R is a unique solution

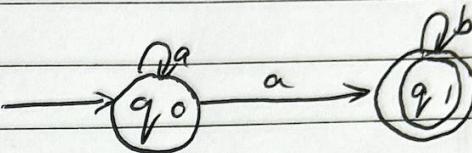
$$R = Q + RP$$

$$R = QP^*$$

Consider the transition diagram given below

→ Construct the regular expression corresponding to this transition diagram.

$$q_0 = \epsilon + q_0 a$$
$$q_1 = q_0 a + q_1 b$$

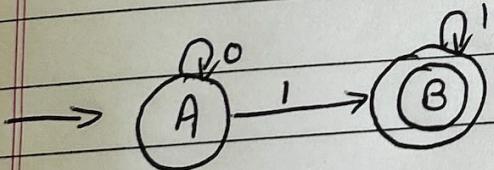


$$q_1 = (\epsilon + q_0 a) a + q_1 b$$

$$q_1 = a^* a + q_1 b$$
$$= a^* ab^* \text{ or } a^* b^*$$

$x^+ \rightarrow$ once to + is
one or many

Q2



$$A = \epsilon + A_0$$

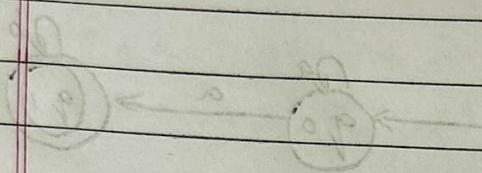
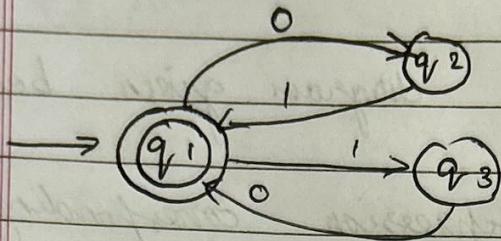
- ①
- ②

$$B = A_0 + B_1$$

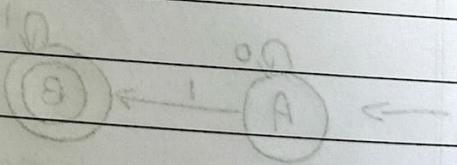
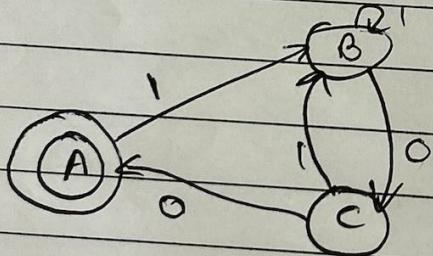
$$B = (\epsilon + A_0)_0 + B_1$$
$$= 0^* 1^*$$

Substituting ① in ②

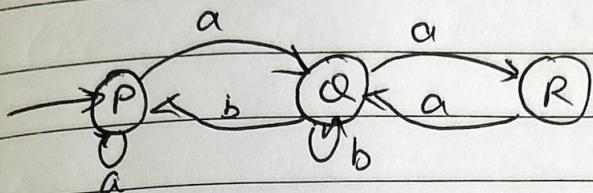
Q3



Q4



Q5



Q6

