Summer of Science
Game Theory
Mid-term report

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Contents

1 Introduction

Game theory is a branch of mathematics and economics that studies strategic decision-making in competitive situations. It provides a framework for analyzing the behavior and outcomes of rational players who are trying to maximize their own benefits while considering the actions and strategies of others. Game theory has applications in various fields, including economics, political science, biology, and computer science.

2 Game Theory Overview

In game theory, a "game" refers to a situation where players make decisions that impact each other's outcomes. These decisions are based on the players' strategies, which are the different options available to them. The outcomes of a game are determined by the combination of strategies chosen by the players.

2.1 The Normal Form

The normal form is a formal representation of a game, where players and their strategies are listed along with the associated payoffs. Payoffs represent the benefits or utilities that players receive based on the outcomes of the game. Strategies that yield the highest payoffs for a player are considered advantageous.

2.2 Nash Equilibrium Introduction

A key concept in game theory is the Nash equilibrium. It is a state in which no player can unilaterally deviate from their chosen strategy to achieve a better outcome for themselves. In other words, a Nash equilibrium represents a stable point where all players are satisfied with their strategies, given the strategies chosen by others.

2.3 Dominant strategies

Dominant strategies are strategies that always yield a better outcome for a player, regardless of the strategies chosen by other players. They provide a straightforward decision rule for players, as they can simply choose the dominant strategy without considering the actions of others. Dominant strategies can simplify the analysis of a game and often lead to the identification of a unique Nash equilibrium.

2.4 Pareto Optimality

Game theory also considers the concept of Pareto optimality, which refers to a state where no player can be made better off without making another player worse off. It represents an efficient outcome where resources are allocated in an optimal manner.

3 Nash Equilibrium

Now we look at nash equilibrium in detail.

3.1 Mixed Strategies and Nash Equilibrium

In game theory, a mixed strategy is a probability distribution over a player's available pure strategies. Unlike a pure strategy, which involves choosing a single action with certainty, a mixed strategy allows for randomness in decision-making. This topic explores how mixed strategies can be used to analyze strategic interactions and determine the Nash equilibrium.

Nash equilibrium is a concept that represents a stable state in a game where no player has an incentive to unilaterally change their strategy. In the context of mixed strategies, a mixed strategy Nash equilibrium occurs when each player's mixed strategy is the best response to the strategies chosen by the other players. This means that no player can improve their expected payoff by unilaterally deviating from their chosen mixed strategy.

3.2 Computing Mixed Nash Equilibrium

Various algorithms and techniques exist for computing mixed Nash equilibrium, including linear programming, Lemke-Howson algorithm, and support enumeration. These methods involve solving mathematical equations and optimizing objective functions to determine the probabilities associated with each mixed strategy.

3.3 Hardness Beyond 2x2 Games

While determining mixed Nash equilibrium is relatively straightforward for 2x2 games (games with two players and two strategies each), the computational complexity increases significantly for games with more players or strategies.We get the basic understanding of the computational hardness that arises when dealing with games beyond the 2x2 format.

As the number of players and strategies increases, the computational complexity of finding mixed Nash equilibrium grows exponentially. We explore the underlying reasons for this hardness and introduces basic concepts related to algorithmic complexity theory, such as polynomial time algorithms and NP-hardness.

Advanced techniques such as reduction, approximation algorithms, and game-specific characteristics are discussed to address the computational challenges posed by larger games. It emphasizes on the trade-offs between computational efficiency and accuracy in finding approximate solutions for games with high computational complexity.

4 Alternate Solution Concepts

This topic delves into the concept of looking beyond the Nash equilibrium, which is the standard solution concept in game theory. It explores situations where players may have incentives to deviate from the Nash equilibrium and analyzes alternative solution concepts that capture such deviations.

4.1 Strictly Dominated Strategies Iterative Removal

Strictly dominated strategies are those that always result in a lower payoff compared to another available strategy, regardless of the actions chosen by the other players. We foucs on the concept of strictly dominated strategies and iterative removal, a process where strictly dominated strategies are iteratively eliminated from consideration. By iteratively removing these dominated strategies, players can refine their strategic choices and potentially converge to a more favorable outcome.

4.2 Maxmin Strategies

Maxmin strategies are strategies that players choose to maximize their minimum possible payoff in a game. We explore the concept of maxmin strategies and their role in strategic decision-making. It discusses how players can determine their maxmin strategies by considering the worst-case scenarios and selecting the strategy that guarantees the highest minimum payoff. Maxmin strategies are particularly relevant in situations where players have risk-averse preferences or face uncertainty regarding their opponents' actions.

4.3 Correlated Equilibrium

Correlated equilibrium is a solution concept that allows for communication or signaling between players before they choose their strategies.Concept of correlated equilibrium and provides an intuitive understanding of how it differs from Nash equilibrium. It explores the idea that players can coordinate their actions based on signals or external information to achieve a more favorable outcome. We see the advantages and limitations of correlated equilibrium and highlights its applications in strategic situations with communication channels or shared information.

5 Extensive-Form Games

5.1 Formalizing Perfect Information Extensive Form Games

This section dives deeper into formalizing perfect information extensive form games. It discusses the elements and components of an extensive form game, including players, actions, information sets, and payoffs.It explains how these elements are structured and interconnected to represent the strategic interactions and decision points in a game with perfect information. Formalizing the extensive form provides a rigorous framework for analyzing and solving such games.

5.2 Subgame Perfection

Subgame perfection is a refinement concept within the extensive form that requires Nash equilibrium to be reached in every subgame of the original game. It explores the idea that subgame perfection ensures a consistent and credible strategy for each decision point, even in the presence of sequential moves and perfect information.

5.3 Backward Induction

Backward induction is a method for solving perfect information extensive form games by working backwards from the final stage. This topic explores the backward induction technique and its application in solving games with perfect information. It demonstrates how players can reason backward, anticipate future actions, and determine the optimal strategies and outcomes at each stage of the game. Backward induction provides a systematic approach to solving sequential games and identifying the subgame perfect equilibrium.

6 Repeated Games

In repeated Games we explore scenarios where players engage in multiple rounds of the same game, allowing for strategic interactions over time. The topic discusses the importance of repeated games in modeling real-world situations and studying the evolution of strategies and outcomes.

6.1 Stochastic Games

Stochastic games are introduced in this topic, expanding the analysis beyond deterministic settings. Stochastic games involve uncertainty or randomness, where the outcomes of players' actions are determined by chance. We discuss the characteristics and challenges associated with stochastic games, such as modeling transitions between states, incorporating probabilities, and analyzing strategic decision-making under uncertainty. It explores how players' strategies and equilibrium concepts adapt to stochastic environments.

6.2 Learning in Repeated Games

This section explores the concept of learning in repeated games. It delves into how players can adapt their strategies and behaviors over time based on their experiences and interactions. We explore how learning processes can lead to the emergence of cooperative strategies, the formation of reputation, and the convergence towards stable equilibria.

6.3 Equilibria of Infinitely Repeated Games

Building upon the earlier topics on repeated and infinitely repeated games, this section focuses on analyzing equilibria in infinitely repeated games. We explore different equilibrium concepts, such as subgame perfect equilibrium and trembling hand perfect equilibrium, and their implications in infinitely repeated interactions. We see strategies that sustain cooperation, deter defection, and promote mutually beneficial outcomes over an infinite time horizon.

6.4 Discounted Repeated Games

This topic explores discounted repeated games in more detail. We see the concept of discounting future payoffs, which allows players to give less weight to future rounds in favor of immediate gains. The topic examines the strategic implications of discounting and how it affects players' decision-making and equilibrium outcomes in repeated interactions. Also it addresses the challenges of coordinating actions and sustaining cooperation in the presence of discounting.

7 Bayesian Games

7.1 Bayesian Games: First Definition (yoav)

This subsection, likely named after the researcher Yoav Shoham who contributed to game theory, may provide additional insights or variations on the first definition of Bayesian games. It might present alternative perspectives, extensions, or refinements to the initial definition, providing a deeper understanding of the concept.

7.2 Bayesian Games: Second Definition

Building upon the first definition, this section presents an alternative formulation or refinement of Bayesian games. This introduces the second definition of Bayesian games, which might offer a different perspective or notation to analyze the strategic interactions and equilibrium concepts in games with incomplete information. We see how this alternative definition captures the essence of Bayesian games and provides a framework for analyzing players' beliefs, strategies, and equilibrium outcomes.

8 Coalitional Games

In game theory, a cooperative game (or coalitional game) is a game with competition between groups of players ("coalitions") due to the possibility of external enforcement of cooperative behavior (e.g. through contract law).

8.1 Shapley value

Shapley value is a prominent solution concept in coalitional game theory. It explains how the Shapley value assigns a unique allocation of payoffs to each player in a cooperative game based on their marginal contributions. The topic looks into the properties of the Shapley value, such as efficiency, fairness, and stability, and explores its applications in various contexts, such as resource allocation, cost sharing, and voting systems.

9 Plan of Action

With reference to my previous plan of action regarding my Summer of Science project, I am running a week behind schedule due to my end semester exams. However my plan of action leaves room to catch up and just shifts all actions one week forward. I expect a timely completion of my project and submission of final report.