Summer of Science
Game Theory
End-term report

Soham Dahane

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Contents

1 Cooperative Game Theory:Transferable Utility, Shapley Value

1.1 Transferable Utility

Transferable Utility (TU) is a fundamental concept in cooperative game theory, where agents can transfer their utility among themselves. It implies that the total value of the grand coalition (all agents together) can be distributed among the agents in various ways. Each agent has a value function that depends on the coalition they belong to. This allows for cooperation, bargaining, and negotiation among agents to achieve a mutually beneficial outcome.

1.2 Transferable Utility Example

Consider a scenario with three players, A, B, and C. They form coalitions with different values: A, B, C, A, B, A, C, B, C, and A, B, C. Each coalition has an associated value reflecting its collective benefit. For instance, A, B may have a value of 20, while A, C may have a value of 25. The goal is to find a fair way to distribute the total value of the grand coalition among the players.

1.3 Shapley Value

The Shapley Value is a prominent solution concept in cooperative game theory that distributes the total value of the grand coalition fairly among the players. It assigns to each player the average marginal contribution they make across all possible permutations of players. In essence, it measures the incremental value an individual brings to different coalitions.

The Shapley Value has appealing properties, such as efficiency, symmetry, and additivity, making it an important tool for cooperative game analysis and fairness evaluation in various real-world applications.

2 Social Choice

2.1 Social Choice: Taste and Voting Scheme

Social Choice involves collective decision-making processes where the preferences of multiple individuals are aggregated to determine a group choice. It deals with selecting one option from a set of alternatives based on the preferences of the participants. Taste refers to the individual preferences of each voter, while a Voting Scheme specifies how these preferences are combined to reach a group decision.

2.2 Paradoxical Outcomes

In social choice theory, various voting methods can lead to paradoxical outcomes, where the collective choice may not align with individual preferences. Examples include the Condorcet Paradox, where a cyclical preference occurs among three or more alternatives, making it challenging to determine a clear winner.

2.3 Impossibility of Non-Paradoxical Social Welfare Functions

Arrow's Impossibility Theorem states that there is no non-dictatorial and Pareto-efficient social welfare function that can consistently convert individual preferences into a group decision without encountering paradoxes. This theorem reveals the inherent difficulties in designing a perfect voting system that satisfies desirable properties such as unanimity, independence of irrelevant alternatives, and non-dictatorship.

3 Arrow's Theorem

3.1 Conditions of Arrow's Theorem

Arrow's Theorem identifies three essential conditions that a desirable voting system should ideally meet:

a. Universal Domain: The voting system should allow any individual preference profile as input, meaning it can handle any possible combination of individual preferences.

b. Non-dictatorship: The voting outcome should not be determined solely by the preferences of a single individual. In other words, no single voter should be a "dictator" who can impose their preferences on the group.

3.2 Arrow's Impossibility Theorem

Arrow's theorem proves that no voting system can simultaneously satisfy all three conditions. In other words, there is no perfect social welfare function that can transform individual preferences into a consistent group ranking while adhering to universal domain, non-dictatorship, and IIA. This implies that no voting system can fully escape the possibility of paradoxical outcomes, where the group preference contradicts individual preferences.

Arrow's Impossibility Theorem is a significant result with profound implications for social choice theory and political decision-making. It challenges us to critically examine the trade-offs between desirable properties in voting systems and the inherent limitations in aggregating individual preferences to achieve a coherent and fair group choice.

4 Basics Of Mechanism Design

4.1 Mechanism Design: Taste

In mechanism design, researchers focus on designing rules and procedures that incentivize self-interested individuals to reveal their true preferences honestly. Mechanism designers aim to achieve desirable outcomes while considering the individuals' incentives to strategically manipulate the system. Preferences play a crucial role in this process, as the mechanism's success depends on understanding individuals' tastes and priorities.

4.2 Revelation Principle

The Revelation Principle is a fundamental concept in mechanism design. It states that in a direct-revelation mechanism, truthful revelation of preferences is always a dominant strategy for every participant. In simpler terms, participants in a well-designed mechanism are better off truthfully revealing their preferences, regardless of others' actions.

4.3 Revelation Principle: Examples

To illustrate the Revelation Principle, consider the classic example of a single-item auction. In a truthful revelation mechanism, bidders have an incentive to bid their true valuations for the item. Suppose a bidder's true valuation is $50, buttheybid40$ to pay a lower price. The mechanism may result in the bidder winning the item at $40, buttheygainmoreutilityiftheywinitat50$.

4.4 Impossibility of General Dominant-Strategy Implementation

The Impossibility of General Dominant-Strategy Implementation theorem reveals that achieving dominant-strategy incentive compatibility for all possible preference profiles is generally impossible. This means that, in certain scenarios, it is not feasible to design a mechanism where truthful revelation is a dominant strategy for all participants, making strategy-proof mechanisms challenging to achieve in some settings.

Understanding these basics of mechanism design lays the foundation for exploring various mechanisms and understanding their strengths and limitations in eliciting truthful preferences from self-interested agents. It also provides insights into how mechanism designers can strike a balance between desirable outcomes and strategic incentives in real-world decision-making processes.

5 Efficient Mechanisms

5.1 VCG: Taste and Definition

The Vickrey-Clarke-Groves (VCG) mechanism is a prominent example of an efficient mechanism. It is designed to achieve Pareto efficiency, where no other feasible outcome can make at least one agent better off without making any other agent worse off. The VCG mechanism operates by having participants submit their private valuations or preferences for the available options.

5.2 VCG: Examples and Limitations

An example of the VCG mechanism is a multi-item auction, where bidders submit their valuations for different items, and the mechanism allocates the items to maximize social welfare. However, the VCG mechanism has limitations, such as computational complexity, as it requires solving optimization problems to determine the optimal allocations.

5.3 VCG: Individual Rationality and Budget Balance in VCG

Individual rationality is a critical aspect of mechanism design, ensuring that participants are not worse off by participating in the mechanism. VCG mechanisms satisfy individual rationality, guaranteeing that no participant has an incentive to withhold their true preferences.

Budget balance is another important property of VCG mechanisms, meaning that the mechanism's payments exactly cover the costs or payments required for the mechanism to operate. Budget balance ensures that the mechanism does not incur losses and is sustainable

5.4 VCG: The Myerson-Satterthwaite Theorem

The Myerson-Satterthwaite theorem explores the limitations of the VCG mechanism. It states that no efficient and individually rational mechanism exists for allocating private goods when participants have private information about their preferences and costs. This theorem highlights the inherent trade-offs and challenges in designing mechanisms that are simultaneously efficient, incentive-compatible, and individually rational.

Understanding the VCG mechanism and its properties provides valuable insights into the design and implementation of efficient mechanisms in various real-world scenarios, considering the interplay between efficiency, individual rationality, and budget balance. It also emphasizes the need for innovative solutions to address the limitations of existing mechanisms in specific settings.

6 Auctions

6.1 Auctions: Taste and Taxonomy

Auctions are widely used mechanisms for allocating goods and services among multiple participants. They come in various forms, each with distinct rules and bidding mechanisms. The taxonomy of auctions categorizes them based on key characteristics, such as bid disclosure, number of bidders, and winner determination.

6.2 Bidding in First-Price and Second-Price Auctions

In a first-price auction, participants submit sealed bids, and the highest bidder wins the item, paying their bid amount. In contrast, a second-price auction, also known as a Vickrey auction, awards the item to the highest bidder but at the price of the second-highest bid. Bidders have an incentive to bid truthfully in a second-price auction, as it promotes dominant-strategy truthfulness.

6.3 Revenue Equivalence

Revenue equivalence is a crucial concept in auction theory, which states that under certain conditions, different auction formats can generate the same expected revenue for the seller. For instance, a first-price sealed-bid auction and a second-price sealed-bid auction can be revenue-equivalent when bidders have independent private values.

6.4 Optimal Auctions

Designing optimal auctions involves maximizing the seller's revenue while considering the bidders' strategic behavior. Optimal auction designs can vary depending on the bidder's valuations, risk preferences, and other factors. The Vickrey-Clarke-Groves (VCG) mechanism is a prominent example of an optimal auction that achieves efficiency.

6.5 More Advanced Auctions

Beyond the traditional first-price and second-price auctions, more advanced auction formats have been developed to suit specific contexts and preferences. Some examples include ascending-bid auctions, descending-bid auctions, and combinatorial auctions, which handle complex scenarios involving multiple items.

Understanding the various auction formats, their properties, and revenue implications is essential for stakeholders involved in auction-based transactions. It allows sellers to optimize their revenue generation, while bidders can strategize effectively to pursue their desired items efficiently. Additionally, auction design continues to evolve, leading to innovative mechanisms for diverse market settings.

7 Revise Everything till now and prepare report

Doing the complete course on game theory was fun. I revised parts of the midterm topics which I felt important and then revised the topics done after the midsem. It was an absolute amazing journey in doing this SOS on game theory. Now I have got interest in it and plan to take the courses CS6001(Game Theory) and EC402(Economics and Game Theory) in my upcoming sems. Really this was a very fun and knowledgeable journey and I would like to thank my mentor Arhaan for all his valuable support without whom and which this would not have been possible.