Control Systems Lab
Experiment 3: Inverted Pendulum

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1 Objectives

The objectives of the experiment were as follows:

(a) To study the basics of non-linear control using an inverted pendulum and implement the LQR control algorithm

(b) To restrict the pendulum arm vibration(α) within ± 3 degrees

(c) To restrict the base angle oscillation(θ) within ± 30 degrees

2 Description of the Setup

1. The primary part of the setup is the inverted pendulum which which consists of a base arm, a pendulum arm and a motor to drive each arm.

2. In order to rotate the motors, we use an **L293D** motor driver takes control signals from an **Arduino Mega** using PWM.

3. The base arm and the pendulum arm are equipped with an **AM22 encoder**. AM22 encoder provides the angle of the arm on a scale of (0-16384).

3 Control Algorithm

In this experiment, we had to implement the **Linear Quadratic Regulator(LQR)** control algorithm to balance the inverted pendulum within the given constraints. For an **LTI system** given in state space by:

$$\dot{x} = Ax + Bu$$

Given any initial state, $x(0) = x_0$, find out u such that:

$$\int_0^\infty (x^T Q x + u^T R u) dt$$

is minimum and $x(t) \longrightarrow 0$ as $t \longrightarrow \infty$.

In our system(inverted pendulum) x is a column vector consisting of the following quantities:

$$x = \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix}$$

The matrices A and B depend on the physical specifications of the system. We had to set Q and R to balance the pendulum. Q is a diagonal matrix $diag(a_1, a_2, a_3, a_4)$ where a_1 is the penalty for θ, a_2 for α, a_3 for $\dot{\theta}$ and a_4 for $\dot{\alpha}$. R is a singleton matrix. On solving the above minimization problem which translates to solving the **Algebraic Ricatti Equation**, we obtain a vector $K = [k_1, k_2, k_3, k_4]$ and our control signal u is given by:

$$u = k_1\theta + k_2\alpha + k_3\dot{\theta} + k_4\dot{\alpha}$$

The Algebraic Ricatti Equation is solved using the **lqr()** function in MATLAB.

4 Challenges Faced

1. **Delay**: Initially, we were providing a delay in our code which would help us observe the outputs, but we realized very late that this delay was causing issues in our control algorithm.

2. **Scaling Problems**: The encoders present on the Inverted Pendulum apparatus provided to us output values between 0 to 16384. Initially, we used these values for computation of the control signal, hence we were getting very large values of the control signal(order of 10^7!!). To tackle this problem we tried scaling by some parameter to restrict the control signals from 0 to 255 however, in this solution, every time we changed the values of k_1, k_2, k_3, k_4, we had to change our scaling factor. Finally, we converted our angles to degrees which gave us the appropriate values for the control signals.

```
alpha=alpha*(360.0/16384.0);
theta=theta*(360.0/16384);
```

3. **Set Zero**: In our setup, the topmost value maps to 0 and a disturbance on the right side leads to small positive values whereas a disturbance on the left side leads to large values(around 16,000). Hence, we had to center the values accordingly.

```
if (alpha>8192){
    alpha=alpha-16384;
}

if (theta>8192){
    theta-=16384;
}
```

4. **Tuning**: Ideally, one should modify the diagonal matrix Q in order to balance the pendulum, however, we were unable to do so for a long time, hence we modified the values of $k_i's$ and reverse calculated Q.

5 Results

We obtained the following value of Q matrix:

$$Q = \begin{bmatrix} 110 & 0 & 0 & 0 \\ 0 & 5300 & 0 & 0 \\ 0 & 0 & 10^{-8} & 0 \\ 0 & 0 & 0 & 10^{-8} \end{bmatrix}$$

which gives

$$K = \begin{bmatrix} -10.5 & 120 & -0.54 & 0.8 \end{bmatrix}$$

6 Observations

- In order to control any parameter better, the penalty corresponding to it must be increased.
- The penalties corresponding to θ, α help in reducing large vibrations of the base angle and top angle respectively.
- For small vibrations, one must modify the penalties corresponding to $\dot{\theta}, \dot{\alpha}$.

7 Conclusions

We were able to balance the pendulum according to the given specifications, α within ± 3 degrees and θ within ± 30 degrees, however after some time the pendulum goes off balance.