ECE 431 Spring 2020 HW2 Soln.

Problem 2.1

a) Area of A = Area of B = $\frac{1}{2}$ × Area of C.

$\phi_A = \phi_B = \frac{1}{2}\phi_c \Rightarrow B_A = B_B = B_c = B_g$

From Ampere's Law, $NI = H_c(l_A + l_c - l_g) + H_g l_g$

$l_A = 0.17\,m \quad l_c = 0.05\,m \quad l_g = 0.004\,m \quad N = 120,$

From Fig 1.10, $H_c = 20$ A·turns/m when $B_c = 1.3\,T$, $\mu_c = \frac{1.3}{20} = 0.065\,H_m^{-1}$

$H_g = \frac{1.3}{\mu_0} = 1.03 \times 10^6$ A·turns/m

$J = \frac{H_c(l_A + l_c - l_g) + H_g l_g}{N} \Rightarrow I = \boxed{34.4\,A}$

b) $W_{gap} = \frac{1}{2} l_g A_g \frac{B_g^2}{\mu_0} = \boxed{4.30\,J}$

$W_{core} = \frac{1}{2}\frac{B_c^2}{\mu_c}(l_A A_A + l_B A_B + (l_c - l_g)A_c) = \boxed{4.49 \times 10^{-3}\,J}$

$W_{total} = W_{gap} + W_{core} = \frac{1}{2} L I^2 \Rightarrow L = \boxed{7.294\,mH}$

c) $\mu_c = \infty \Rightarrow W_{core} = 0$

$L = \frac{2W_{gap}}{I^2} = \boxed{7.286\,mH}$

Problem 2.2

$V = N\frac{d\phi}{dt}$

$= NA\frac{dB}{dt}$

$= NA\frac{d}{dt}(B_{max}\sin(\omega t))$

$= NA\,\omega B_{max}\cos(\omega t)$

$V_{max} = NA\omega B_{max}$

$= 2\pi f NA B_{max}.$

60 Hz:

Primary $V_{max} = 120\pi \cdot 1200 \cdot 60\times10^{-4} \cdot 1.45$

$= \boxed{3936\,V}$

Secondary $V_{max} = 3936\frac{80}{1200} = \boxed{262\,V}$

50 Hz:

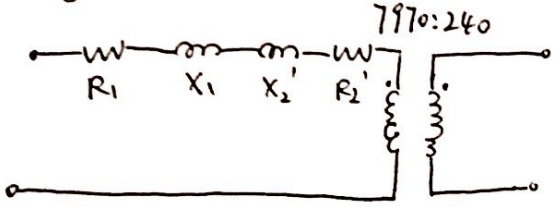
Primary $V_{max} = 100\pi \cdot 1200 \cdot 60\times10^{-4} \cdot 1.45$

$= \boxed{3280\,V}$

Secondary $V_{max} = 3280\frac{80}{1200} = \boxed{219\,V}$

Problem 2.3.

a) High Side:

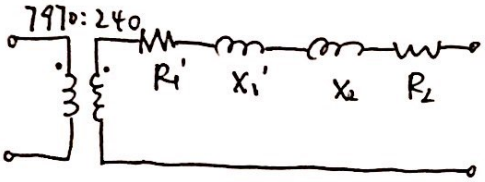


$V_1 = 7970\,V$ $R_1 = 40\,\Omega$ $R_1' = R_1\left(\frac{N_2}{N_1}\right)^2 = 36.3\,m\Omega$

$V_2 = 240\,V$ $X_1 = 42\,\Omega$ $X_1' = X_1\left(\frac{N_2}{N_1}\right)^2 = 38.1\,m\Omega$

$\frac{N_1}{N_2} = \frac{7970}{240}$

Low side:



$R_2 = 37\,m\Omega$ $R_2' = R_2\left(\frac{N_1}{N_2}\right)^2 = 40.8\,\Omega$

$X_2 = 40\,m\Omega$ $X_2' = X_2\left(\frac{N_1}{N_2}\right)^2 = 44.1\,\Omega$

b) i) $I_{load} = \frac{40,000}{240} \angle(-\cos^{-1}0.87) = 166.67 \angle -29.54^\circ A$

$V_{load} = 240 \angle 0^\circ\ V.$

$V_{high} = \frac{N_1}{N_2}\left(V_{load} + I_{load}(R_1' + R_2 + jX_1' + jX_2)\right) = \boxed{8538 \angle 1.18^\circ\ V}$

ii) $I_{load} = \frac{40,000}{240} \angle(\cos^{-1}0.87) = 166.67 \angle 29.54^\circ A$

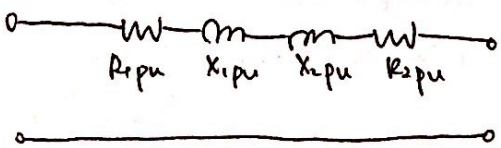
$V_{high} = \frac{N_1}{N_2}\left(V_{load} + I_{load}(R_1' + R_2 + jX_1' + jX_2)\right) = \boxed{8130 \angle 4.06^\circ\ V}$

c) $S_{base} = 40\,kVA.$ $V_{Hbase} = 7970\,V$ $V_{Lbase} = 240\,V$

$Z_{Hbase} = \frac{V_{Hbase}^2}{S_{base}} = 1588\,\Omega$ $Z_{Lbase} = \frac{V_{Lbase}^2}{S_{base}} = 1.44\,\Omega$

$R_{1pu} = \frac{R_1}{Z_{Hbase}} = 0.0252$ $R_{2pu} = \frac{R_2}{Z_{Lbase}} = 0.0257$

$X_{1pu} = \frac{X_1}{Z_{Hbase}} = 0.0264$ $X_{2pu} = \frac{X_2}{Z_{Lbase}} = 0.0278$



Problem 2.4

a) $I_{rated,H} = \frac{25\ MVA}{\sqrt{3} \times 69\ kV} = 209\ A$

$Z_{eq\,H} = \left(\frac{N_H}{N_L}\right)^2 Z_{eq\,L} = \left(\frac{69}{13.8}\right)^2 (60 + j350) = 1.5 + j\,8.75\ \Omega$

$V_{H,\phi} = I_{rated\,H} \, |Z_{eq\,H}| = 1855\ V$

$V_{H,L-L} = \sqrt{3}\, V_{H\phi} = \boxed{3214\ V.}$

b) $S = 18\ MW / 0.8 = 22.5\ MVA$

$I_{load} = \frac{22.5\ MVA}{\sqrt{3} \times 69\ kV} = 188\ A$

$\theta = -\cos^{-1}(0.8) = -36.9^\circ \Rightarrow \bar{I}_{load} = 188 \angle -36.9^\circ\ A$

$V_{H\phi\,rated} = \frac{69\ kV}{\sqrt{3}} = 39.8\ kV.$

$V_{load,H\phi} = |V_{H\phi\,rated} - \bar{I}_{load} Z_{eq\,H}| = 38.6\ kV.$

$V_{load,L\phi} = \left(\frac{13.8}{69}\right) V_{load\,H} = 7.72\ kV.$

$V_{Load,L,L-L} = 7.72 \times \sqrt{3} = \boxed{13.4\ kV.}$