Galactic Dynamo Simulations, Task-1 Report

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(Dated: March 13, 2024)

I. INTRODUCTION

In this report, we delve into the dynamics of the galactic dynamo by focusing on the simplified dynamo equation in radial coordinates (r). By solving this equation numerically, we aim to investigate the evolution of the magnetic field strength (B_r) as a function of radial distance (r) and time (t). Through computational simulations, we seek to elucidate the underlying mechanisms driving the amplification or decay of magnetic fields in galactic environments.

The numerical solution of the dynamo equation allows us to explore how various factors, such as turbulent magnetic diffusivity and boundary conditions, influence the evolution of the magnetic field. By analyzing the simulated results, we can gain valuable insights into the dynamics of magnetic fields in galaxies and their implications for astrophysical phenomena.

Overall, this report serves as a comprehensive investigation into the behavior of magnetic fields in galaxies, shedding light on the intricate interplay between plasma dynamics and magnetic field evolution through the lens of the galactic dynamo theory.

The github repository with all the respective codes are provided in this link Click here The link is also attached in the google classroom

II. RUNGE-KUTTA METHOD

The fourth-order Runge-Kutta (RK4) method is a numerical technique commonly used for solving ordinary differential equations (ODEs) of the form $\frac{dy}{dt} = f(t, y)$, where y is the unknown function and $f(t, y)$ is a known function representing the derivative of y with respect to t. The general form of the RK4 method involves four steps to compute the next value of y based on the current value:

1. Compute the increment k_1 using the initial values:
$$k_1 = h \cdot f(t_n, y_n)$$

2. Compute the increment k_2 using the midpoint values:
$$k_2 = h \cdot f\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

3. Compute the increment k_3 using the midpoint values:
$$k_3 = h \cdot f\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

4. Compute the increment k_4 using the endpoint values:
$$k_4 = h \cdot f(t_n + h, y_n + k_3)$$

5. Use the weighted sum of these increments to update the value of y:
$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Here, t_n represents the current time, y_n represents the current value of the unknown function y, h is the step size, and y_{n+1} is the next value of y. The increments k_1, k_2, k_3, and k_4 represent estimates of the change in y over the time step h based on the derivative function f evaluated at different points. The RK4 method is fourth-order accurate, meaning that the error in each step is proportional to h^5, providing accurate results with relatively large step sizes compared to lower-order methods.

III. STEPS OF SOLVING AND RESULTS

A. Solving the diffusion equation in 'r'

The dynamo equation describes the evolution of magnetic fields in astrophysical systems, particularly in galaxies, where the magnetic field is generated and amplified by the motion of conducting fluids (e.g., plasma) through a process known as dynamo action.

The simplified form of the dynamo equation in polar coordinates (r) considers the radial component of the magnetic field (B_r) and its evolution over time (t). It can be expressed as:

$$\frac{\partial B_r}{\partial t} = \eta_t \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial B_r}{\partial r}\right) - \frac{\pi^2}{4h^2}B_r\right) \tag{1}$$

where:

- B_r is the radial component of the magnetic field.
- t is time.
- r is the radial distance from the galactic center.

- η_t is the magnetic diffusivity, representing the ability of the plasma to diffuse magnetic fields.

- $\frac{\partial}{\partial t}$ denotes the partial derivative with respect to time.

- $\frac{\partial}{\partial r}$ denotes the partial derivative with respect to radial distance r.

This equation describes how the magnetic field evolves over time due to the interplay between magnetic diffusion and the stretching and folding of the magnetic field lines by turbulent motions in the galactic plasma. The first term on the right-hand side represents the diffusion of the magnetic field, while the second term represents the stretching effect.

Solving the dynamo equation numerically allows us to explore the behavior of magnetic fields in galaxies and understand the mechanisms responsible for their generation, amplification, and decay. This equation serves as a fundamental tool for studying the dynamics of magnetic fields in astrophysical environments and their role in shaping the structure and evolution of galaxies.

Methodology:
The provided code implements the numerical solution of the dynamo equation using the fourth-order Runge-Kutta method. Here's a detailed breakdown:

1. **Setting Parameters:**
   - The turbulent magnetic diffusivity (η), maximum radial distance (R_{max}), number of grid points (N_{grid}), and total time (`Total_Time`) are specified.

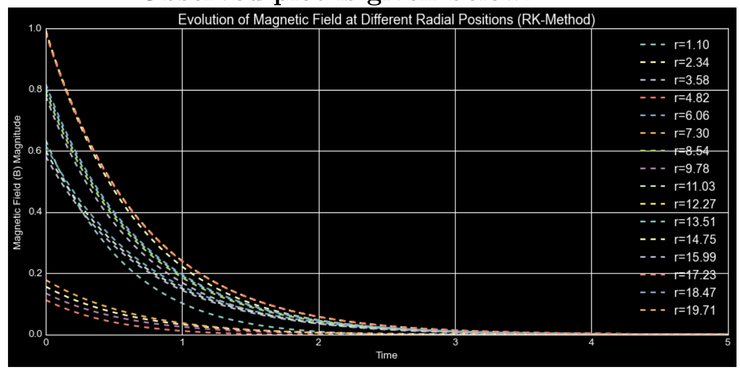
2. **Initialization:**
   - The radial grid points (r) are generated, and an initial value of the radial magnetic field (B_r) is set using a sinusoidal function.

3. **Defining Functions:**
   - `Laplacian_of`: Computes the radial Laplacian of the magnetic field (B_r).
   - `RK_time_step`: Implements the fourth-order Runge-Kutta method to solve the diffusion equation over a time step (Δt).

4. **Numerical Integration:**
   - The Runge-Kutta method is applied iteratively over multiple time steps to solve the diffusion equation and obtain the evolution of the magnetic field (B_r) at different radial positions over time.

5. **Plotting Results:**
   - The evolution of the magnetic field magnitude at select radial positions is plotted over time using Matplotlib.
   - Click here for the code

Observed plot is given below:



B. Exploring the evolution of the magnetic field magnitude and of the exponential decay rate

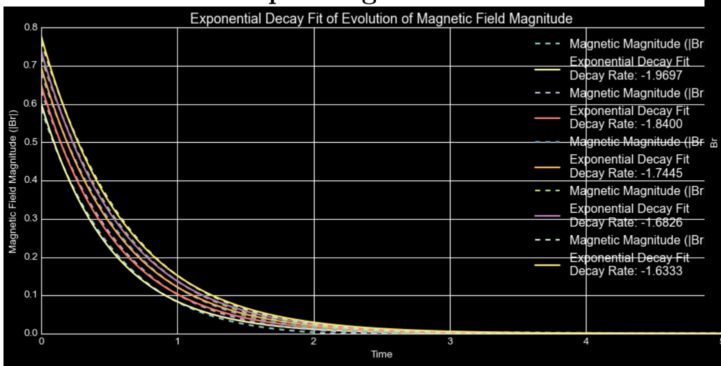
The provided code snippet demonstrates fitting an exponential decay function to the evolution of the magnetic field magnitude over time. Here's a detailed breakdown:

1. **Define Exponential Decay Function:**
   - The function `Expo_decay_func` is defined to model the exponential decay process, taking time (`t`), amplitude, and decay rate as parameters.

2. **Extract Data:**
   - Time points (`Time`) and magnetic field magnitude data are extracted from the previously computed evolution of magnetic field.

3. **Plotting and Fitting:**
   - For visualization, the evolution of magnetic field magnitude at select radial positions is plotted over time.
   - For each plotted curve, an exponential decay model is fitted to the data using the `curve_fit` function from SciPy.
   - The fitted exponential decay curve is then plotted along with the original data.
   - Click here for the code

Observed plot is given below:

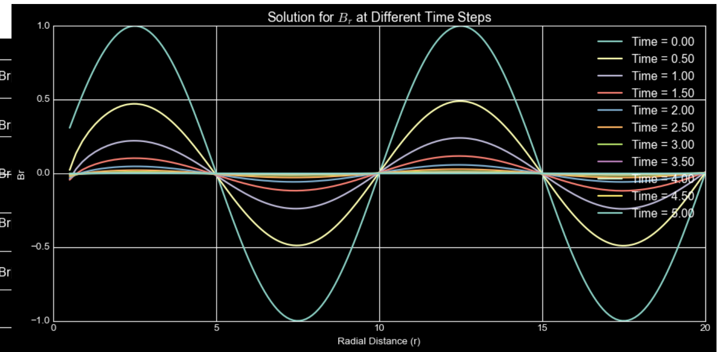


C. Explore the evolution of the spatial solution for B_r and B_ϕ, and of the pitch angle of the mean magnetic field

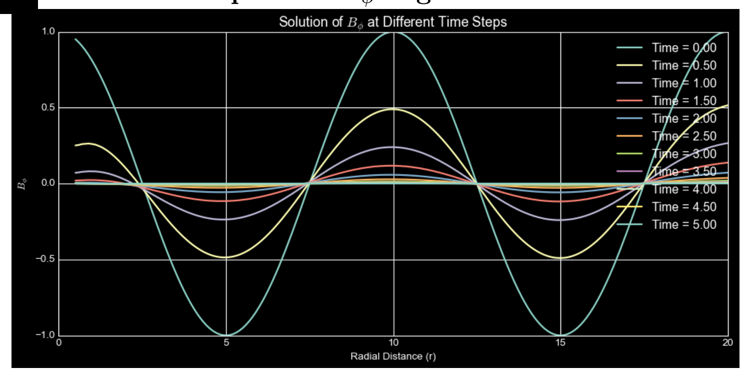
The provided code snippet demonstrates the numerical solution of the dynamo equation considering both radial (B_r) and azimuthal (B_ϕ) components of the magnetic field. Here's a detailed breakdown:

1. **Initialization:**
    - The magnetic field components (B_r and B_ϕ) are initialized using sinusoidal functions at various radial distances (R).
2. **Runge-Kutta Method:**
    - The function `RK_step_diff` implements the fourth-order Runge-Kutta method to solve the diffusion equation for both B_r and B_ϕ.
    - The method computes the evolution of B_r and B_ϕ over multiple time steps.
3. **Storage of Evolution:**
    - Arrays `evolution_Br` and `evolution_Bphi` are used to store the evolution of B_r and B_ϕ, respectively, at different time steps.
4. **Plotting Results:**
    - Two separate plots are generated to visualize the evolution of B_r and B_ϕ over time, respectively.
    - Each plot shows the variation of the magnetic field components at different radial distances for selected time steps.
    - Click here for the code

Observed plot for B_r is given below:



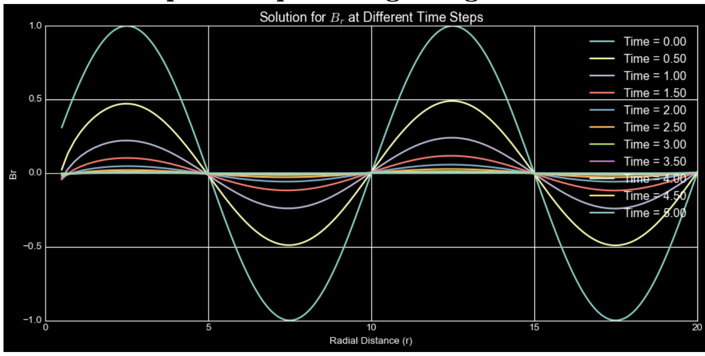
Observed plot for B_ϕ is given below:



The provided code snippet calculates the pitch angle of the magnetic field vector at different radial positions and time steps. Here's a detailed breakdown:

1. **Pitch Angle Computation:**
    - The function `compute_pitch_angle` computes the pitch angle (p) using the arctan2 function, which is the angle between the azimuthal (B_ϕ) and radial (B_r) components of the magnetic field.
2. **Storage of Evolution:**
    - An array `evolution_pitch_angle` is used to store the evolution of the pitch angle at different time steps and radial positions.
3. **Time-stepping Scheme:**
    - The pitch angle is computed for each time step using the Runge-Kutta method applied to the evolution of the radial and azimuthal components of the magnetic field.
4. **Plotting Results:**
    - A plot is generated to visualize the spatial solution for the pitch angle at different time steps.
    - The pitch angle is plotted against radial distance (R), showing its evolution over time.
    - Click here for the code

Observed plot for pitch angle is given below:



IV. CONCLUSION

In conclusion, this report has presented a numerical approach for solving the dynamo equation in a galactic context. The solution method employed the fourth-order Runge-Kutta scheme to evolve the magnetic field components over time. We began by initializing the magnetic field with specified initial conditions and then iteratively applied the Runge-Kutta method to calculate the evolution of the magnetic field components at different radial positions.

The results demonstrated the evolution of the magnetic field magnitude and pitch angle over time and space. Through visualization of the magnetic field's behavior, we observed patterns of exponential decay in the magnetic field magnitude and the evolution of the pitch angle across radial distances. These findings provide valuable insights into the dynamics of magnetic fields in galactic systems.

Overall, the numerical solution presented in this report offers a powerful tool for studying the behavior of magnetic fields in galactic environments. Further research and refinement of the computational techniques presented here could lead to a deeper understanding of the complex interplay between magnetic fields and astrophysical processes in galaxies.