Assignment - 2

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Theory :

1. $u(t) = [20 + 2\cos(3k\pi t) + 10\cos(6k\pi t)]\cos(2\pi f_c t)$

$f_c = 10^5$ Hz

(a) $u(t) = 20\cos(2\pi f_c t) + 2\cos(3k\pi t)\cos(2\pi f_c t)$

$+ 10\cos(6k\pi t)\cos(2\pi f_c t)$

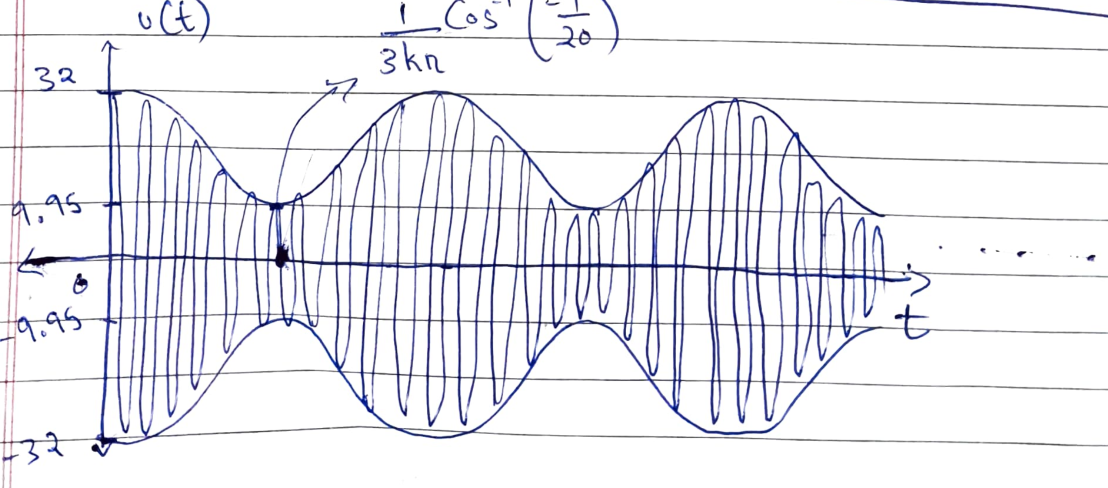
$\therefore$ $\boxed{u(t) = 20\cos(2\pi f_c t) + \cos(203k\pi t) + \cos(197k\pi t) + 5\cos(206k\pi t) + 5\cos(194k\pi t)}$

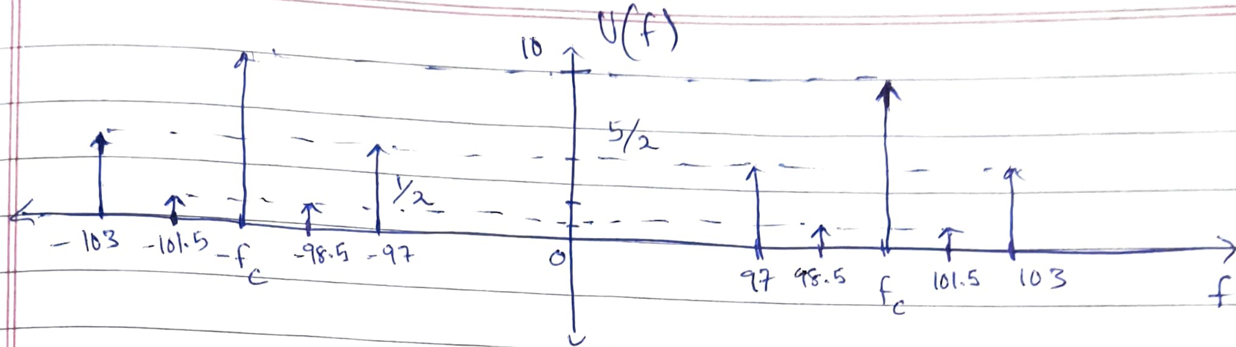
$\therefore$ $U(f) = 10[\delta(f+f_c) + \delta(f-f_c)] + \frac{1}{2}[\delta(f+101.5k)$

$+ \delta(f-101.5k)] + \frac{5}{2}[\delta(f+103k)$

$+ \delta(f-103k)] + \frac{1}{2}[\delta(f+98.5k) + \delta(f-98.5k)]$

$+ \frac{5}{2}[\delta(f+97k) + \delta(f-97k)]$





(b) Power in each freq^n. component.

(i) $f_c \rightarrow P_{f_c} = 200\,W$

(ii) $P_{101.5} = \frac{1}{2}\,W$ (iv) $P_{103} = 12.5\,W$

(iii) $P_{98.5} = \frac{1}{2}\,W$ (v) $P_{97} = 12.5\,W.$

(Since all are sinusoids $P = \frac{(\text{Amplitude})^2}{2}$)

(c)

Let $m(t) = 2\cos(3k\pi t) + 10\cos(6k\pi t)$

$\therefore v(t) = 20\,[1 + k_a \cdot m(t)]\cos(2\pi f_c t)$

Now $\mu = k_a \left| \min\{m(t)\} \right|$

$k_a = \frac{1}{20}$ & $\min\{m(t)\} = -10.05$

$\Rightarrow \mu = \frac{1005 \times 10^{-2}}{\cancel{20}} \times \cancel{100}^{5}$

$\Rightarrow$ $\boxed{\mu = 50.25\,\%}$

(d) we know that total power of a signal is equally divided into its ~~USB~~ USBs & LSBs

$\therefore P_{USB} = \frac{1}{2} + \frac{25}{2} = \cancel{6.5}\ 13\ W$

& $P_{LSB} = \frac{1}{2} + \frac{25}{2} = \cancel{6.5}\ 13\ W$

$\Rightarrow P_{total} = P_{carrier} + P_{sidebands}$

$\Rightarrow P_{total} = 200 + 26$

$\Rightarrow \boxed{P_{total} = 226\ W}$

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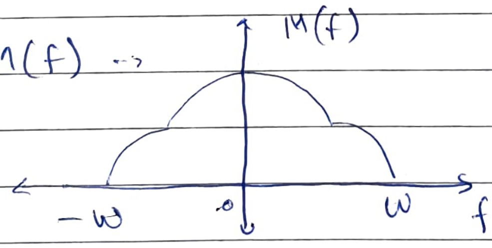
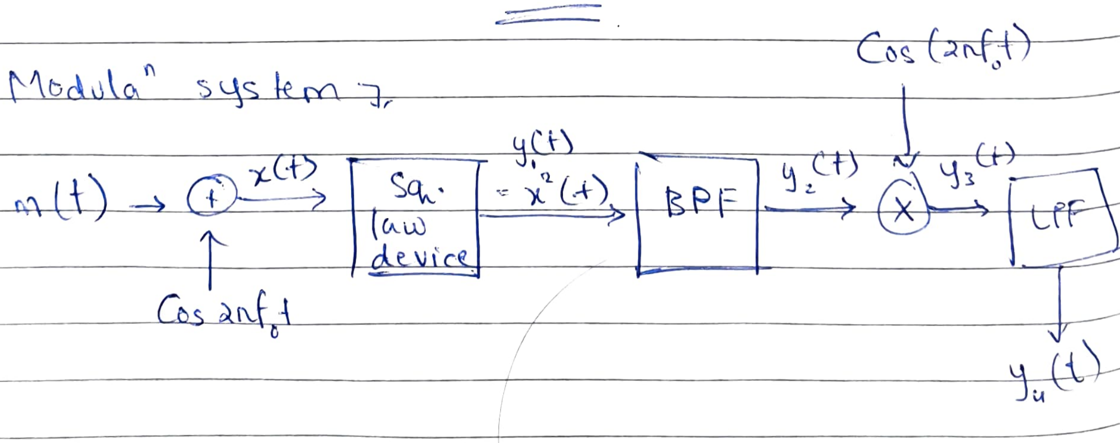
~~m(t)~~ M(f) -->



$\Rightarrow$ BW of M(f) = W

Modula^n system :-



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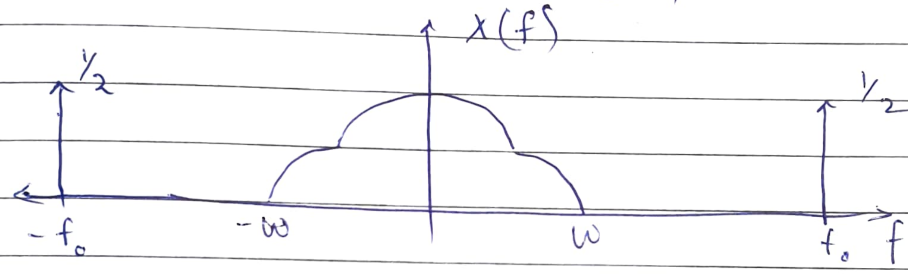
Now $x(t) = m(t) + \cos(2\pi f_0 t)$.

when two signals added, ~~th~~ the bandwidth of resultant signal is max ~~th~~ of bandwidth of two added ones.

$$X(f) = M(f) + \frac{\delta(f+f_0)}{2} + \frac{\delta(f-f_0)}{2}$$

Assuming $f_0 >> W$ we get,



$$\Rightarrow \boxed{BW_{x(t)} = f_0}$$

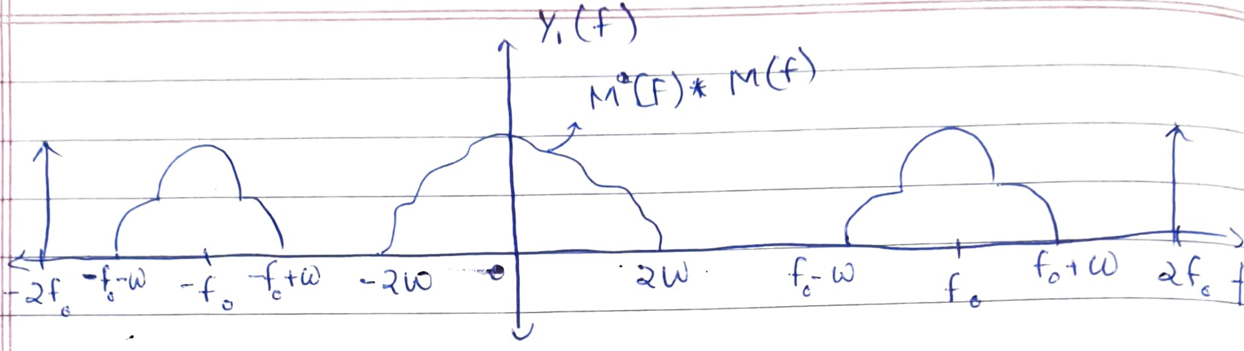
$$y_1(t) = x^2(t) = [m(t) + \cos(2\pi f_0 t)]^2$$

$$\therefore y_1(t) = m^2(t) + 2m(t)\cos(2\pi f_0 t) + \cos^2(2\pi f_0 t)$$

Multiplying two signals in time domain adds their bandwidths in freq. domain

$\therefore$ $BW_{m^2} = 2W$, $BW_{m \cdot \cos} = 2W$ (centered around f_0)

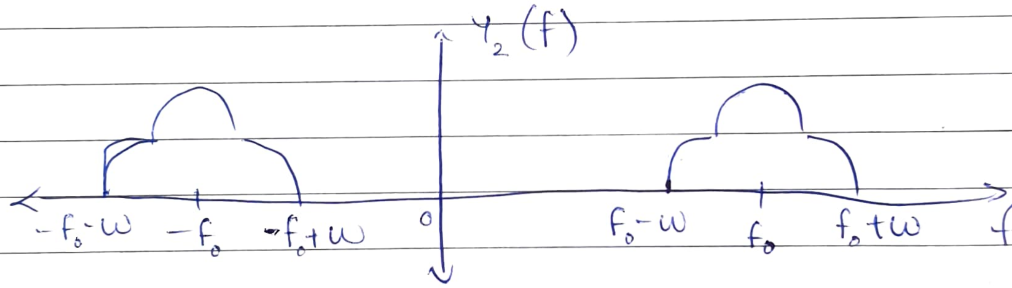
& $BW_{\cos} = 2f_0$



$\therefore \boxed{BW_{y_1(t)} = 2f_0}$

Now we pass $y_1(t)$ through a BPF with bandwidth of $2W$ centered around f_0

$\therefore$



$\Rightarrow \boxed{BW_{y_2(t)} = 2W}$ $\Rightarrow y_2(t) = 2m(t)\cos(2\pi f_0 t)$

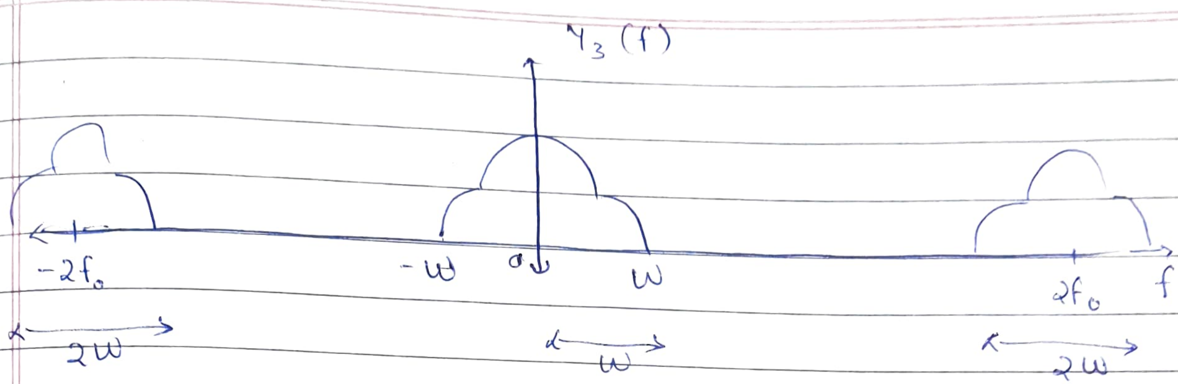
$y_3(t) = y_2(t)\cos(2\pi f_0 t)$

$\therefore y_3(t) = 2m(t)\cos^2(2\pi f_0 t)$

$\therefore \boxed{y_3(t) = m(t) + m(t)\cdot\cos(4\pi f_0 t)}$

$\therefore Y_3(f) = M(f) + \frac{1}{2}\left(M(f+2f_0) + M(f-2f_0)\right)$

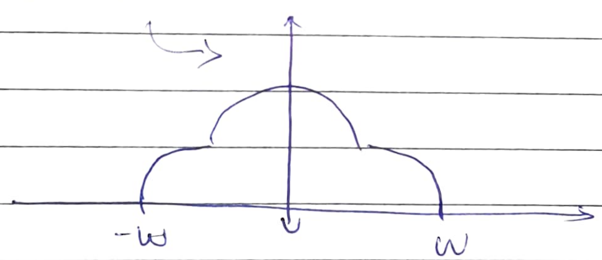
$\Rightarrow \boxed{BW_{y_3(t)} = 2f_0 + W}$



Now $y_3(t)$ is passed through an LPF of $BW = W$

$\Rightarrow$ $\boxed{y_4(t) = m(t).}$ ($\because$ LPF will remove |freq| above > W)

$\Rightarrow$ $Y_4(f) = M(f)$



& $\boxed{BW_{y_4(t)} = W}$

4. $M(f) = \begin{cases} j2\pi f & |f| < 1 \\ 0 & o.w \end{cases}$

$m(t)$ is sent through an FM modulator with $K_f = 1$

$\phi_{FM}(t) = A\cos(2\pi f_c t + \phi(t))$ $\rightarrow f_c$ = carrier freq.

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(a) we have $M(f) = \begin{cases} j2\pi f & ; |f|<1 \\ 0 & ; o.w \end{cases}$

$$\therefore m(t) = \int_{-1}^{1} j2\pi f \, e^{j2\pi ft} \, df$$

$$\therefore m(t) = \frac{\cancel{j2\pi} f}{\cancel{j2\pi} t} \cdot e^{j2\pi ft} \Big|_{-1}^{1} - \frac{1}{t}\int_{-1}^{1} e^{j2\pi ft} \, df$$

$$\therefore m(t) = \frac{1}{t}\left(e^{j2\pi t} + e^{-j2\pi t}\right) - \frac{1}{j2\pi t^2}\left(e^{j2\pi t} - e^{-j2\pi t}\right)$$

$$\therefore \boxed{m(t) = \frac{2\cos(2\pi t)}{t} - \frac{\sin(2\pi t)}{\pi t^2}}$$

we know that,

$$\sin(t) = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \ldots$$

$$\& \quad \cos(t) = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \ldots$$

$$\Rightarrow \frac{2\cos(2\pi t)}{t} = 2\left[\frac{1}{t} - \frac{(2\pi)^2 \cdot t}{2!} + \frac{(2\pi)^4 t^3}{4!} - \ldots\right]$$

$$\& \quad \frac{\sin(2\pi t)}{\pi t^2} = \frac{1}{\pi}\left[\frac{2\pi}{t} - \frac{(2\pi)^3 \cdot t}{3!} + \frac{(2\pi)^5 \cdot t^3}{5!} - \ldots\right]$$

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$\therefore m(t) = \frac{2}{\pi}\left[(2\pi)^2 \cdot t\left(\frac{1}{2!} - \frac{1}{2!}\right) + (2\pi)^4 t^3\left(\frac{1}{4!} - \frac{1}{5!}\right) + \cdots\right]$

$\therefore m(t) = \frac{-2 \cdot 2\pi}{\pi}\left[(2\pi \cdot t) \cdot \frac{2}{3!} - (2\pi t)^3 \frac{4}{5!} + \cdots\right]$

$\therefore \phi(t) = \int_{-\infty}^{t} m(\tau)\, d\tau$

$\therefore \phi(t) = -4 \int_{-\infty}^{t} \left[(2\pi\tau)\frac{2}{3!} - \frac{(2\pi\tau)^3\, 4}{5!} + \cdots\right] d\tau$

$\therefore \phi(t) = -4\left[2\pi \cdot \frac{\tau^2}{3!} - (2\pi)^3 \cdot \frac{\tau^4}{5!} + \cdots\right]_{-\infty}^{t}$

$\therefore \phi(t) = -4\left[\frac{\sin(2\pi\tau)}{(2\pi)^2\tau} - \frac{1}{2\pi}\right]\Bigg|_{-\infty}^{t}$

Now $\lim_{x \to -\infty} \frac{\sin(x)}{x} = 0 \quad \because \sin(x) \ \forall\, x \in [-1, 1]$ & $x \to -\infty$

$\Rightarrow \phi(t) = -4 \cdot \frac{\sin(2\pi t)}{4\pi^2 t}$

$\Rightarrow \boxed{\phi(t) = -\frac{\sin(2\pi t)}{\pi^2 t}}$

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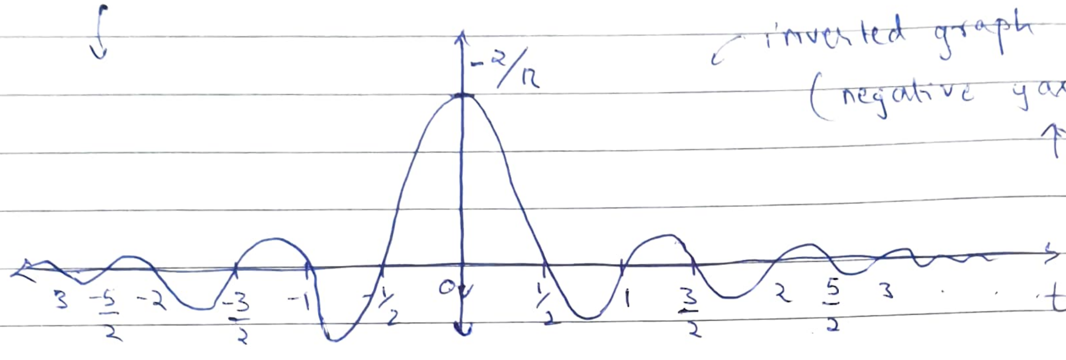
$$\Rightarrow \boxed{\phi(t) = -\text{Sinc}(2\pi t)\cdot\frac{2}{\pi}}$$

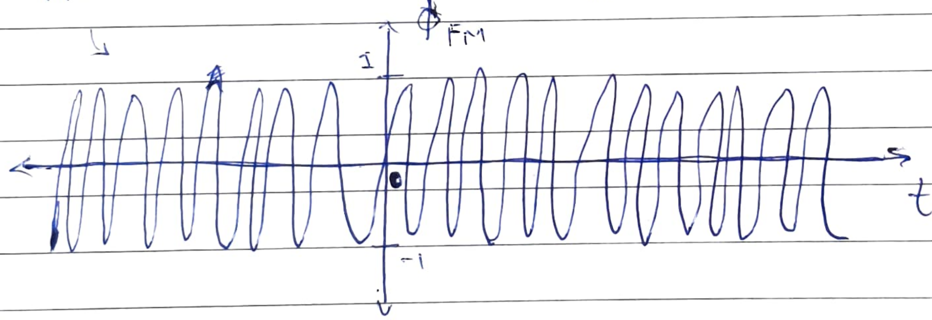
$\left(\text{Sinc}(t) = \frac{\text{Sin}(t)}{t}\right)$



inverted graph
(negative y axis is ↑)

(b) $\phi_{FM}(t) = A\cos(2\pi f_c t + \phi(t))$

$\therefore \phi_{FM}(t) = A\cos\left(2\pi f_c t - \frac{2}{\pi}\text{Sinc}(2\pi t)\right)$



(c) $\Delta f_{FM} = \frac{k_f \cdot m(t)}{2\pi}$

$$\therefore \Delta f_{FM}\Big|_{t=1/4} = \frac{1}{2\pi}\cdot\frac{\text{Sin}(2\pi\cdot 1/4)}{\pi^3\cdot 1/4^3}\left[\frac{2\cos(2\pi\cdot 1/4)}{1/4} - \frac{\text{Sin}(2\pi\cdot 1/4)}{\pi\cdot\frac{1}{16}}\right]$$

$$\Rightarrow \boxed{\Delta f_{FM}\Big|_{t=1/4} = \frac{-8}{\pi^2}}$$

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(d) we have $k_f = 1$ & $a(t)$ (in our case $\phi(t)$)

$$= -\frac{2}{\pi} \text{sinc}(2\pi t)$$

$$\Rightarrow \left| k_f \cdot a_{max}(t) \right| = \left| \frac{2}{\pi} \right| \approx 0.64 < 1$$

$$\Rightarrow \Delta f_{FM} = \frac{k_f \{m(t)\}_{max}}{2\pi}$$

$$\Rightarrow \cancel{\Delta f_{FM} = \frac{1}{2\pi}}$$

$$\because m'(t) = 0$$

$$\Rightarrow -\frac{2}{t^2} \cos(2\pi t) - \frac{4\pi \sin(2\pi t)}{t} + \frac{2\sin(2\pi t)}{\pi^2 t^3} - \frac{2\cos(2\pi t)}{t^2} = 0$$

$$\Rightarrow \frac{\cancel{2}\sin(2\pi t)}{\cancel{t}} \left[\frac{1}{\pi t^2} - 2\pi \right] = \frac{\cancel{4}^{2} \cos(2\pi t)}{t^{\cancel{2}}}$$

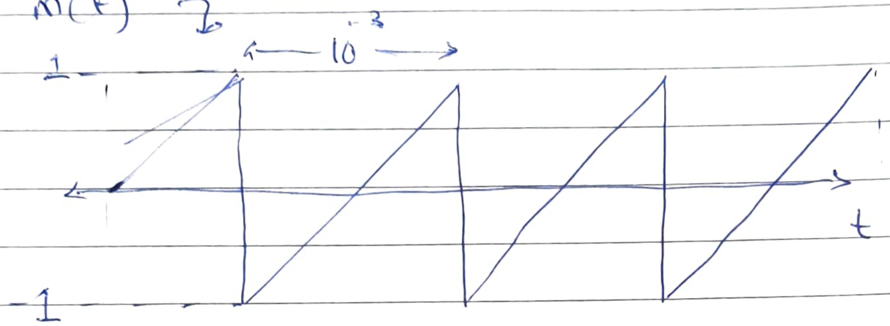
using online tool hereon, I get $m(t)_{max} = 5.481$

$$\Rightarrow \Delta f_{FM} = \frac{5.481}{2\pi} = 0.93$$ which is not insignificant $\Rightarrow$ This is WBFM

$$\Rightarrow BW_{\phi_{FM}(t)} = 2(\Delta f + B)$$ & $B = 1$ (Given)

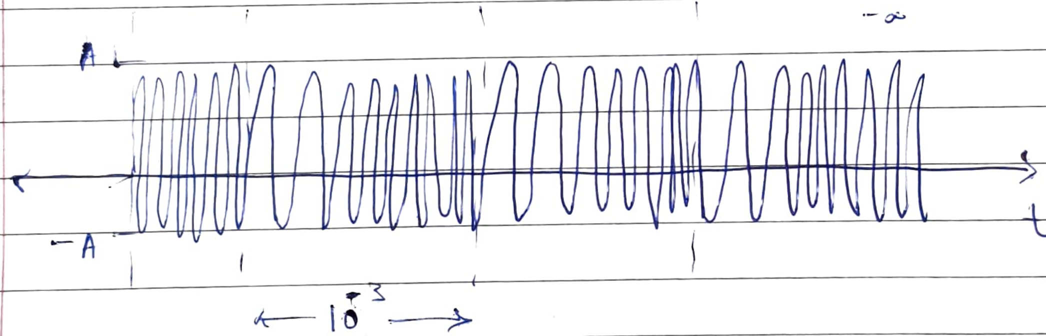
$$\Rightarrow \boxed{BW_{\phi_{FM}(t)} = 2(0.93 + 1) = 3.86 \text{ Hz}}$$

Q5 $m(t) \Rightarrow$



(a) $\omega_c = 2\pi \times 10^6$, $k_f = 2000\pi$ & $k_p = \frac{\pi}{2}$

$$\phi_{FM}(t) = A\cos\left(2\pi \times 10^6 t + 2000\pi \int_{-\infty}^{t} m(t)\right)$$

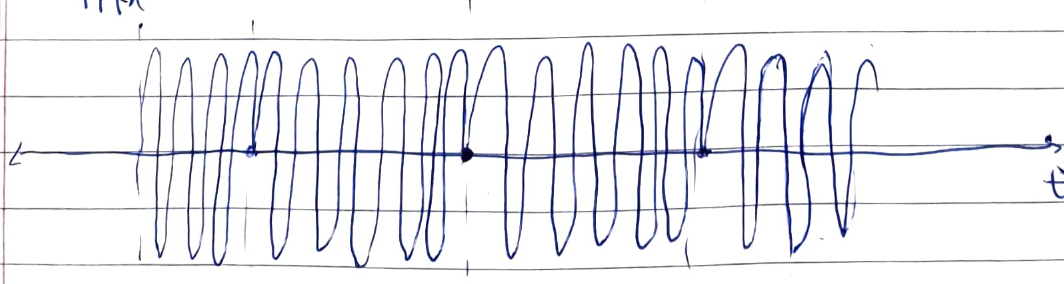


& for PM, at places where $m(t)$ is continuous

$$f_i = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt} = \left(10^6 + \frac{2\cdot 10^3}{4}\right) \text{Hz}$$

at discontinuities, we get a $\underbrace{\text{phase shift}}_{= k_p \cdot m_d}$ proportional to jump in $m(t) \rightarrow m_d$

$\therefore \phi_{PM}(t) \Rightarrow$



Now the phase shift at discont. is

$$\Delta\theta = k_p \cdot m_d \quad \text{where} \quad m_d = 2.$$

$\therefore$ to avoid the ambiguity at Rx end due to when phase deviations which may occur if $\Delta\theta > \pi$ or $\Delta\theta < -\pi$

$$(\because \Delta\theta = \Delta\theta + 2n\pi)$$

we need

$$k_p m_d \leq \pi$$

$$\Rightarrow \boxed{k_p < \frac{\pi}{2}.}$$

(b) we have $m(t)$ is periodic with $T = 10^{-3}$ s.

$\Rightarrow$ ~~m(t) =~~ $m(t) = \frac{2t}{10^{-3}} \quad \forall \; t \in [a, a+T]$ s.t. $a \in \mathbb{R}$

$$\Rightarrow FS\{m(t)\}_n = \frac{2 \cdot 2}{10^{-6}} \int_{-10^{-3}/2}^{10^{-3}/2} t \cdot e^{-j\frac{2\pi n t}{10^{-3}}} \, dt$$

$$\Rightarrow x_n = \frac{2 \cdot 2}{10^{-6}} \int_{-10^{-3}/2}^{10^{-3}/2} t \cdot e^{-j\frac{2\pi n t}{10^{-3}}} \, dt$$

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$$\Rightarrow x_n = \frac{2\cdot\cancel{\pi}}{10^{-3}}\left[\frac{-t\cdot 10^{-3}}{j2\pi n}\cdot e^{\frac{-j2\pi n t}{10^{-3}}}\Bigg|_{-\frac{10^{-3}}{2}}^{\frac{10^{-3}}{2}} + \left(\frac{10^{-3}}{j2\pi n}\right)^2 e^{\frac{-j2\pi n t}{10^{-3}}}\Bigg|_{-\frac{10^{-3}}{2}}^{\frac{10^{-3}}{2}}\right]$$

$$\Rightarrow x_n = \frac{2\cdot\cancel{\pi}}{\cancel{10^{-6}}}\left[-\frac{\cancel{10^{-6}}}{j\cancel{4}\pi n\cancel{2}}\cos\left(\frac{2\pi n}{\cancel{10^{-3}}}\right) + \frac{\cancel{10^{-6}}\cdot\cancel{2}j}{\cancel{2}j\,\cancel{4}\pi^2 n^2}\sin\left(\frac{2\pi n}{\cancel{10^{-3}}}\right)\right]$$

$$\Rightarrow x_n = \frac{2\cdot j\,\pi n\cos\underbrace{\left(\frac{2\pi n}{\cancel{10^{-3}}}\right)}_{(-1)^n} + 2\sin\underbrace{\left(\frac{2\pi n}{\cancel{10^{-3}}}\right)}_{\to 0}}{\pi^2 n^2}$$

$$\Rightarrow \boxed{x_n = \frac{2j(-1)^n}{\pi n}} \longrightarrow \text{FS Coeff for } m(t).$$

$$\& \quad \boxed{x_0 = 0}$$

$$\Rightarrow m(t) = \sum_{n=-\infty}^{\infty} x_n = \frac{2}{\pi}\sum_{n=-\infty}^{\infty}\sin\left(\frac{2\pi n t}{10^{-3}}\right)$$

$\Rightarrow$ at 5^{th} harmonic, $BW_{m(t)} = 5\times 10^{3} = 5\,kHz.$

$$\Rightarrow \boxed{BW_{\phi_{FM}} = 2(\Delta f_{FM} + B) = 2(1000 + 5000) = 12\,kHz}$$

$$\& \quad \boxed{BW_{\phi_{PM}} = 2(\Delta f_{PM} + B) = 2\left(\frac{\cancel{\pi}\cdot 10^{3}}{\cancel{\pi}2} + 5000\right) = 11\,kHz}$$

($\because$ wide band BW because $|k_i\,m(t)|$ is large enough).

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$$x_h(t) = x(t) * \frac{1}{\pi t}$$

$$\begin{array}{ccccc} x(t) & \longrightarrow & \boxed{HT} & \longrightarrow & x_h(t) \\ & & \downarrow & & \\ & & \frac{1}{\pi t} & & \end{array}$$

Signum

$$H(f) = FT\{h(t)\} = FT\left\{\frac{1}{\pi t}\right\} = -j\,\mathrm{Sgn}(f)$$

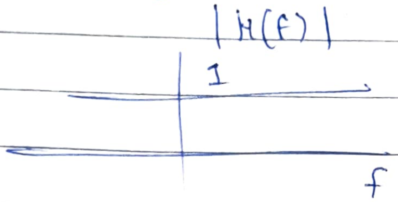
$$\Rightarrow \boxed{X_h(f) = -j\,X(f)\,\mathrm{Sgn}(f)}$$

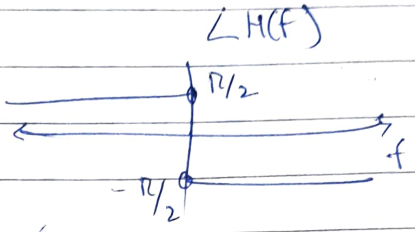
Now $\rightarrow$

$$H(f) = \begin{cases} -j & ; \ f > 0 \\ 0 & ; \ f = 0 \\ j & ; \ f < 0 \end{cases}$$

which can also be written as,

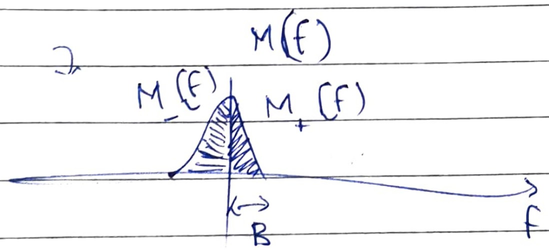
$$H(f) = \begin{cases} e^{-j\pi/2} & f > 0 \\ e^{j\pi/2} & f < 0 \end{cases}$$





$\Rightarrow$ Hilbert transform shifts all frequencies in the FT of a signal by $\pi/2$

$\Rightarrow$ let



$$\therefore M_+(f) = M(f) \cdot u(f)$$

$$= M(f) \cdot \frac{1}{2} \left[1 + \mathrm{Sgn}(f) \right]$$

$$\& \quad M_H(f) = -j M(f) \mathrm{Sgn}(f)$$

$$\Rightarrow \boxed{M_+(f) = \frac{M(f)}{2} + \frac{j}{2} M_h(f)}$$

Similarly,

$$\boxed{M_-(f) = \frac{M(f)}{2} - \frac{j}{2} M_h(f)}$$

& $$\phi_{USB}(f) = M_+(f-f_c) + M_-(f+f_c)$$

$$= \frac{1}{2}\left[M(f-f_c) + jM_h(f-f_c) + M(f+f_c) - jM_h(f+f_c) \right]$$

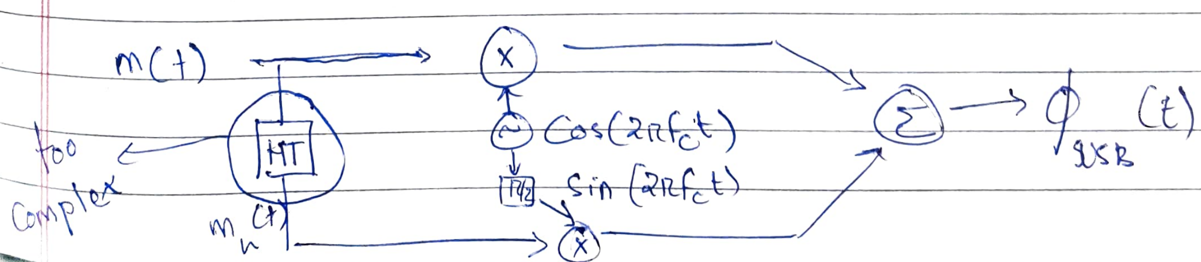
$$\boxed{\phi_{USB}(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c) \right] - \frac{j}{2}\left[M_h(f+f_c) - M_h(f-f_c) \right]}$$

$$\Rightarrow \boxed{\phi_{USB}(t) = m(t)\cdot \cos(2\pi f_c t) - m_h(t)\sin(2\pi f_c t)}$$

Similarly

$$\boxed{\phi_{LSB}(t) = m(t)\cdot \cos(2\pi f_c t) + m_h(t)\sin(2\pi f_c t)}$$

Physical realisation. ?



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Demodulation for this process

$$\phi_{USB}(t) \times \cos(2\pi f_c t) = \left[m(t)\cos(2\pi f_c t) - m_h(t) \sin(2\pi f_c t) \right] \times \cos(2\pi f_c t)$$

$$= \frac{1}{2}\left[m(t) + \underbrace{m(t)\cos(4\pi f_c t) - m_h(t)\sin(4\pi f_c t)}_{\text{same as } \phi_{USB} \text{ but shifted to } 2f_c} \right]$$

$m(t) \rightarrow$ LPF $\rightarrow$ $\boxed{\frac{m(t)}{2}}$.

⇒ Implentation at Rx end stays same.