The Darcy Friction Factor

An Investigation into Laminar Flow Dynamics

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Scope of Research:

Determine the correlation between the Darcy friction factor and the Reynolds Number in smooth pipe environments.

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1 Introduction

Understanding how the Darcy friction factor changes with the Reynolds number is a fundamental aspect of fluid mechanics with profound implications for various engineering applications. Understanding this factor is crucial for designing, analysing, and optimising fluid flow systems in real-world scenarios. In industries such as chemical engineering, HVAC (heating, ventilation, and air conditioning) [1], and water distribution systems, accurate pressure drop and flow rate prediction is essential for optimal system design and operation. The Darcy friction factor directly influences these parameters, making it vital for engineers and designers seeking to maximise efficiency and minimise energy consumption. Additionally, in petroleum engineering, understanding frictional losses in pipelines is vital for efficient oil and gas transportation. Therefore, in my experiment, I will investigate the relationship between Reynolds number and the Darcy friction factor to deepen my understanding of fluid flow behaviour.

2 Background Science

2.1 Laminar Flow

Laminar flow is when fluids flow smoothly in layers with little or no lateral mixing [2] There are no cross-currents perpendicular to the direction of fluid flow or any swirls formed[3] . At low velocities, when the Reynolds Number is below 2300, it is considered that the fluid is in laminar flow[4].

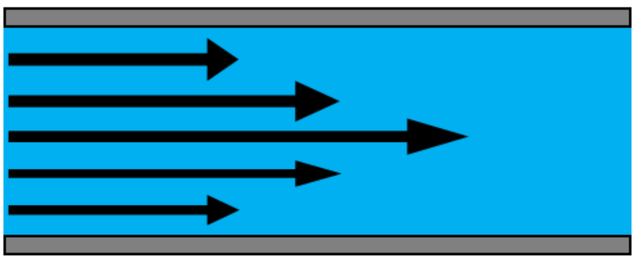

Figure 1: *Laminar Flow in a pipe*

2.2 Darcy-Weisbach Equation

A relationship between the pressure difference of the pipe, the properties, and the friction factor, can be derived, allowing the friction factor to be calculated.

The friction factor for fluid flow in a pipe can be calculated by considering a steady flow of an incompressible fluid through a uniform horizontal pipe. This pipe has a fixed diameter d, length L, and cross-sectional area A. Fluid flows due to a pressure difference between two sections of the pipe, as shown in Figure 2.

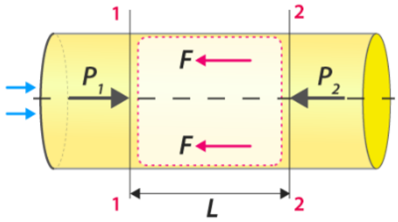

Figure 2: *Uniform horizontal pipe with steady fluid flow.*

The two sections of the pipe, S_1 and S_2, are separated by a distance L. At S_1, the pressure is P_1, the velocity is v_1, and the elevation is h_1. At S_2, the corresponding values are P_2, v_2, and h_2. Since the fluid is flowing steadily and the pipe is horizontal, $h_1 = h_2$, and the diameter remains constant, implying $v_1 = v_2$. These simplifications are applied in subsequent steps.

The energy distribution and pressure differences between S_1 and S_2 are analysed by applying Bernoulli's equation.

This principle arises from the conservation of energy, requiring that the sum of kinetic energy, potential energy, and internal energy remains constant in a streamlined flow [5]. So, an increase in the speed of the fluid, implies an increase in its kinetic energy, resulting in a simultaneous decrease in the sum of its potential energy and internal energy [6]. So, for a steady flow of an incompressible fluid, an increase in the fluid's velocity is accompanied by a decrease in pressure, and vice versa. Bernoulli's equation is expressed as:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

where:

$$
\begin{aligned}
P_1 &: \text{Pressure at point 1 (Pa)} \\
P_2 &: \text{Pressure at point 2 (Pa)} \\
\rho &: \text{Density of the fluid (kg/m}^3\text{)} \\
v_1 &: \text{Velocity at point 1 (m/s)} \\
v_2 &: \text{Velocity at point 2 (m/s)} \\
g &: \text{Acceleration due to gravity (m/s}^2\text{)} \\
h_1 &: \text{Height of the fluid above a reference point at point 1 (m)} \\
h_2 &: \text{Height of the fluid above a reference point at point 2 (m)}
\end{aligned}
$$

Applying Bernoulli's principle to S_1 and S_2:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Dividing through by ρg to express terms in head units results in:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + H_f$$

Here, H_f represents the head loss due to friction. For a horizontal pipe with constant diameter:

- $h_1 = h_2$ (no elevation difference)
- $v_1 = v_2$ (velocity remains constant due to steady flow and uniform cross-section)

Substituting these into the equation and simplifying yields:

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + H_f$$

Rearranging for H_f, the head loss due to friction is expressed as:

$$H_f = \frac{P_1 - P_2}{\rho g}$$

The pressure difference $P_1 - P_2$ arises due to frictional resistance along the pipe's length. The frictional force F acting along the pipe can be expressed as[7]:

$$F = f' \cdot A_{\text{wetted}} \cdot v^2$$

Here, f' represents the frictional resistance per unit wetted area per unit velocity squared. The wetted area, A_{wetted}, is the surface area of the pipe in contact with the fluid, directly influencing the resistance to flow. The velocity term, v^2, reflects the fact that frictional losses increase with velocity, as higher velocities generate greater shear stresses at the pipe wall. The wetted area for a pipe with perimeter $P = 2\pi r$ and length L is $P \cdot L$. Substituting this:

$$F = f' \cdot P \cdot L \cdot v^2$$

The net force driving the flow is the difference in pressure forces at S_1 and S_2, subtracting the frictional force. Resolving forces:

$$P_1 A - P_2 A - F = 0$$

Substituting for F and solving for $P_1 - P_2$:

$$P_1 - P_2 = \frac{f' P L v^2}{A}$$

Using the expression for $P_1 - P_2$ in the head loss equation:

$$H_f = \frac{f' P L v^2}{\rho g A}$$

The term $\frac{P}{A}$ represents the ratio of the wetted perimeter to the cross-sectional area of the pipe. For a circular pipe:

$$\frac{P}{A} = \frac{\pi d}{\pi d^2/4} = \frac{4}{d}$$

Substituting this into the equation for H_f:

$$H_f = \frac{f'}{\rho g} \cdot \frac{4 L v^2}{d}$$

To simplify further, the Darcy friction factor f is introduced, related to f' by [8]:

$$f = 2 \cdot \frac{f'}{\rho}$$

Substituting this into the equation for H_f results in the Darcy-Weisbach equation:

$$H_f = \frac{f}{2g} \cdot \frac{Lv^2}{d}$$

where:

$$\begin{aligned} H_f &: \text{head loss due to friction (m)} \\ f &: \text{Darcy-Weisbach friction factor (dimensionless)} \\ L &: \text{pipe length (m)} \\ d &: \text{pipe diameter (m)} \\ v &: \text{flow velocity (m/s)} \\ g &: \text{acceleration due to gravity (m/s}^2\text{)} \end{aligned}$$

2.3 Reynolds Number

In fluid dynamics, the Reynolds number (Re) is a dimensionless quantity that helps predict fluid flow patterns by measuring the ratio between inertial and viscous forces [9]. The flow is considered laminar at low Reynolds numbers, while at high Reynolds numbers, the flow is considered to be turbulent.

To understand the origin of the Reynolds Number, the incompressible form of the Navier–Stokes equations for a Newtonian fluid, expressed in terms of the Lagrangian derivative, is considered.

The Navier-Stokes equations establish the fundamental mechanics of fluid dynamics through partial differential equations that govern the motion of incompressible [10]. These equations mathematically represent momentum balance for Newtonian fluids and incorporate conservation of mass. They are derived from the continuity equation for conservation of mass, Newton's Second Law of Motion, and the First Law of Thermodynamics to describe fluid transportation.

The form of the Navier–Stokes equations used here is the special, but common, case of incompressible flow. This simplifies the momentum equations significantly by making the following assumptions[11]:

- Viscosity μ is constant.
- The second viscosity effect $\lambda = 0$.
- The simplified mass continuity equation $\nabla \cdot \mathbf{u} = 0$ applies.

Under these assumptions, the incompressible form of the Navier–Stokes equations for a Newtonian fluid, expressed in terms of the Lagrangian derivative, is[5]:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{f}, \tag{1}$$

where:

$$\begin{aligned} \mathbf{v} &: \text{velocity vector,} \\ p &: \text{pressure,} \\ \rho &: \text{fluid density,} \\ \mu &: \text{dynamic viscosity,} \\ \mathbf{f} &: \text{external force field,} \\ \frac{D}{Dt} &: \text{Lagrangian derivative.} \end{aligned}$$

Each term in the Navier-Stokes equation has units of force per unit volume, with dimensions equivalent to density times acceleration. Since each term depends on the exact measurements of the flow, the equation is made dimensionless by multiplying it by a factor with inverse units of the base equation. The non-dimensionalization is achieved by multiplying the equation by:

$$\frac{L}{\rho V^2},$$

where:

$$V: \text{mean velocity (m/s)},$$
$$L: \text{characteristic length (m)},$$
$$\rho: \text{fluid density (kg/m}^3\text{)}.$$

The non-dimensionalized variables are defined as follows:

$$\mathbf{v}' = \frac{\mathbf{v}}{V}, \qquad p' = p\frac{1}{\rho V^2},$$
$$\mathbf{f}' = \mathbf{f}\frac{L}{V^2}, \qquad \frac{\partial}{\partial t'} = \frac{L}{V}\frac{\partial}{\partial t},$$
$$\nabla' = L\nabla.$$

Substituting these into the Navier-Stokes equation results in the non-dimensional form:

$$\frac{D\mathbf{v}'}{Dt'} = -\nabla' p' + \frac{\mu}{\rho L V}\nabla'^2\mathbf{v}' + \mathbf{f}',$$

where the term $\frac{\mu}{\rho L V} = \frac{1}{Re}$.

Finally, dropping the primes for ease of readability produces:

$$\frac{D\mathbf{v}}{Dt} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{v} + \mathbf{f}.$$

The Reynolds Number is thereby introduced, defined as[12]:

$$Re = \frac{\rho \cdot v \cdot L}{\mu}$$

where:

$$\begin{aligned}
Re &: \text{Reynolds number}, \\
\rho &: \text{density of the fluid} \quad (\mathrm{kg/m^3}), \\
v &: \text{velocity of the fluid} \quad (\mathrm{m/s}), \\
L &: \text{characteristic length} \quad (\mathrm{m}), \\
\mu &: \text{dynamic viscosity of the fluid} \quad (\mathrm{Pa \cdot s}).
\end{aligned}$$

This dimensionless number characterizes the flow regime, distinguishing between laminar and turbulent flow.

2.4 Measuring the Pressure Drop

The experiment will use straw drilled into the side of the pipe to measure the pressure drop along the pipe. The straws will act a manometers, due to manometers being unavailable.

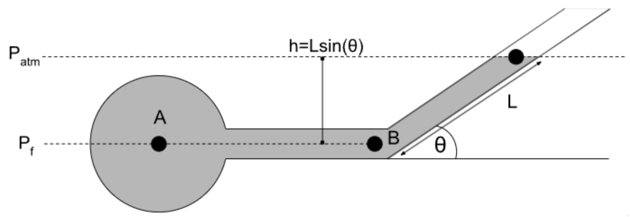

Figure 3: *An Inclined manometer*

Assuming the pressure at point A is P_f, and that it is pushing fluid up the manometer. Assume that P_f is greater than the atmospheric pressure P_{atm} at Point C, where the fluid is being pushed down. This difference in pressure causes a resultant force which accelerates the water up the manometer.

As the fluid rises and the column of fluid in the manometer increases, the pressure P_w due to the downward force of the weight of the column increases. This occurs until the water reaches a vertical height of h, where all 3 forces are balanced, and the water in the manometer is stationary. By measuring the length the water travels up the pipe, we can determine the pressure at that point, allowing us to calculate the pressure difference.

From Stevin's Law[13], the pressure on a fluid at a given point is dependent only on the depth of that point. Therefore, the pressure at point B is the same as other pressure at Point A. Therefore, the pressure P_w is shown to be:

$$P_w = \rho g h$$

This only depends on the vertical depth of h, which can be calculated by h = Lsin(θ), where L is the distance the water has travelled up the pipe.
Once the fluid reaches the height of h, the 3 forces are in equilibrium:

$$P_f \cancel{A_p} = P_w \cancel{A_p} + P_{atm} \cancel{A_p}$$

$$\Rightarrow P_f = pgL \sin\theta + P_{atm}$$

where A_p is the cross-sectional area of the manometer. The manometers will be inclined at an angle $\theta = 60°$

2.5 Hagen–Poiseuille correlation

The relationship between the Darcy friction factor and the Reynolds number for laminar flow ($Re < 2100$) is fundamental in fluid mechanics. It describes how energy losses due to friction depend on the flow properties. Therefore, to understand the relationship the Hagen–Poiseuille correlation will be considered.

Fluid motion is governed by the Navier-Stokes equations, which describe the conservation of momentum in a fluid. For steady, incompressible flow in a pipe, these equations reduce to:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \mu \nabla^2 \mathbf{v}.$$

Here, the first term represents unsteady effects, the second term represents convective acceleration, and the terms on the right-hand side represent pressure forces, viscous effects, and body forces. For steady, incompressible, and fully developed flow in a circular pipe, several simplifying assumptions are applied:

- The flow is steady, so $\frac{\partial \mathbf{v}}{\partial t} = 0$.
- The velocity depends only on the radial position ($v = v(r)$), and not on the axial position or time.
- The pressure gradient ($-\nabla p$) along the length of the pipe is constant.
- The flow is axisymmetric, so there is no tangential velocity.
- The continuity equation ($\nabla \cdot \mathbf{v} = 0$) holds, ensuring conservation of mass.

Applying these assumptions simplifies the Navier-Stokes equation to:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv}{dr}\right) = \frac{1}{\mu}\frac{dp}{dz},$$

where:

$$\begin{aligned}
v(r) &: \text{axial velocity as a function of radial position,} \\
\frac{dp}{dz} &: \text{pressure gradient along the pipe's length,} \\
\mu &: \text{dynamic viscosity of the fluid,} \\
r &: \text{radial distance from the pipe's centerline.}
\end{aligned}$$

This is a second-order ordinary differential equation for $v(r)$, which describes how the velocity varies across the pipe's cross-section. Solving this equation will yield the velocity profile, a key step in deriving the relationship between the friction factor and the Reynolds number[14].

To solve the equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv}{dr}\right) = \frac{1}{\mu}\frac{dp}{dz},$$

multiply through by r to simplify:

$$\frac{d}{dr}\left(r\frac{dv}{dr}\right) = \frac{r}{\mu}\frac{dp}{dz}.$$

Integrating both sides with respect to r gives:

$$r\frac{dv}{dr} = \frac{1}{\mu}\frac{dp}{dz} \cdot \frac{r^2}{2} + C_1,$$

where C_1 is an integration constant. Dividing through by r and integrating again results in:

$$v(r) = \frac{1}{4\mu}\frac{dp}{dz} \cdot r^2 + C_1 \ln(r) + C_2,$$

where C_2 is another integration constant.

To determine the constants C_1 and C_2, the boundary conditions for the flow are applied. First, the no-slip condition at the pipe wall states that the fluid velocity at the wall ($r = R$) is zero, i.e., $v(R) = 0$. Second, the velocity at the center of the pipe ($r = 0$) must remain finite, which eliminates the term containing $\ln(r)$ (i.e., $C_1 = 0$). Substituting the no-slip condition ($v(R) = 0$) into the velocity equation:

$$0 = \frac{1}{4\mu}\frac{dp}{dz} \cdot R^2 + C_2.$$

Solving for C_2:

$$C_2 = -\frac{1}{4\mu}\frac{dp}{dz} \cdot R^2.$$

The velocity profile becomes:

$$v(r) = \frac{1}{4\mu}\frac{dp}{dz} \cdot (r^2 - R^2).$$

This is the parabolic velocity profile for fully developed laminar flow in a circular pipe. It shows that the velocity is maximum at the centerline ($r = 0$) and decreases parabolically to zero at the pipe wall ($r = R$) due to viscous effects[15]. The maximum velocity occurs at the centerline ($r = 0$):

$$v_{\max} = -\frac{1}{4\mu}\frac{dp}{dz} \cdot R^2.$$

The average velocity (v_{avg}) is determined by integrating the velocity profile over the pipe's cross-sectional area. The average velocity is given by:

$$v_{\text{avg}} = \frac{1}{\pi R^2} \int_0^R 2\pi r v(r)\, dr.$$

Substituting $v(r) = \frac{1}{4\mu} \frac{dp}{dz} \cdot (r^2 - R^2)$:

$$v_{\text{avg}} = \frac{1}{\pi R^2} \int_0^R 2\pi r \left(\frac{1}{4\mu} \frac{dp}{dz} \cdot (r^2 - R^2) \right) dr.$$

Simplifying and evaluating the integral:

$$v_{\text{avg}} = -\frac{1}{8\mu} \frac{dp}{dz} \cdot R^2.$$

The pressure drop ΔP across a pipe of length L is related to the pressure gradient by:

$$\Delta P = -\frac{dp}{dz} \cdot L.$$

Substituting this into the expression for v_{avg}:

$$v_{\text{avg}} = \frac{\Delta P \cdot R^2}{8\mu L}.$$

The Darcy-Weisbach friction factor (f) relates the pressure drop to energy losses:

$$\Delta P = \frac{f L \rho v_{\text{avg}}^2}{2d}.$$

Substituting $d = 2R$ and solving for f:

$$f = \frac{64\mu}{\rho v_{\text{avg}} d}.$$

Using the definition of the Reynolds number:

$$Re = \frac{\rho v_{\text{avg}} d}{\mu},$$

the friction factor becomes:

$$f = \frac{64}{Re}.$$

This derivation demonstrates that the friction factor for laminar flow is inversely proportional to the Reynolds number.

2.6 Flow Velocity

The time taken for the fluid to travel over a known distance is measured to determine the flow velocity of a fluid moving through a pipe. The flow velocity is then calculated using the following equation:

$$v = \frac{d}{t},$$

where:

- v: flow velocity (m/s),
- d: distance traveled by the fluid (measured in meters, m),
- t: time over which the distance is measured (s).

3 Hypothesis

In laminar flow through a smooth pipe, the Darcy friction factor (f) decreases as the Reynolds number (Re) increases, following an inverse proportionality given by:

$$f = \frac{64}{Re}.$$

This equation, derived from the Hagen-Poiseuille law, predicts that doubling the Reynolds number will halve the friction factor.

This relationship arises because, in laminar flow, viscous forces dominate, and the shear stress at the pipe wall follows a predictable pattern. As Re increases, either due to increased velocity or decreased viscosity, the influence of viscous resistance per unit of inertial force decreases, leading to a reduction in f.

When plotted on a logarithmic scale, the friction factor should form a straight line with a slope of -1, confirming the inverse proportionality. Comparing experimental data with this theoretical trend will validate whether the expected relationship holds under real conditions.

4 Method

4.1 Preliminary Experiment

Conducting a preliminary experiment is essential to determine the suitable range of parameters for the independent variable. This involves analysing the dependent variable's values across its minimum and maximum range to ensure significant variation. This step ensures the experiment's validity, feasibility, and reliability. For a flow rate of 0.007143 ± 0.000008 m^3s^{-1}, the Reynolds number was 92.98 and the measured Darcy Friction Factor was 0.6 ± 0.2, and for a flow rate of 0.1563 ± 0.0006 m^3s^{-1}, the Reynolds number was 2033.97 and the measured Darcy Friction Factor was 0.032 ± 0.001. Therefore, the preliminary experiment showed sufficient variance to proceed. However, an issue with water leakage caused inconsistent flow rates. To address this, stronger waterproof tape was used to prevent leaks.

4.2 Variables

4.2.1 Independent Variable

Here, the independent variable is the flow rate measured in m^3s^{-1}. By changing the flow rate, the Reynolds number can be changed, allowing the Darcy friction factor to be calculated in laminar flow. For this study, the Reynolds number will range from 100 ± 10 to 2000 ± 10 with intervals of 100 for laminar flow, achieved using flow rates within the range of 0.007 ± 0.0005 m^3s^{-1} and 0.156 ± 0.0005 m^3s^{-1} with intervals of 0.01 m^3s^{-1}. The flow rate will be adjusted with the tap faucet.

4.2.2 Dependant Variable

The Darcy friction factor is the dependent variable. It is calculated by measuring the pressure drop, viscosity, flow velocity, pipe length, and diameter with the Darcy Weisbach equation.

4.2.3 Control Variables

The following variables are controlled to isolate the effect of changing the type of flow via the Reynolds number on the Darcy Friction Factor:

- **Pipe Dimensions**
  - A change in diameter directly affects the velocity of the fluid (as velocity is inversely proportional to the cross-sectional area), which in turn influences the Reynolds number and the pressure drop. Similarly, the pipe length impacts the pressure drop, which is central to calculating the Darcy Friction Factor. Consistent pipe dimensions ensure the results are comparable across trials.
  - **Control method:** The diameter is measured with a vernier calliper to minimize error and ensure precision. The same physical pipe is used throughout all trials to maintain a consistent pipe length.
  - The pipe diameter is kept at $0.018 \pm 0.0005\,\mathrm{m}$, and the length remains constant at $0.5 \pm 0.0005\,\mathrm{m}$
- **Fluid Properties**
  - Any variation in fluid properties will alter the Reynolds number, making it impossible to isolate the effect of flow type on the Darcy Friction Factor. Keeping fluid properties constant ensures that only changes in the flow regime are investigated.
  - **Control method:** Before each trial, the density of the water is calculated by measuring the mass and volume. Similarly, the dynamic viscosity is measured using a viscometer to detect and account for any variations caused by temperature changes.
  - The dynamic viscosity remained at $0.00138 \pm 0.000005\,\mathrm{Pa \cdot s}$, and the density remained at $998 \pm 0.06\,\mathrm{kg\,m^{-3}}$
- **Atmospheric Pressure**

- If atmospheric pressure changes during the experiment, it could alter the fluid's behavior and the observed friction factor. Consistent atmospheric pressure ensures that the calculated Darcy Friction Factor depends only on flow type and not on variations in external conditions.
- **Control method:** All experiments are conducted on the same day under similar environmental conditions to minimise the impact of atmospheric pressure variations. This avoids discrepancies caused by multi-day trials or changes in weather.

4.2.4 Equipment

- Drill
- Straws
- Water tap
- Smooth pipe of known length and diameter
- Water source
- Measuring cylinder (increments of 1 mL)
- Vernier caliper (increments of 0.01 mm)
- Ruler (increments of 1 mm)
- Viscometer (increments of $0.001\,\mathrm{Pa}\cdot\mathrm{s}$)
- Mass balance (increments of 0.01 g)
- Tape

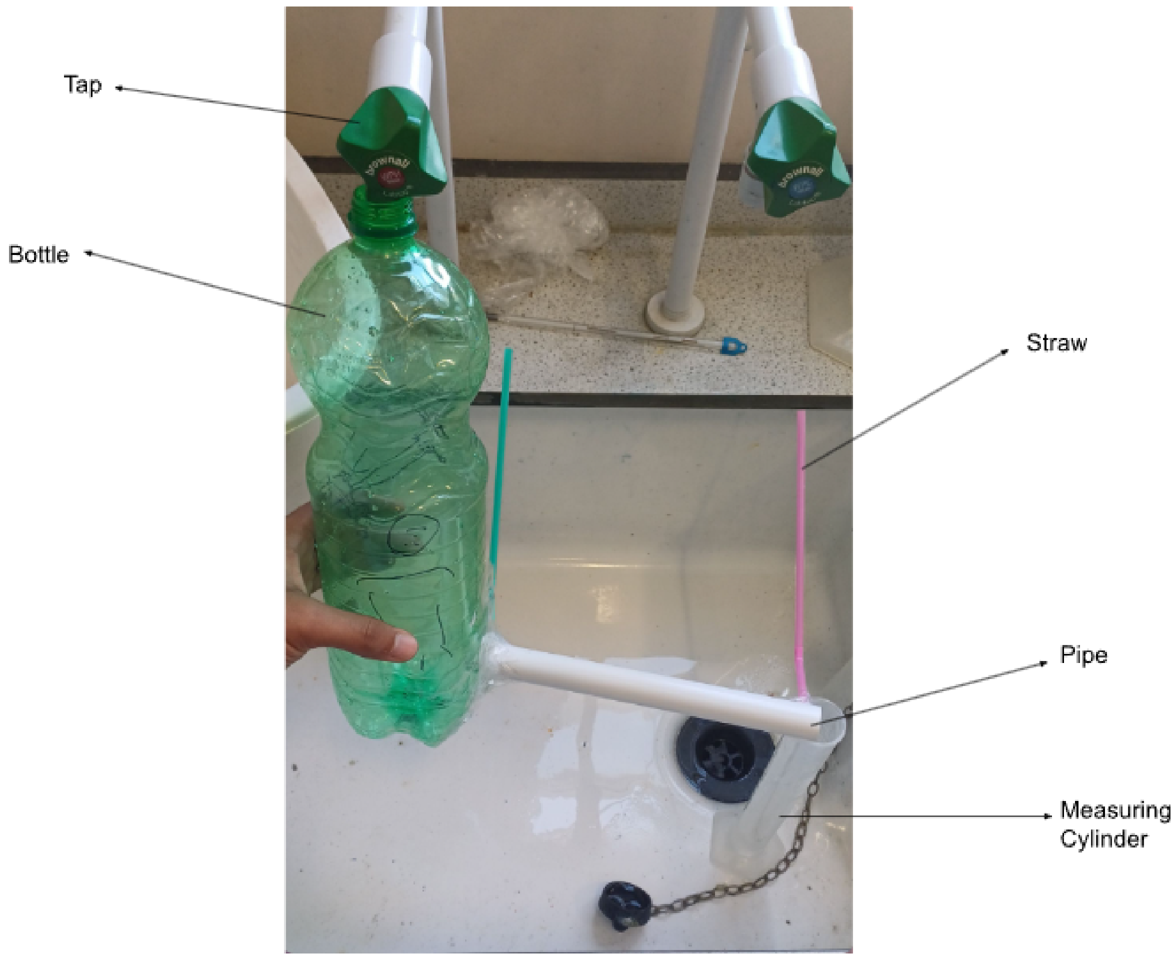

Figure 4: *Experiment set up*

4.3 Experimental procedure

1. Measure the diameter of the pipe using a vernier calliper.

2. Measure the length of the pipe.

3. Measure the mass and volume of water to calculate the density of water.

4. Measure the dynamic viscosity of water using a viscometer.

5. Set up apparatus as shown in Figure 4.

6. Turn on the tap and let water flow through the pipe.

7. Calculate flow velocity by measuring the time the water takes to come out of the pipe.

8. Calculate Reynolds Number and note it

9. Measure the distance travelled up by the water in the pipe and use the Equation 29 to calculate the pressure at one end of the pipe

10. Calculate the head loss

11. Calculate Darcy friction factor with Darcy Weisbach equation

12. Iterate steps 5-8 with flow rates within the range of $0.007 \pm 0.0005\ m^3s^{-1}$ and $0.156 \pm 0.0005\ m^3s^{-1}$ with intervals of $0.01\ m^3s^{-1}$ and Reynolds Number within the range of 100 ± 10 and 2000 ± 10 with intervals of 100.

13. Record all values and Plot a graph of Darcy friction factor and Reynolds Number

4.4 Risk Assessment

Environmental Considerations: Wastewater from the experiment is collected and reused to minimise waste. This reduces water consumption and limits environmental impact.

4.4.1 Risks

- Risk of water spillage: Ensure proper drainage systems are in place.
- Risk of electric shock: Use waterproof and insulated equipment for electrical connections.
- Risk of injury due to drill: Have to supervise adult help using the drill

4.4.2 Safety Equipment

- Safety goggles to protect eyes from splashes and drilling.
- Rubber gloves to handle the equipment.
- Lab coat to protect clothing.

5 Data and Results

5.1 Raw Data

Diameter of pipe ($\pm 5.00 \times 10^{-4}$ **m**)	**Mass of water** ($\pm 1.00 \times 10^{-5}$ **kg**)	**Volume of water** ($\pm 5.00 \times 10^{-8}$ **m**^{3})	**Dynamic Viscosity** ($\pm 5.00 \times 10^{-4}$ **Pa s**)	**Length of pipe** ($\pm 5.00 \times 10^{-4}$ **m**)
0.018	0.998	0.001	0.0014	0.5

Table 1: *Fixed Measurements*

Time taken for water to flow through pipe ($\pm 1.00 \times 10^{-2}$ **s**)	**Pressure Difference** ($\pm 1.00 \times 10^{-5}$ **Pa**)
70	0.44851
32	1.07562
21	1.68683
16	2.15808
13	2.60306
11	3.06835
9	3.93801
8	4.34297
7	5.01617
6	6.19130
5.5	6.67510
5.2	6.85494
5	6.84462
4.5	7.85805
4.2	8.39407
4	8.63504
3.7	9.56305
3.5	10.07335
3.4	10.10806
3.2	10.81207

Table 2: *Variable Measurements*

5.2 Example Calculations

Density of Water (ρ)

$$\rho = \frac{\text{Mass of water}}{\text{Volume of water}}$$

$$e.g\ \rho = \frac{0.998 \text{ kg}}{0.001 \text{ m}^3} = 998 \text{ kg/m}^3$$

Uncertainty in Density ($\Delta\rho$)

$$\Delta\rho = 0.06 \text{ kg/m}^3$$

Flow Speed (v)

$$v = \frac{L}{t}$$

$$e.g\ v = \frac{0.5 \text{ m}}{70 \text{ s}} = 0.007143 \text{ m/s}$$

Uncertainty in Flow Speed (Δv)

$$\frac{\Delta v}{v} = \left(\frac{\Delta L}{L}\right) + \left(\frac{\Delta t}{t}\right)$$

$$e.g\ \frac{\Delta v}{0.007143 \text{ m/s}} = \left(\frac{0.0005 \text{ m}}{0.5 \text{ m}}\right) + \left(\frac{0.01 \text{ s}}{70 \text{ s}}\right)$$

$$\Delta v = 0.007143 \times 0.001 = 0.000008 \text{ m/s}$$

Reynolds Number (Re)

$$Re = \frac{\rho v D}{\mu}$$

$$e.g\ Re = \frac{998 \text{ kg/m}^3 \times 0.007143 \text{ m/s} \times 0.018 \text{ m}}{0.00138 \text{ Pa} \cdot \text{s}} = 92.98$$

Uncertainty in Reynolds Number (ΔRe)

$$\frac{\Delta Re}{Re} = \left(\frac{\Delta\rho}{\rho}\right) + \left(\frac{\Delta v}{v}\right) + \left(\frac{\Delta D}{D}\right) + \left(\frac{\Delta\mu}{\mu}\right)$$

$$e.g\ \frac{\Delta Re}{92.98} = \left(\frac{0.6}{998}\right) + \left(\frac{0.000008}{0.007143}\right) + \left(\frac{0.0005}{0.018}\right) + \left(\frac{0.0000005}{0.00138}\right)$$

$$\Delta Re = 92.98 \times 0.0006 = 0.06$$

Darcy Friction Factor (f)

$$f = \frac{2\Delta PD}{\rho v^2 L}$$

$$e.g\ f = \frac{2 \times 0.44851 \text{ Pa} \times 0.018 \text{ m}}{998 \text{ kg/m}^3 \times (0.007143 \text{ m/s})^2 \times 0.5 \text{ m}} = 0.6$$

Uncertainty in Darcy Friction Factor (Δf)

$$\frac{\Delta f}{f} = \left(\frac{\Delta P}{\Delta P}\right) + \left(\frac{\Delta D}{D}\right) + \left(\frac{\Delta \rho}{\rho}\right) + \left(\frac{2\Delta v}{v}\right) + \left(\frac{\Delta L}{L}\right)$$

$$e.g\ \frac{\Delta f}{0.6} = \left(\frac{0.00001}{0.44851}\right) + \left(\frac{0.0005}{0.018}\right) + \left(\frac{0.6}{998}\right) + \left(\frac{2 \times 0.000008}{0.007143}\right) + \left(\frac{0.0005}{0.5}\right)$$

$$\Delta f = 0.6 \times 0.34 = 0.2$$

5.3 Final Processed Data

Density of water ($\pm 0.06 kg/m^3$)	Average flow speed (m/s)	Uncertainty in flow speed (m/s)	Re	Uncertainty in Re	Friction Factor	Uncertainty in Friction Factor	Theoretical Re	Theoretical Friction Factor(F)
998	0.007143	0.000008	92.98	0.06	0.6	0.2	100	0.64
998	0.01563	0.00002	203.40	0.06	0.32	0.04	200	0.32
998	0.02381	0.00004	309.94	0.06	0.21	0.02	300	0.21
998	0.03125	0.00005	406.79	0.06	0.16	0.01	400	0.16
998	0.03846	0.00007	500.67	0.06	0.127	0.009	500	0.13
998	0.04545	0.00009	591.70	0.06	0.107	0.007	600	0.11
998	0.0556	0.0001	723.19	0.06	0.092	0.005	700	0.09
998	0.0625	0.0001	813.59	0.06	0.080	0.004	800	0.08
998	0.0714	0.0002	929.81	0.06	0.071	0.004	900	0.07
998	0.0833	0.0002	1084.78	0.06	0.064	0.003	1000	0.06
998	0.0909	0.0003	1183.40	0.06	0.058	0.003	1100	0.06
998	0.0962	0.0003	1251.67	0.06	0.053	0.002	1200	0.05
998	0.1000	0.0003	1301.74	0.06	0.049	0.002	1300	0.05
998	0.1111	0.0004	1446.38	0.06	0.046	0.002	1400	0.05
998	0.1190	0.0004	1549.69	0.06	0.043	0.002	1500	0.04
998	0.1250	0.0004	1627.17	0.06	0.040	0.002	1600	0.04
998	0.1351	0.0005	1759.11	0.06	0.038	0.002	1700	0.04
998	0.1429	0.0006	1859.63	0.06	0.036	0.002	1800	0.04
998	0.1471	0.0006	1914.32	0.06	0.034	0.001	1900	0.03
998	0.1563	0.0006	2033.97	0.06	0.032	0.001	2000	0.03

Table 3: *Processed Data*

Ln(Re)	Ln(Uncertainty in Re)	Ln(Friction Factor)	Ln(Uncertainty in Friction)	Ln(Theoretica Re)	Ln(Theoretical Friction Factor)
4.5324	0.0006	-0.5	0.3	4.6052	-0.4
5.3152	0.0003	-1.1	0.1	5.2983	-1.1
5.7364	0.0002	-1.54	0.09	5.7038	-1.54
6.0083	0.0001	-1.84	0.08	5.9915	-1.83
6.2159	0.0001	-2.06	0.07	6.2146	-2.06
6.3830	0.0001	-2.23	0.06	6.3969	-2.24
6.58367	0.00008	-2.39	0.06	6.55108	-2.39
6.70145	0.00007	-2.52	0.05	6.68461	-2.53
6.83498	0.00007	-2.65	0.05	6.80239	-2.64
6.98913	0.00006	-2.74	0.05	6.90776	-2.75
7.07615	0.00005	-2.84	0.05	7.00307	-2.84
7.13224	0.00005	-2.93	0.05	7.09008	-2.93
7.17146	0.00005	-3.01	0.05	7.17012	-3.01
7.27682	0.00004	-3.08	0.04	7.24423	-3.09
7.34581	0.00004	-3.15	0.04	7.31322	-3.15
7.39460	0.00004	-3.22	0.04	7.37776	-3.22
7.47256	0.00003	-3.28	0.04	7.43838	-3.28
7.52813	0.00003	-3.34	0.04	7.49554	-3.34
7.55712	0.00003	-3.39	0.04	7.54961	-3.39
7.61774	0.00003	-3.44	0.04	7.60090	-3.44

Table 4: *Logarithmic processed data*

5.4 Graphs

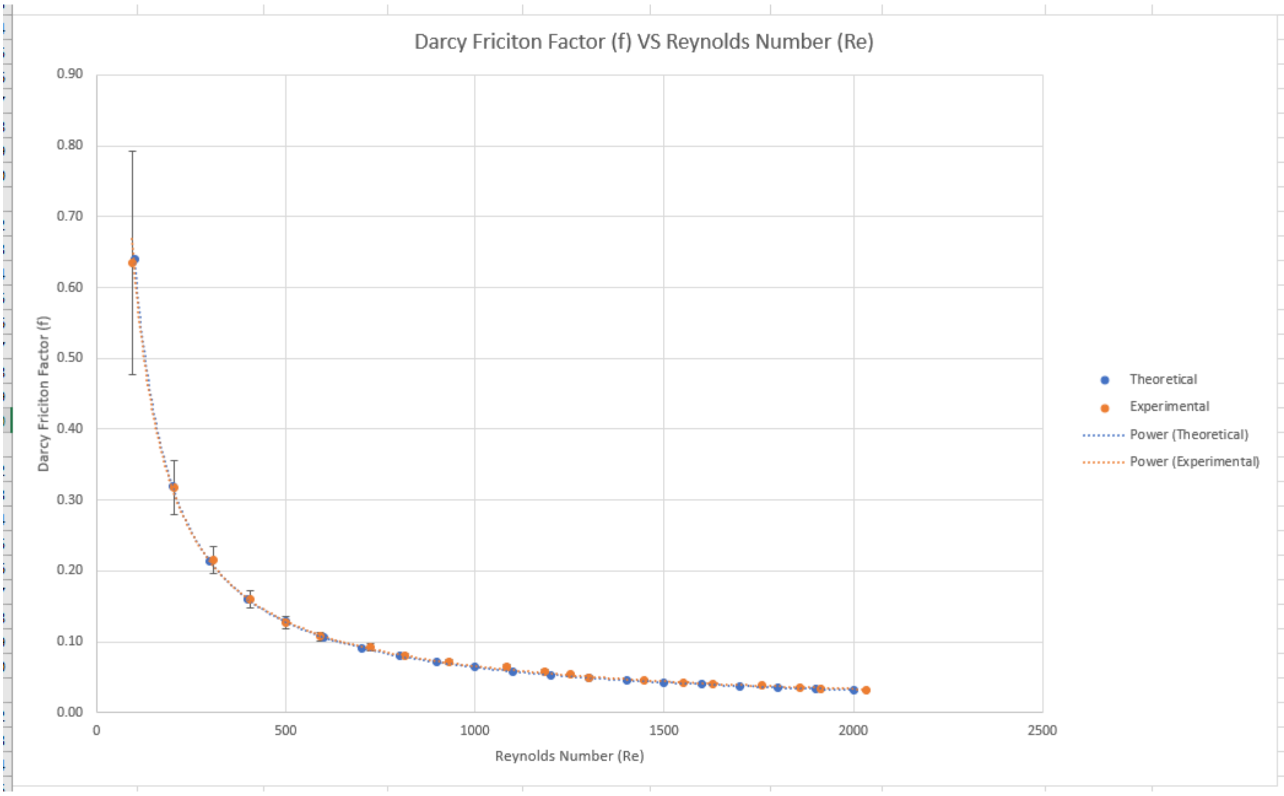

Figure 5: *Darcy Friction Factor VS Reynolds Number*

The first graph illustrates the inverse relationship between the Darcy friction factor and the Reynolds number in laminar flow. As expected from the theoretical relationship $f = \frac{64}{Re}$, the friction factor decreases as the Reynolds number increases. The curve in the graph visually confirms this inverse proportionality, aligning with the Hagen-Poiseuille equation.

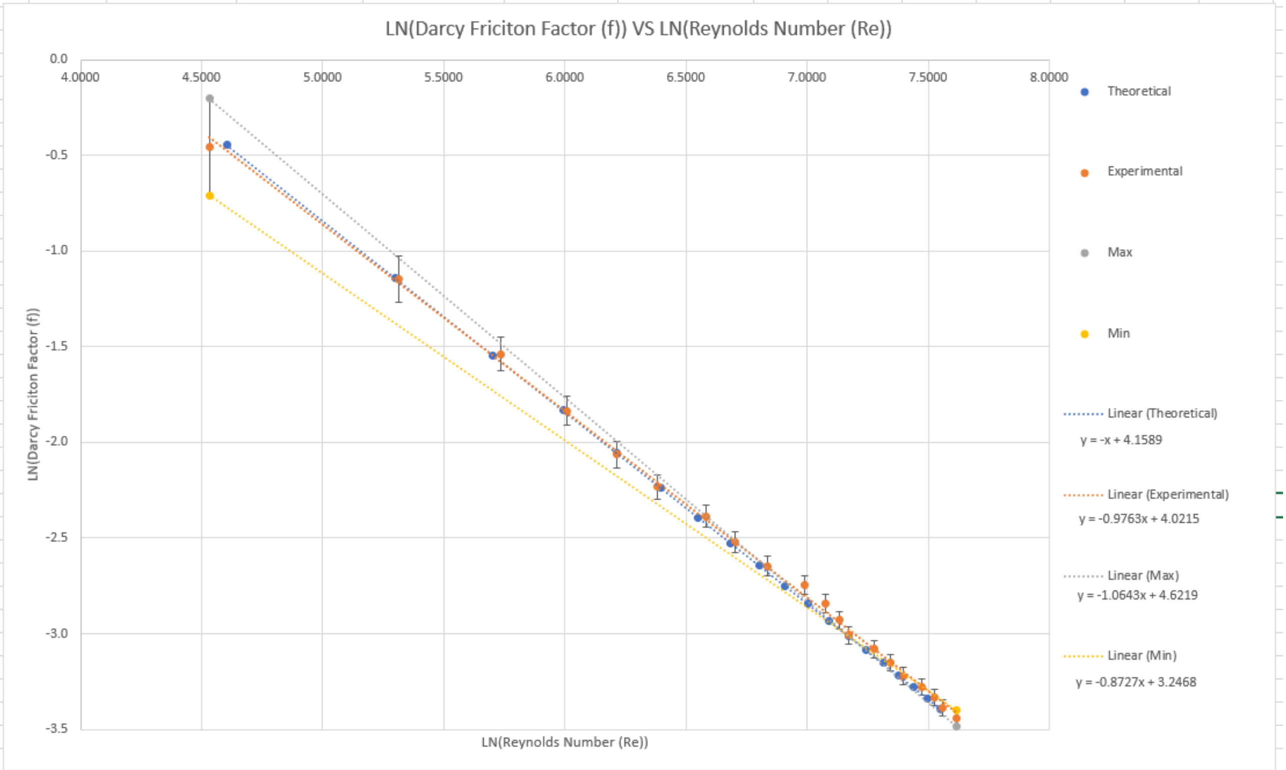

Figure 6: *LN(Darcy Friction Factor) VS LN(Reynolds Number)*

The second graph presents a logarithmic transformation of the data, plotting $\ln(f)$ against $\ln(Re)$. This transformation linearizes the relationship, allowing for a direct comparison of the experimental and theoretical gradients. Since the expected relationship is $f \propto Re^{-1}$, the theoretical gradient should be -1. The experimental gradient, determined from the linear fit, is found to be -0.9763, which is close to the theoretical expectation.

To quantify the uncertainty in the gradient, the maximum and minimum gradient lines were calculated as follows:

$$y = -1.0643x + 4.6219, \quad y = -0.8727x + 3.2468.$$

The error in the gradient is determined using the formula:

$$\text{Error} = \frac{\max - \min}{2}$$

$$= \frac{1.0643 - 0.8727}{2} = \frac{0.1916}{2} = 0.0958.$$

Thus, the experimental gradient is -0.9763 ± 0.0958. Since this range includes the theoretical value of -1, the results strongly agree with the expected inverse relationship.

6 Conclusion

The experiment aimed to determine the Darcy friction factor and Reynolds number in laminar flow, comparing experimental data with theoretical predictions. The results, as depicted in the graph, show a strong similarity between the experimental values and the theoretical curve across the range of Reynolds numbers tested (92.98 to 2033.97). For example, at a Reynolds number of 100, the experimental friction factor was 0.6, closely matching the theoretical value of 0.64. Similarly, at a Reynolds number of 1000, the experimental friction factor was 0.064 compared to the theoretical value of 0.06. The experimental data successfully depict this reciprocal relationship, as hypothesised. The relationship is also consistent with the Hagen-Poiseuille correlation, demonstrating that the friction factor is inversely proportional to the Reynolds number.

In addition, by examining the results on a logarithmic scale, we can plot a straight line of best fit and compare gradients to further validate our results. The theoretical results had a gradient of -1, while the experimental data had a gradient of -0.9673. These values are extremely close, further emphasizing the strong similarity between the experimental and theoretical data and providing additional support for our hypothesis.

The close alignment of these data points validates the experimental methodology and confirms the theoretical understanding of the friction factor in laminar flow. The error bars indicate very low uncertainty in the measurements, demonstrating high precision and reliability in the experimental setup. The slight deviations observed at lower Reynolds numbers are minimal and within acceptable limits, possibly due to minor measurement inaccuracies or slight variations in flow conditions.

7 Evaluation

Strengths	**Justification**
High reproducibility	The overall method and apparatus setup is simple, accessible and feasible, making an effective experiment that can be reproduced.
Low uncertainty	Error bars on the graph indicate low uncertainty, reflecting high precision and reliability in the measurements.
Alignment with theoretical predictions	Results align well with theoretical predictions and established science, validating the experimental methodology and measurements.
Effective variable control	Variables such as atmospheric pressure, pipe dimensions, and fluid properties were controlled effectively, ensuring focus on the Reynolds Number's effect on the friction factor.
Quantitative nature of results	The experiment provided clear, quantitative data that allowed for easy mathematical manipulation, such as plotting the relationship between the Darcy friction factor and Reynolds number. This strengthens the evidence of theoretical alignment.

Table 5: *Strengths of the Experiment*

Source of Error	Impact of Error	Method to Improve
Manual measurement of pipe dimensions and fluid properties	Introduces human error and variability, leading to less precise and reproducible results.	Use automated measuring tools such as laser calipers for pipe dimensions and digital sensors for fluid properties.
Flow rate inconsistency	Inconsistent flow rate affects the Reynolds number, leading to variability in the results.	Use a digital pump to maintain stable and reproducible flow rates.
Surface roughness of the pipe	Surface roughness, though minor in laminar flow, could introduce inaccuracies in the results.	Sand the pipe surface to reduce roughness or calculate the Darcy Friction Factor with roughness taken into account.
Pressure measurement precision	Manual or low-resolution pressure measurements can reduce reliability and precision.	Use a digital manometer to improve the accuracy and precision of pressure measurements.

Table 6: *Weaknesses of the Experiment*

8 Extension

To extend the knowledge of the Darcy friction factor in laminar flow, another experiment that could be performed would involve investigating the impact of various pipe materials and surface roughness on the friction factor. This experiment would include measuring the friction factor in laminar flow conditions using pipes made from different materials such as PVC, copper, steel, and glass and pipes with varying degrees of surface roughness. Through the results, a relationship between the material and surface roughness of pipes and the friction factor in laminar flow could be explored, providing valuable insights for optimizing fluid transport systems in engineering and industrial applications.

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