1) Dynamic Analysis of an Internal Combustion Engine

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Term Project

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Problem Statement

Dynamic analysis of an IC engine:

- **Model** and **analyse** a single cylinder IC engine considering practical values of mass of piston and links, cylinder pressure variation (to the extent possible) etc.

- Check the **ground shaking forces** and performance **without balancing**, **with balancing** and flywheel.

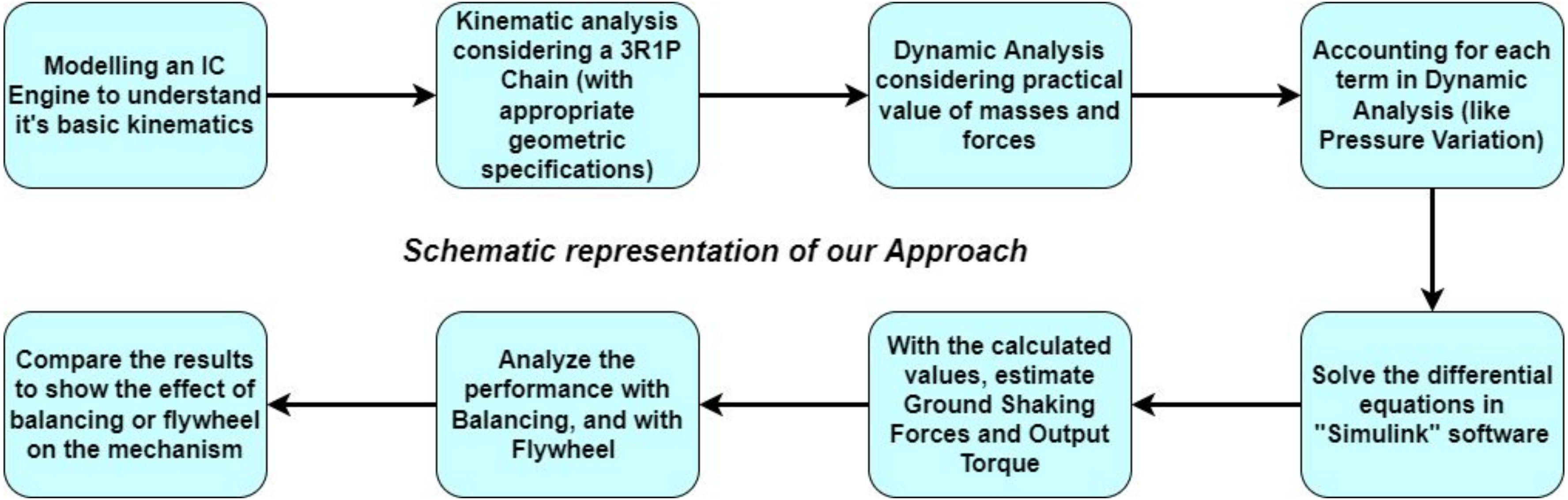
- Plot, compare and discuss all results.

Importance of Problem

- **Analyze 3R1P Mechanism**: **Real life applications** like IC Engine, Punching press, Compressors, etc.
- **Model an IC Engine**: Create a working model of an IC Engine with **technical specifications** of a real engine and considering **highly accurate Pressure variation** (of Otto Cycle)
- **Estimating Ground Shaking Forces**: These need to be accounted for to **avoid unnecessary vibrations**, (which is essential for designing and stabilizing an engine).
- **Engine Balancing**: Helps in **cancellation of vibration** inducing forces (thus, increasing performance)
- **Flywheel**: Effect of flywheel on **stabilizing engine's speed** and increasing its performance.
- **Comparing** all results for concluding how introducing various parts in a machine can provide more optimal results.

Overview of Approach

This is our approach for solving this problem:



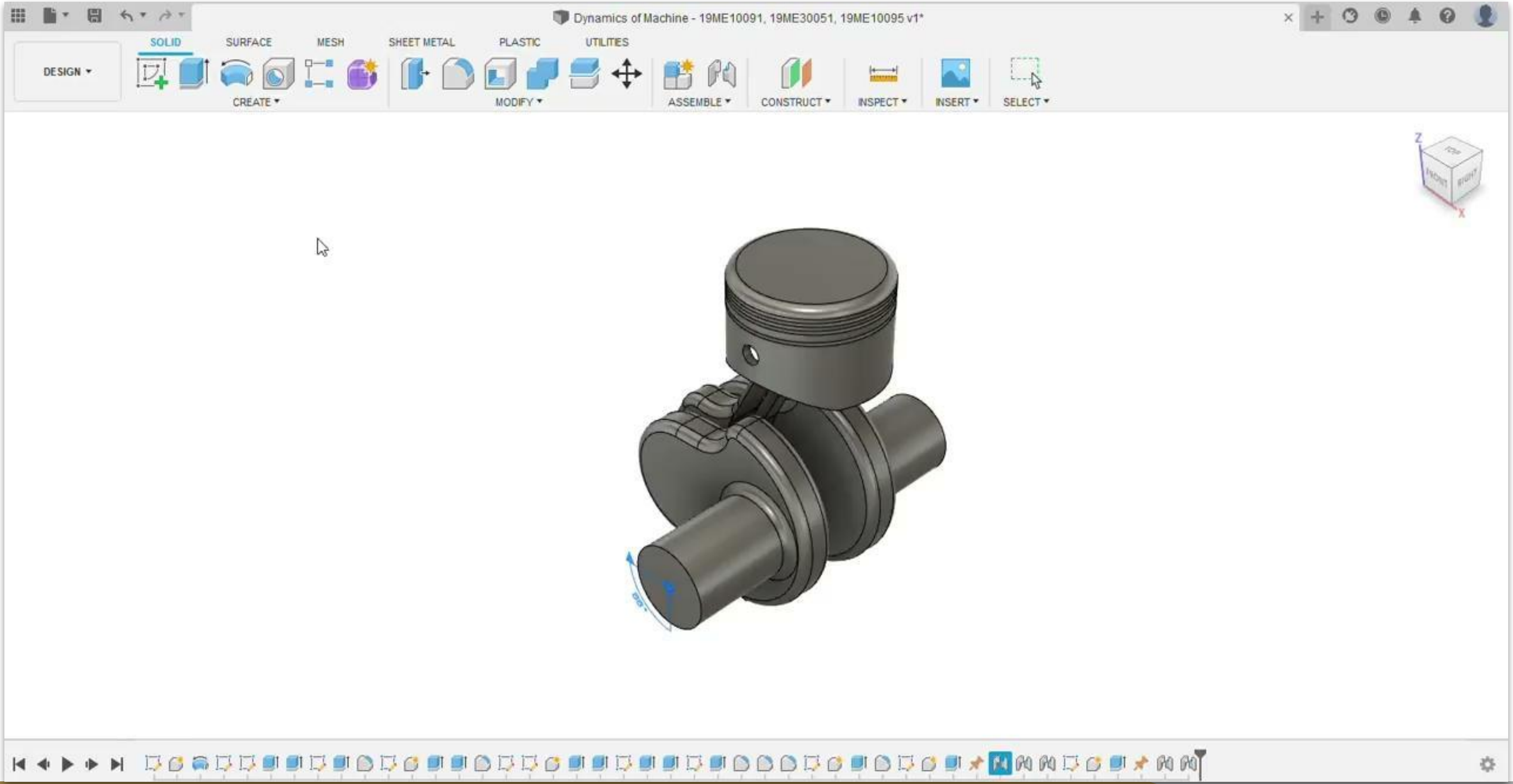
Schematic representation of our Approach

Technical Specifications

These are the technical and geometric specifications of the IC Engine we have used:

Connecting Rod Length (l)	120mm	**Total Volume**	45.8 cc
Crank Radius (r)	20mm	**Compression Ratio**	9 : 1
Stroke Length	40mm	**Viscous Load coefficient**	0.1 N.m.sec/rad
Piston Bore Diameter (b)	36mm	**Viscous Friction at piston head**	10 N.sec/m
Maximum Pressure	500 kPa	**Clearance Length**	5 mm
Minimum Pressure	5 kPa	**Mass of Piston**	1 kg
Mass of Coupler	(0.8 + 0.2) kg	**Moment of Inertia of Crank**	0.01 kgm^2

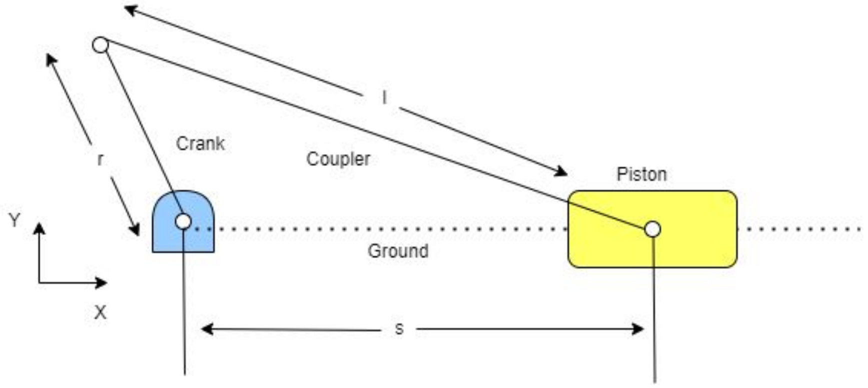
Model Simulation in Fusion 360



Kinematic Analysis

- A typical Internal Combustion Engine is modelled as a **3R-1P mechanism**.

- Consider
  - θ = Angle made by crank with ground
  - Φ = Angle made by coupler with ground
  - λ = r / l **(Here λ = 20/120 = 0.167)**



- The relation between the piston displacement s, θ and Φ is as follows

$$s = rcos\theta + lcos\phi$$

- Also, we have a relation between θ and Φ as $rsin\theta = lsin\phi$

- So, s in terms of θ is $s = rcos\theta + l\sqrt{1 - \lambda^2 sin^2\theta}$

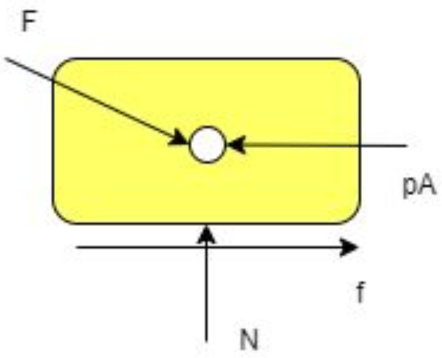
Dynamic Analysis

- Drawing the FBD of the Piston and balancing forces and moments, we get the following equations.

$$m_4\ddot{s} = Fcos\phi + f - pA$$

$$0 = -Fsin\phi + N$$

$$I_{O_2}\ddot{\theta} = Frsin(\theta + \phi) + \tau$$



where,
m_4 is the mass of the piston
f is the friction force
p is the pressure variation
A is the area of cross section of piston
F is the Force due to link on the piston
N is the normal reaction on the piston
τ is considered to be load due viscous force

Dynamic Analysis

- Simplifying the equations, we get the following equation:

$$I_{O_2}\ddot{\theta} = \begin{cases} \frac{m_4\ddot{s}-f+pA}{\sqrt{1-\lambda^2 sin^2\theta}}\left(sin\theta\sqrt{1-\lambda^2 sin^2\theta}+\lambda sin\theta cos\theta\right)r+\tau & \text{if } \dot{s}<0 \\ \frac{m_4\ddot{s}+f-pA}{\sqrt{1-\lambda^2 sin^2\theta}}\left(sin\theta\sqrt{1-\lambda^2 sin^2\theta}+\lambda sin\theta cos\theta\right)r+\tau & \text{if } \dot{s}\geq 0 \end{cases}$$

- For simplicity, we consider the **load torque τ as viscous load**. The expression is given below.

$$\tau = c_d\dot{\theta}$$

where cd is the damping factor.

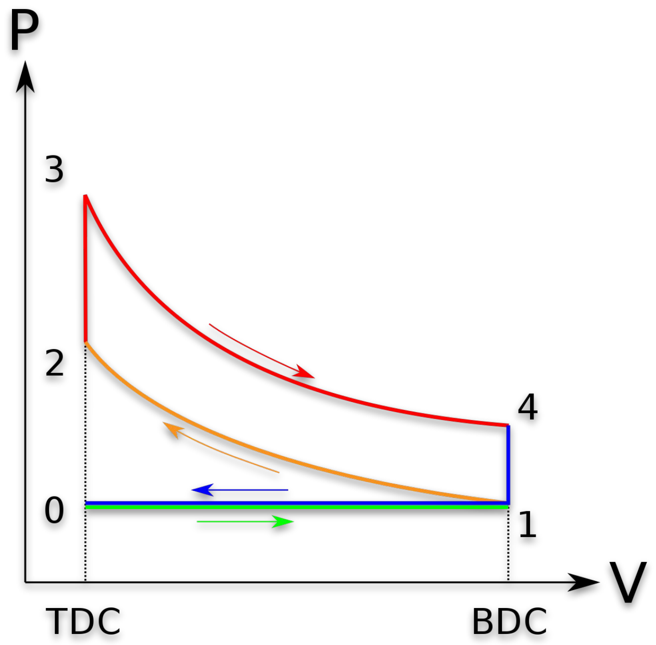
Pressure Variation

- For the dynamic analysis, we consider the standard **Otto cycle** (generally used in **Petrol engines**) to model the pressure-volume relation.

- The four stroke Otto Cycle is made up of the following four internally reversible processes:

  - **1-2 : Isentropic Compression**
  - **2-3 : Constant Volume Heat Addition**
  - **3-4 : Isentropic Expansion**
  - **4-1 : Constant Volume Heat Rejection**

- The pressure vs volume graph of Otto Cycle is shown



Pressure Variation

- The relation between volume and displacement is ***V = (L - s + c) A***, where **c is the clearance** and **L is the length of the cylinder.**

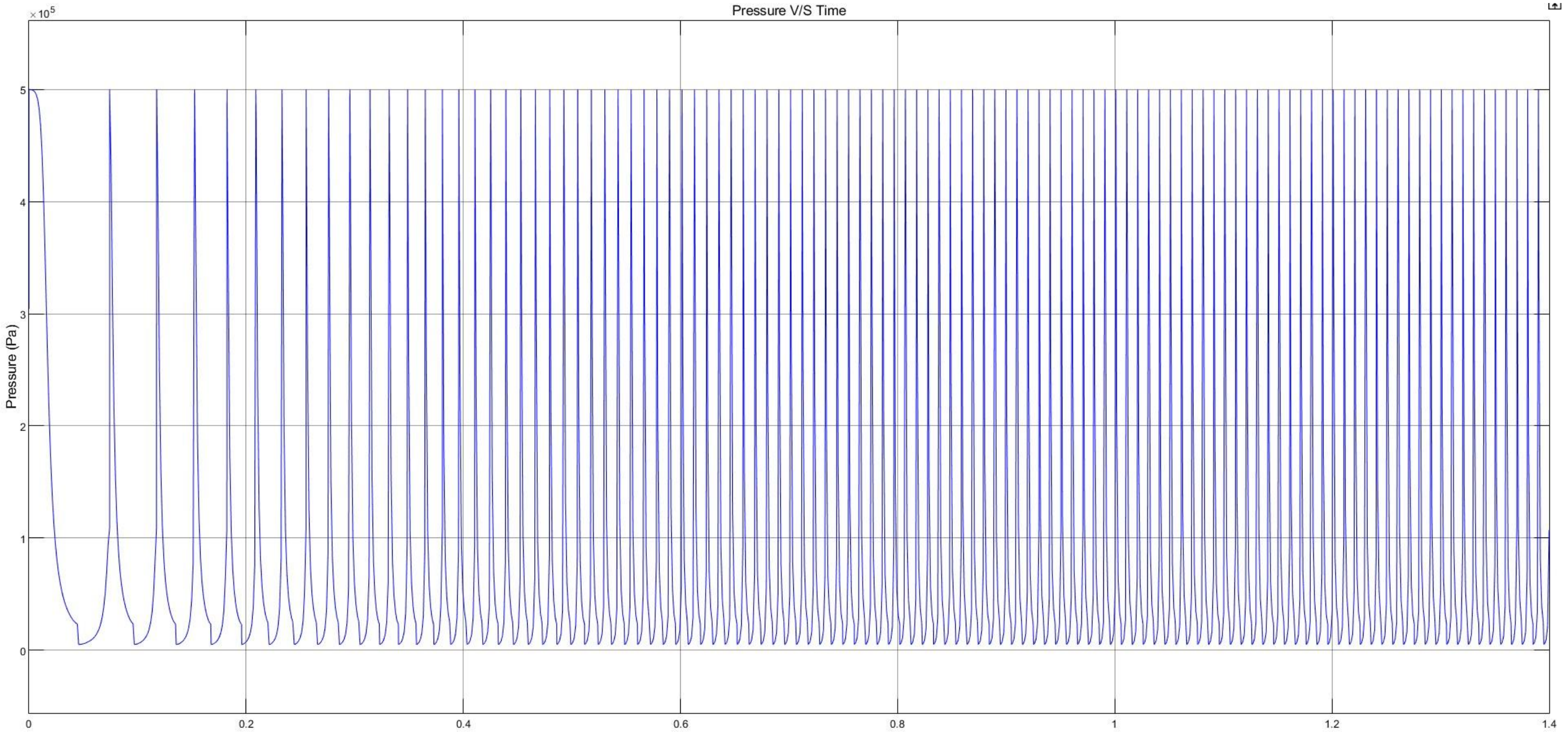
- There are **two adiabatic processes**, the pressure-volume relations of which are mentioned below.

$$p = \begin{cases} \frac{p_{min}[(L+c)A]^\gamma}{[(L-s+c)A]^\gamma} & \text{if } 0 < \theta < 2n\pi \ \text{(Compression)} \\ \frac{p_{max}(cA)^\gamma}{[(L-s+c)A]^\gamma} & \text{if } 2n\pi < \theta < 4n\pi \ \text{(Power Stroke)} \end{cases}$$

- We choose Pmax and Pmin to be 500 kPa and 5 kPa. (γ is the adiabatic constant (or specific heat ratio).

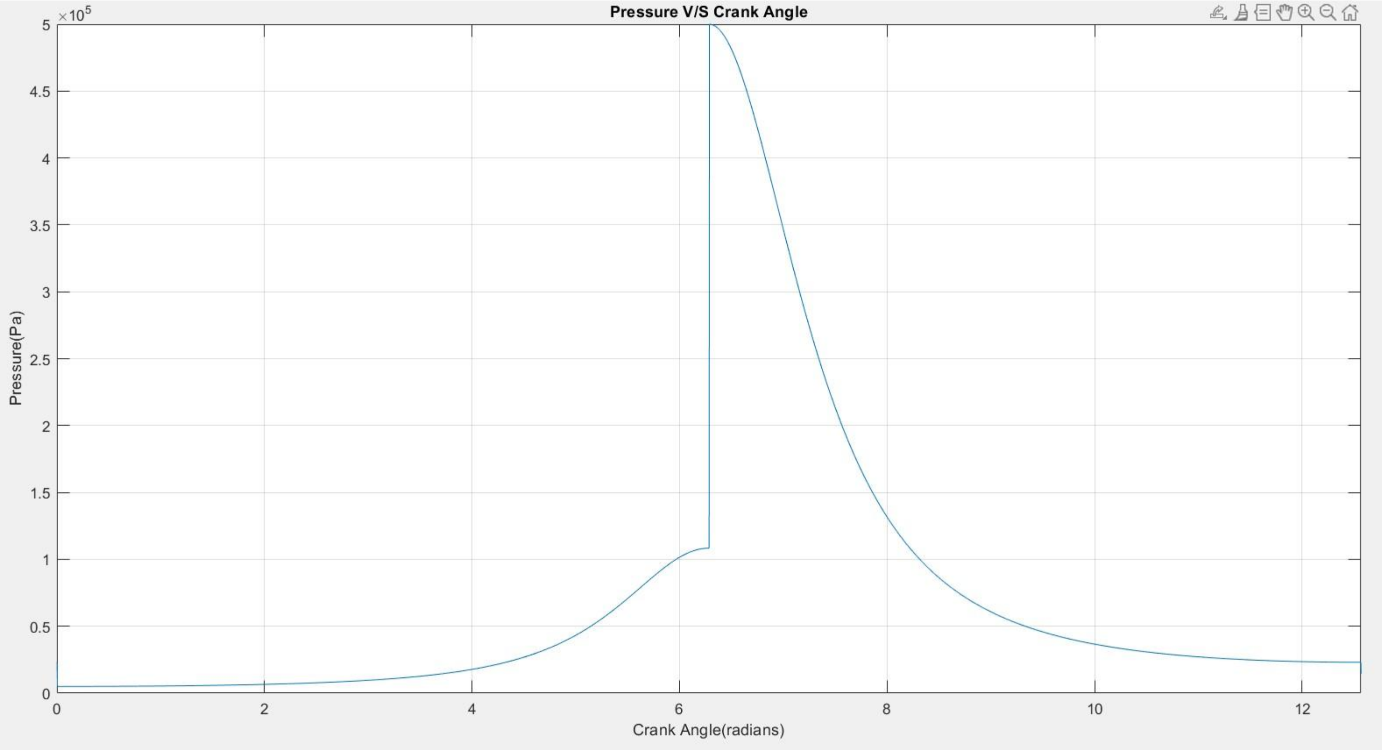
Pressure Variation

- The graph of In-cylinder Pressure and time is shown:



Pressure Variation

- The graph of In-cylinder pressure vs crank angle is shown below:



Solving the Differential Equation

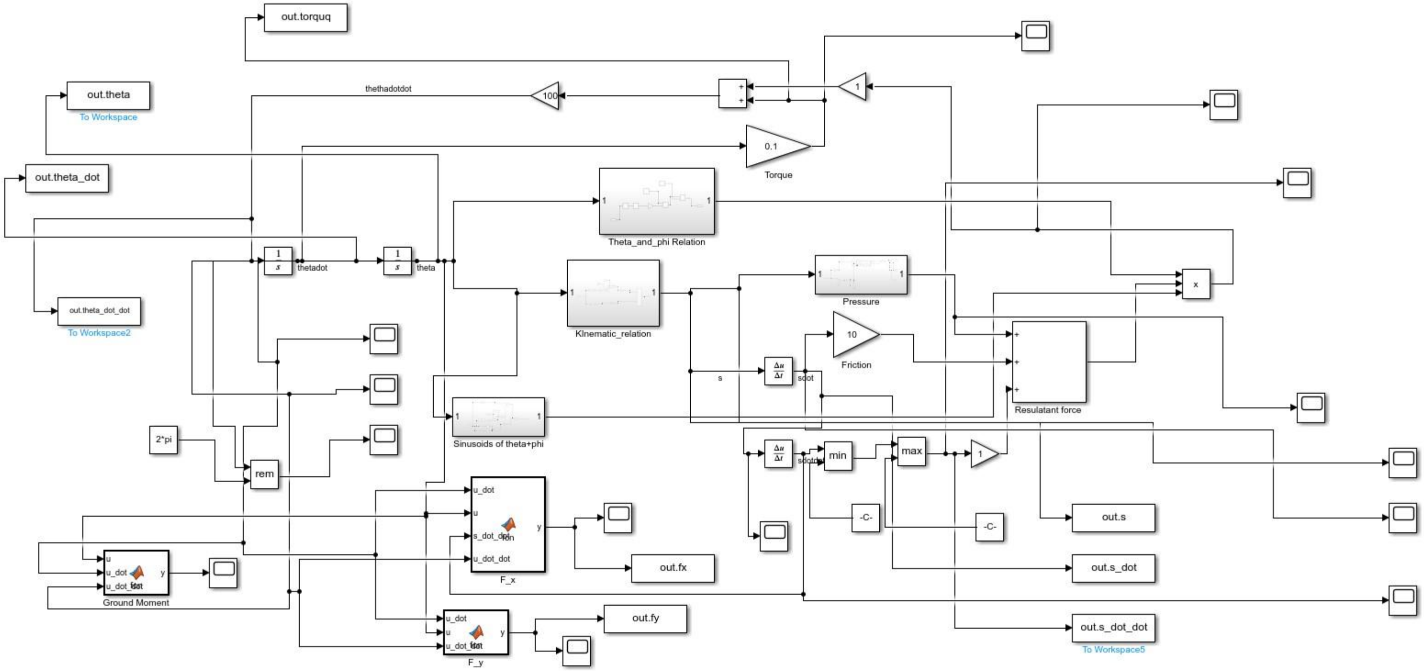
- We see that in order to get the values of s and θ, we have to **solve differential equations** like:

$$I_{O_2}\ddot{\theta} = \begin{cases} \frac{m_4\ddot{s}-f+pA}{\sqrt{1-\lambda^2 sin^2\theta}}\left(sin\theta\sqrt{1-\lambda^2 sin^2\theta}+\lambda sin\theta cos\theta\right)r+\tau & \text{if } \dot{s}<0 \\ \frac{m_4\ddot{s}+f-pA}{\sqrt{1-\lambda^2 sin^2\theta}}\left(sin\theta\sqrt{1-\lambda^2 sin^2\theta}+\lambda sin\theta cos\theta\right)r+\tau & \text{if } \dot{s}\geq 0 \end{cases}$$

- In our project, we have used a software called "**Simulink**" (**offered by MATLAB**) to solve these equations.
- Simulink automatically solves the equations and gives the variations of the dependent variables with respect to the independent variable.
- The entire Simulink code can be found in the repository link given at the end.

Solving the Differential Equation

The **Computation Graph** made using **Simulink** is shown below.



Ground Shaking Forces

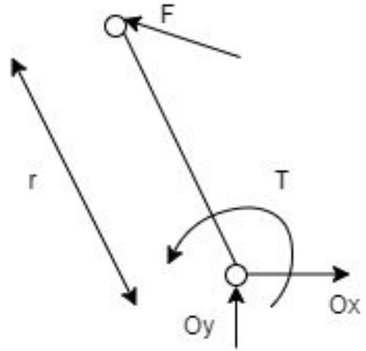
- Calculating ground shaking forces is a major part of dynamic analysis and estimating the **stability of the engine**.

- By **apportionation of mass of coupler**,

$$(m_2\rho_2 + m_A r)cos\theta + (m_A + m_B)\ddot{s} = O_x + f - pA$$

$$(m_2\rho_2 + m_A r)sin\theta = O_y + N$$

$$(I_{O_2} + m_A r^2)\ddot{\theta} + I'_{G_3}\ddot{\phi} = \tau + sN + r_p f$$



$$F_x = \begin{cases} O_x + f - pA & \text{if } \dot{s} \geq 0 \\ O_x - f + pA & \text{if } \dot{s} < 0 \end{cases}$$

$$F_y = O_y + N$$

$$M = \begin{cases} sN + r_p f & \text{if } \dot{s} \geq 0 \\ sN - r_p f & \text{if } \dot{s} < 0 \end{cases}$$

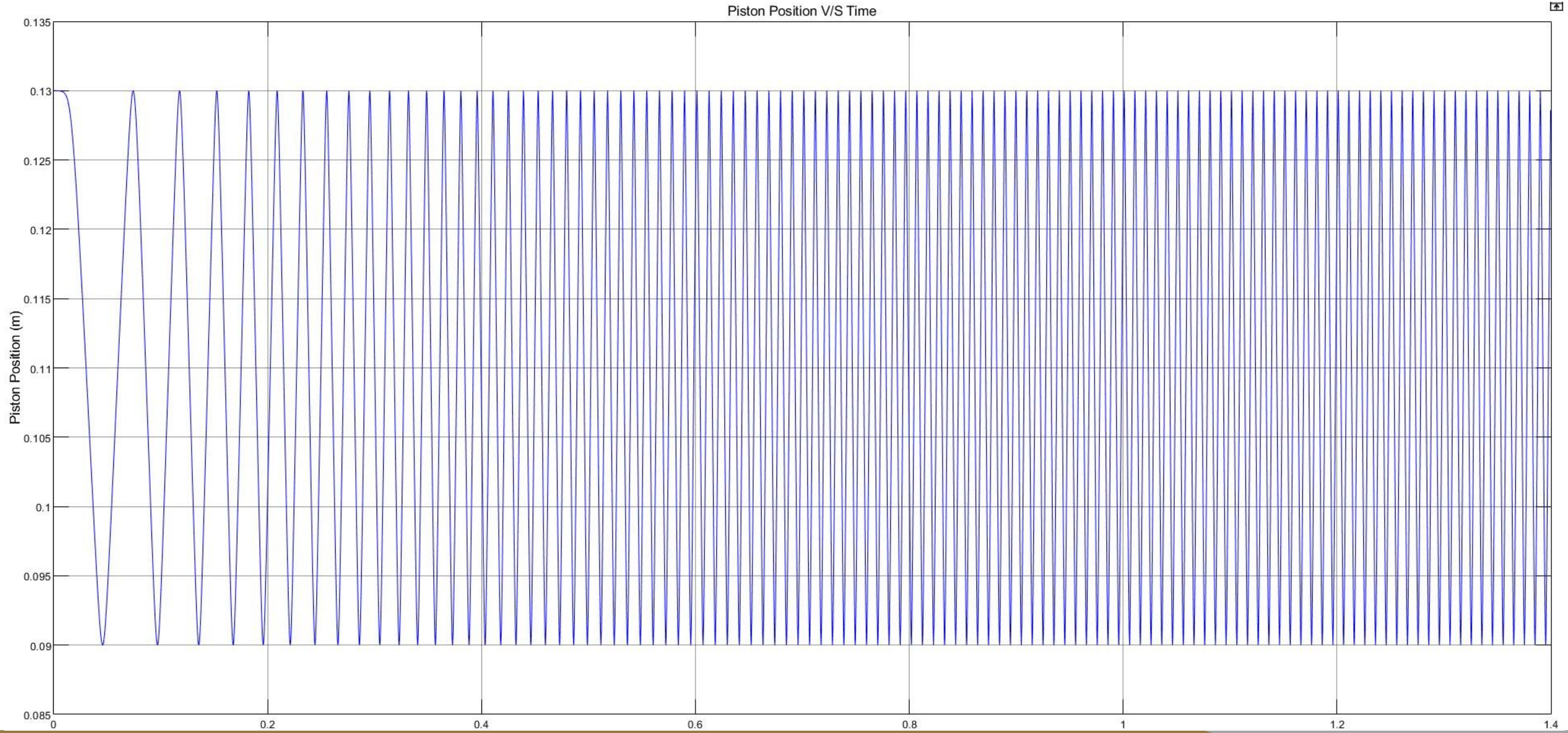
- By solving the above system of equations, we get Ox, Oy and N. Using the relations as shown below, we can get the **ground shaking Forces (Fx and Fy) and moment (M).**

Without Balancing

- All the equations we have derived so far are valid for IC Engine without balancing.
- We will use "**Simulink**" software (Offered by MATLAB) to numerically solve the differential equations we got from kinematic and dynamic analysis
- We will calculate and plot the graph of **d^2s/dt^2, ds/dt, s** and **$d^2\theta/dt^2$, $d\theta/dt$**,
- We will also calculate and plot the **Ground Shaking Forces** and **Moments** and will also find the plot of the **output torque** (which is assumed to be a viscous load).
- Using the ground shaking forces and output torque, we can draw a comparison between engine without balancing with engine with balancing and flywheel (as discussed in subsequent slides).

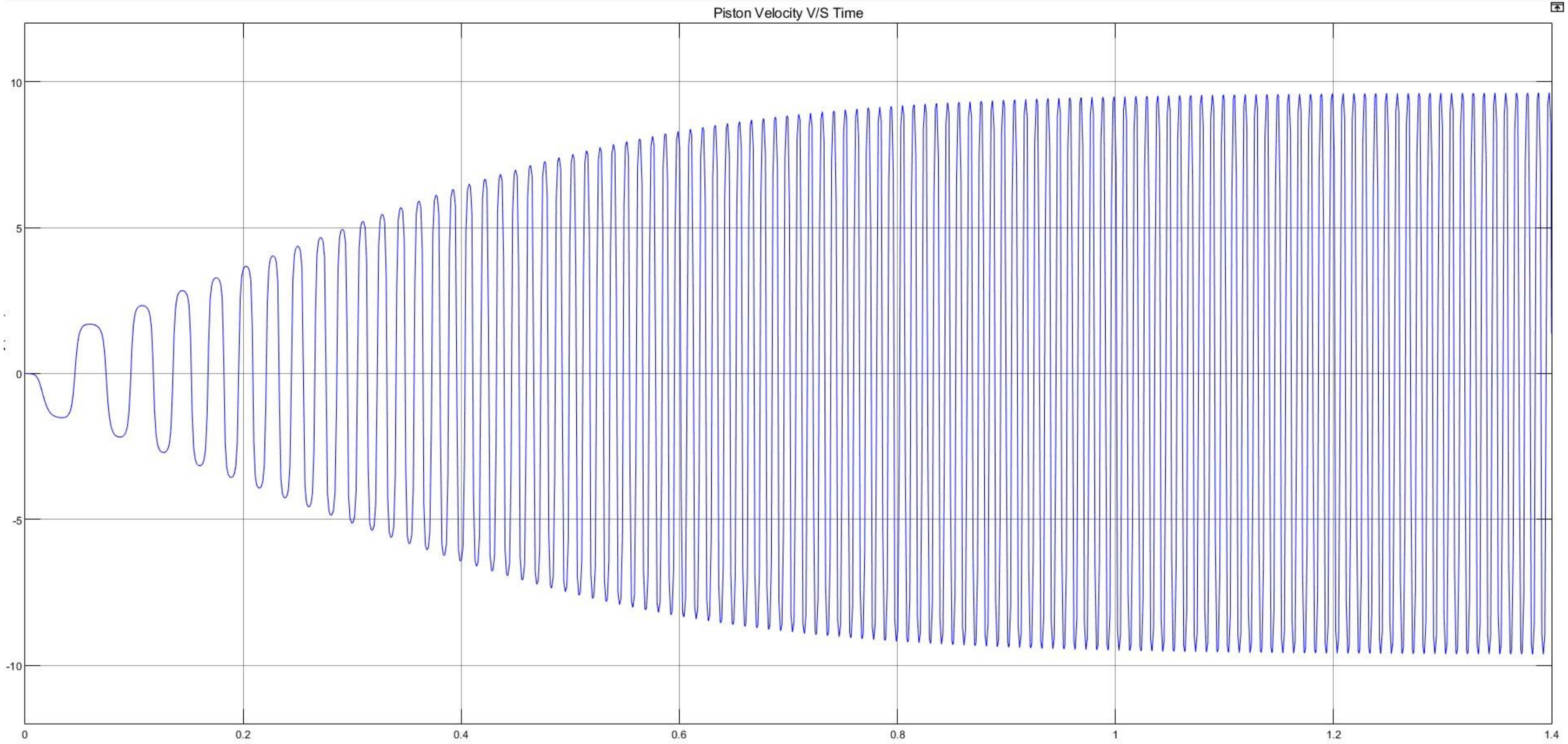
Without Balancing

- The graph of Piston Position (s) vs time is shown:



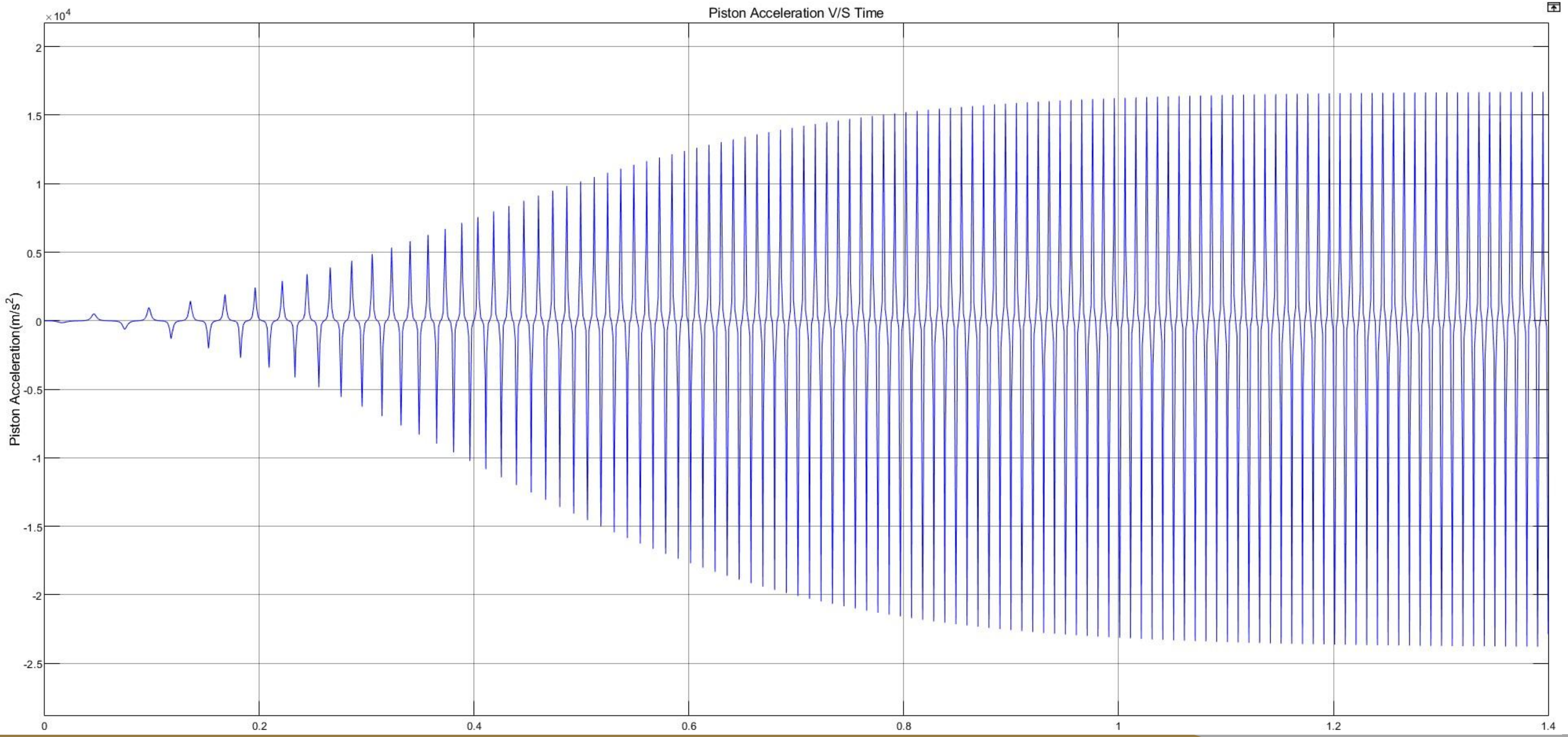
Without Balancing

- The graph of Piston Velocity (ds/dt) vs time is shown:



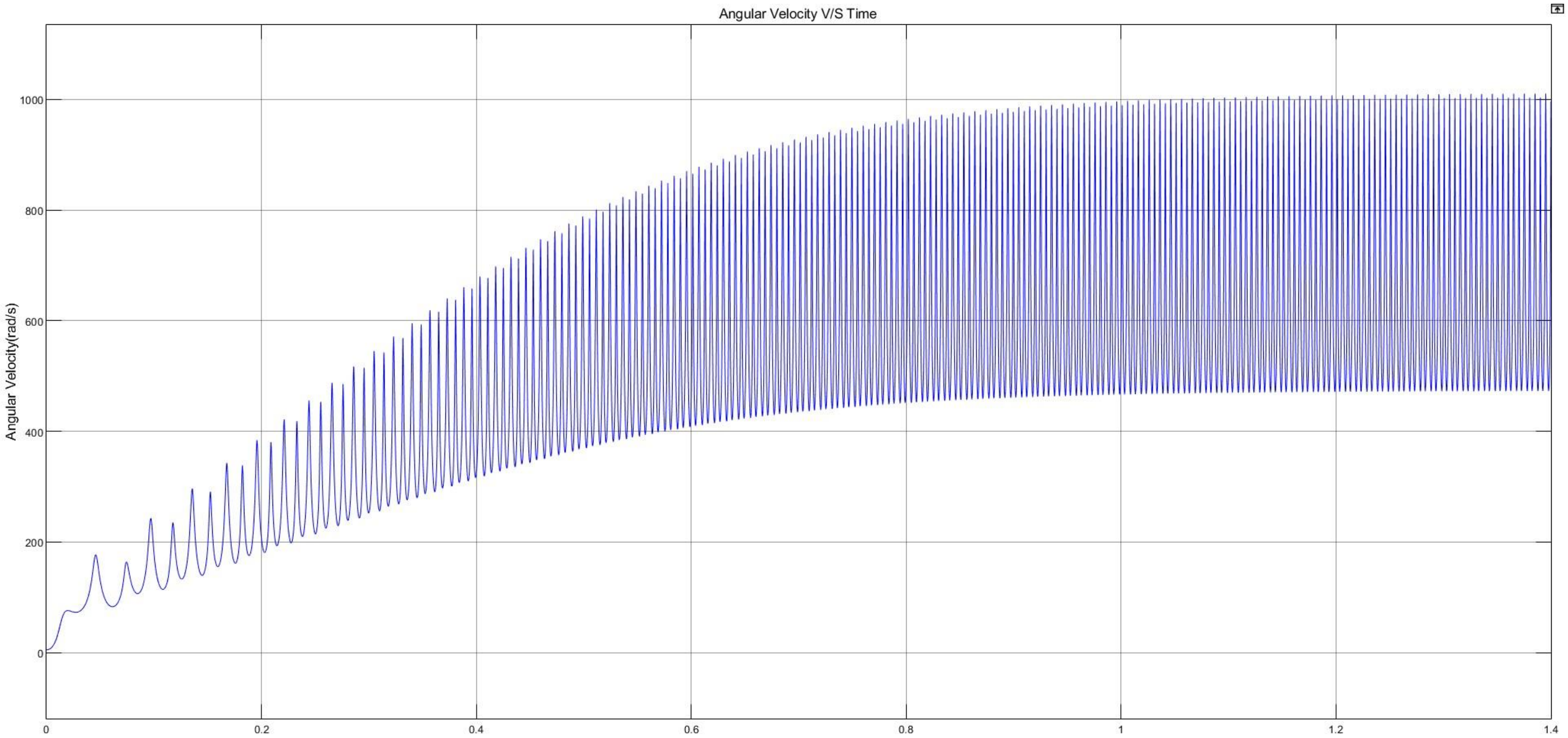
Without Balancing

- The graph of Piston Acceleration (d^2s/dt^2) vs time is shown:



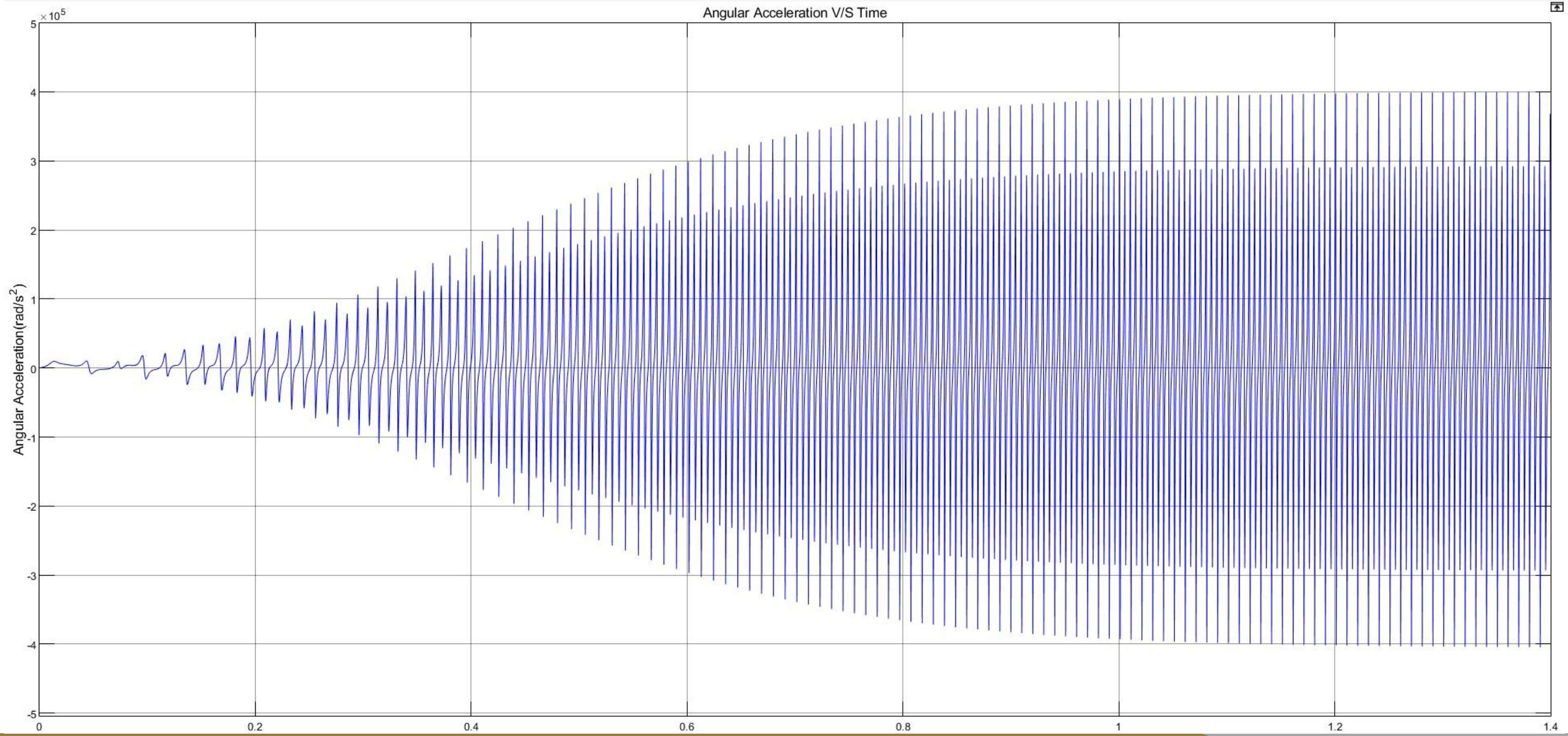
Without Balancing

- The graph of Angular velocity (dθ/dt) vs time is shown:



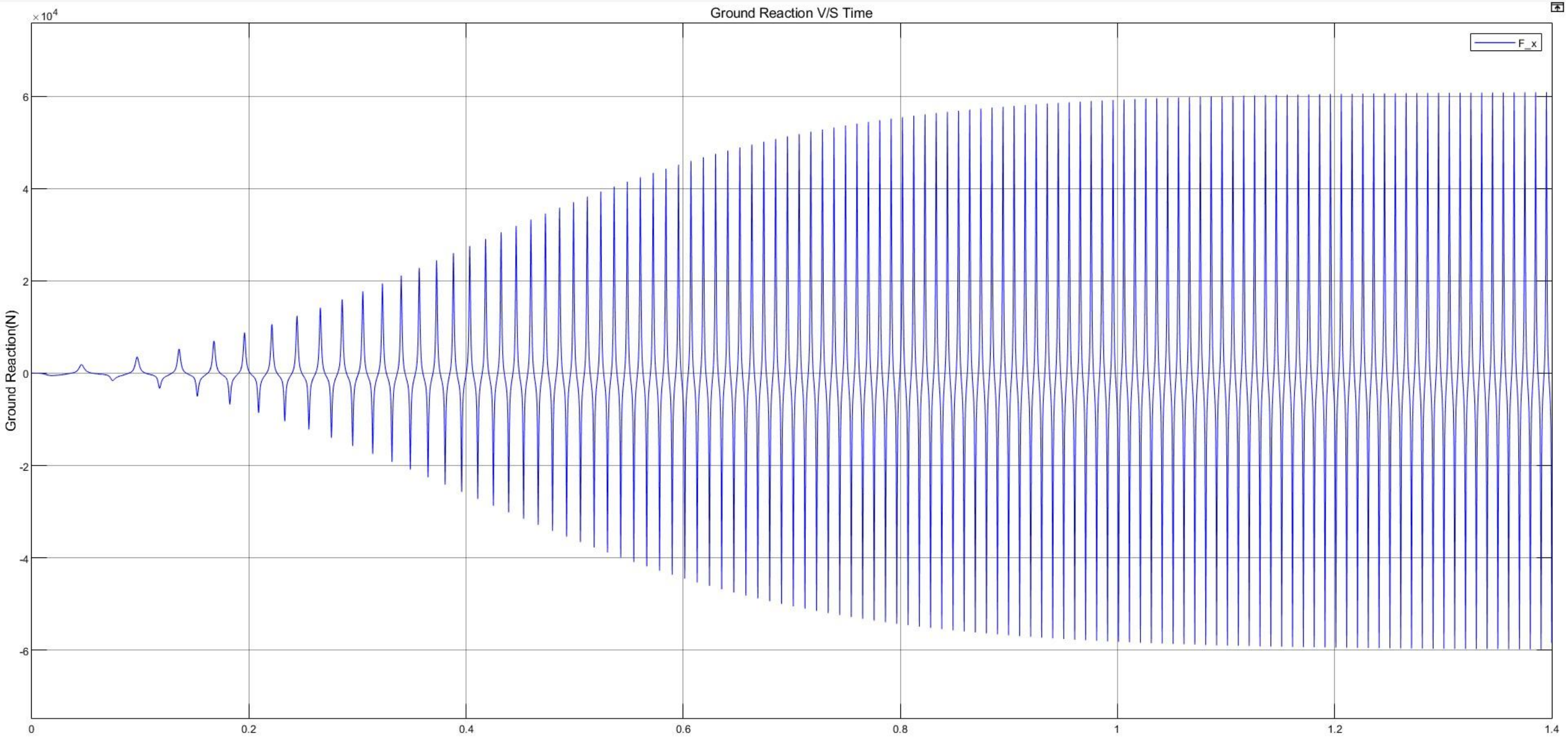
Without Balancing

- The graph of Angular Acceleration $(d^2\theta/dt^2)$ vs time is shown:



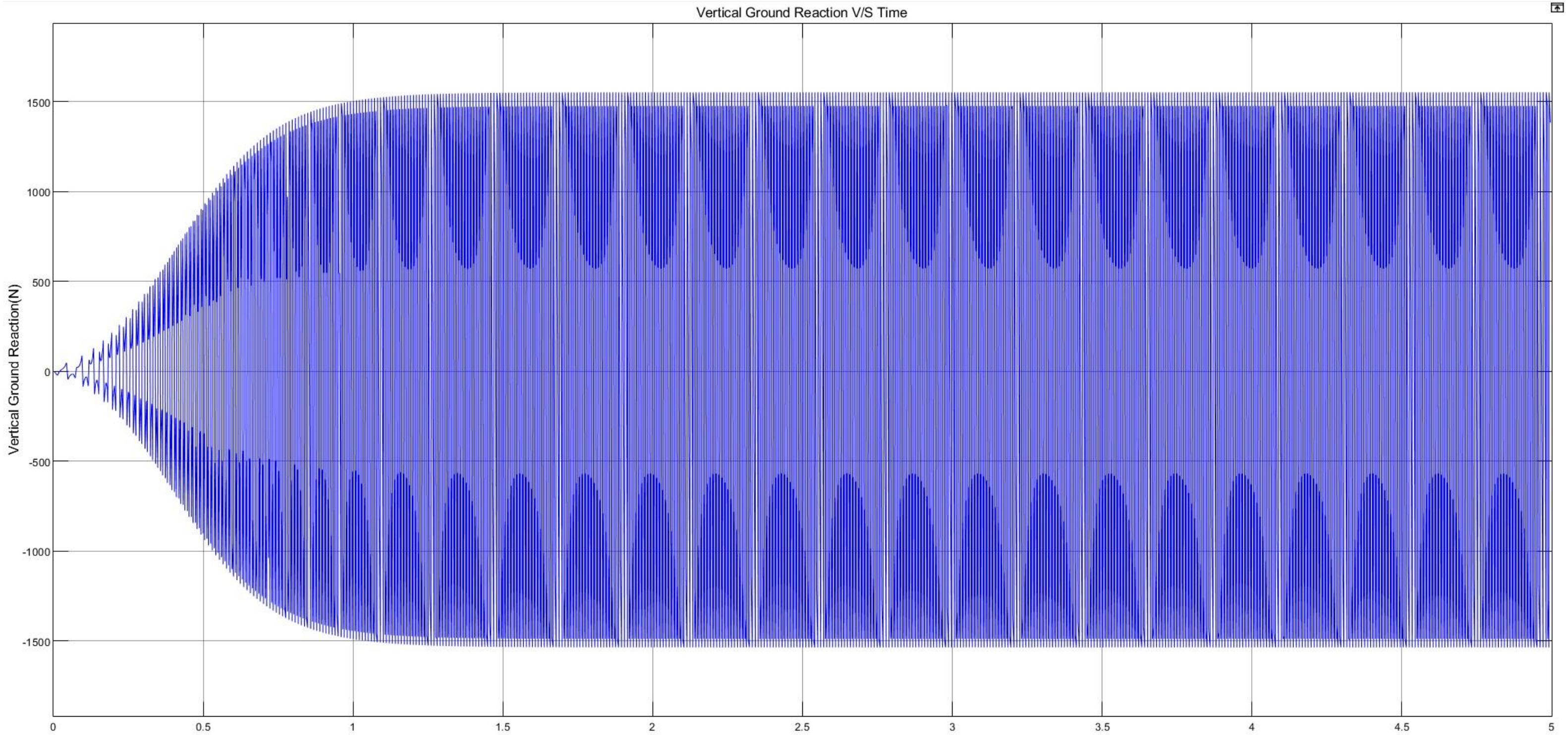
Without Balancing

- The graph of Fx ground reaction vs time is shown:



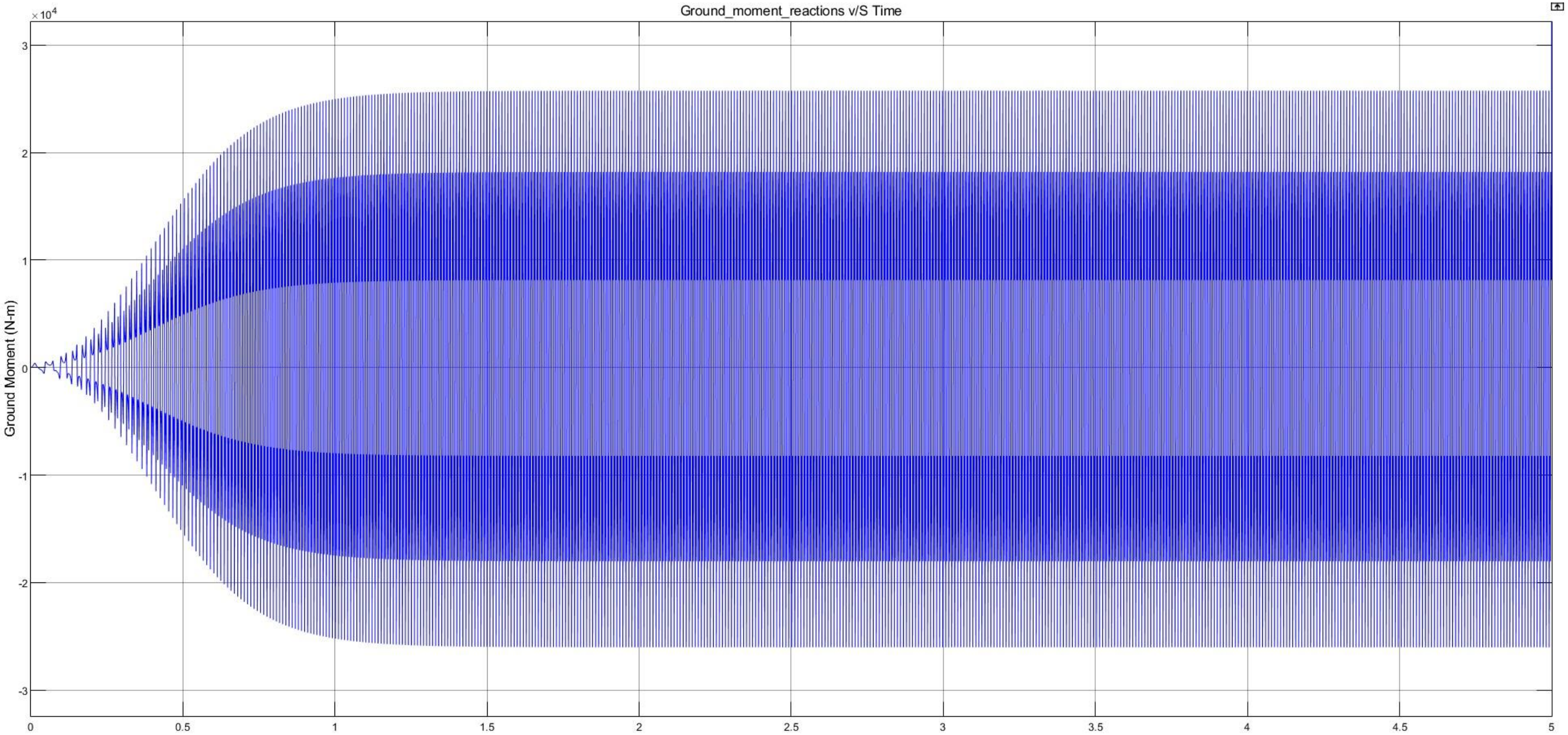
Without Balancing

- The graph of Fy ground reaction vs time is shown:



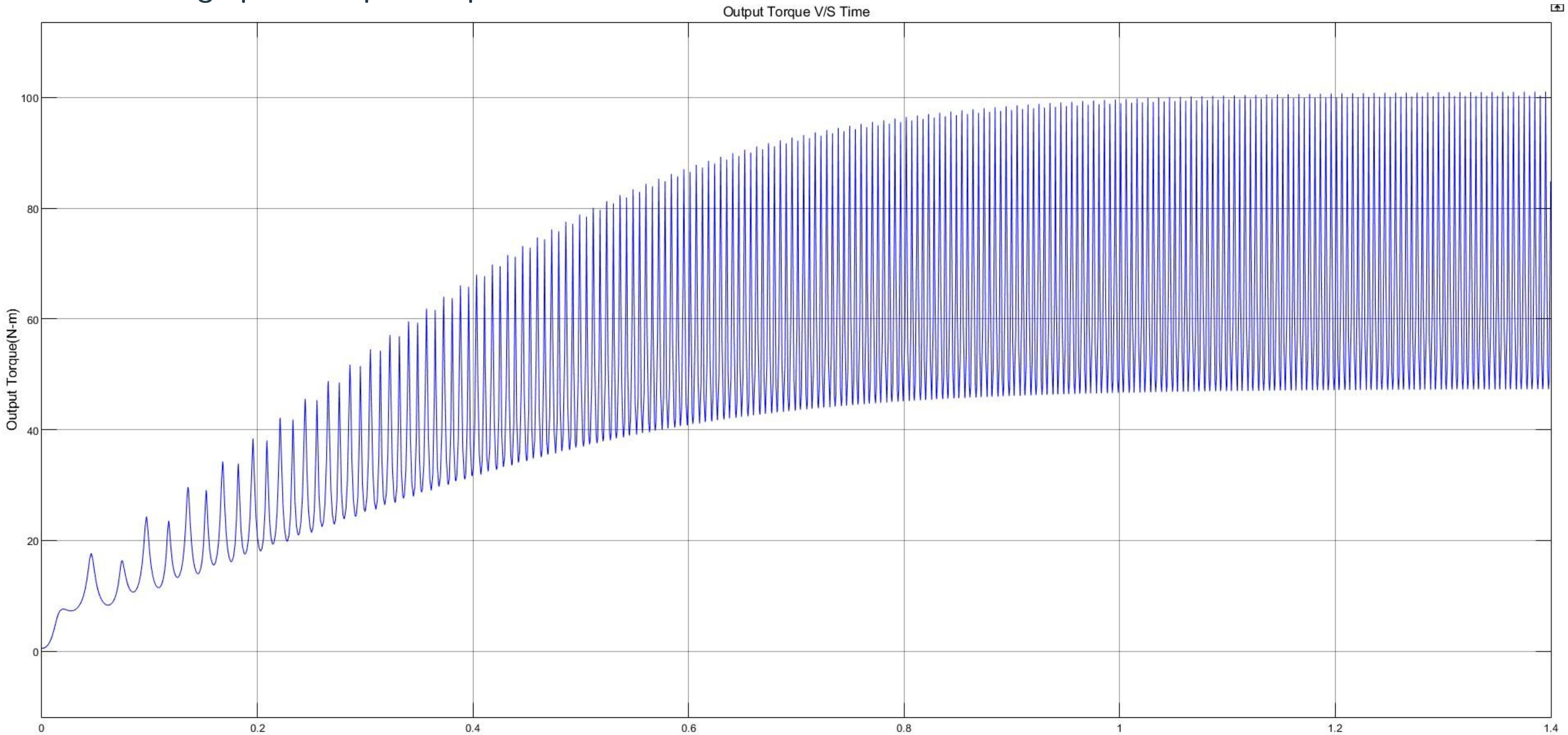
Without Balancing

- The graph of Ground Moment reaction vs time is shown:



Without Balancing

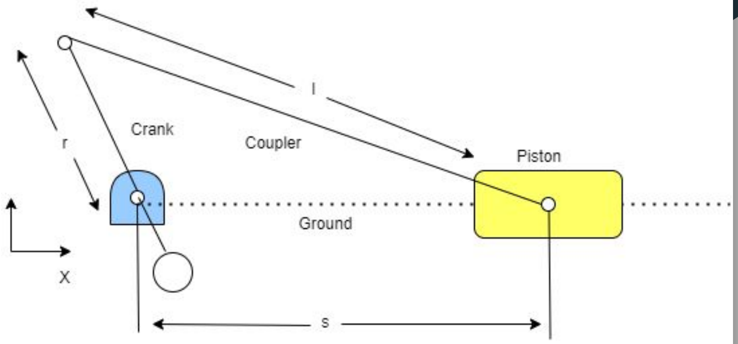
- The graph of Output Torque vs time is shown:



With Balancing: Balanced Crank

- To minimize forces on engine body, we use a **balanced crank.**
- We adjust mass of crank such that **Center of Mass of Crank lies on the Center of rotation**, ie,

$$m_2\rho_2 + m_A r = 0 \quad \text{where} \quad m_A = \frac{m_3(l-a)}{l}$$



- This is possible if $\rho_2 = -\dfrac{m_3(l-a)r}{m_2 l}$
- When this condition is satisfied, we say that the crank is balanced.

The values we are assuming are as follows.

$$m_3 = 1,\ l = 0.12,\ a = 0.024,\ r = 0.02,\ m_2 = 2,\ \rho = -0.008$$

With Balancing: Balanced Crank

- With a balanced crank, we see that Fy gets nullified. So, the **transverse forces in an engine is completely balanced**

$$F_y = 0$$

- The expression for Fx for a balanced crank is as follows (by using expansions of theta):

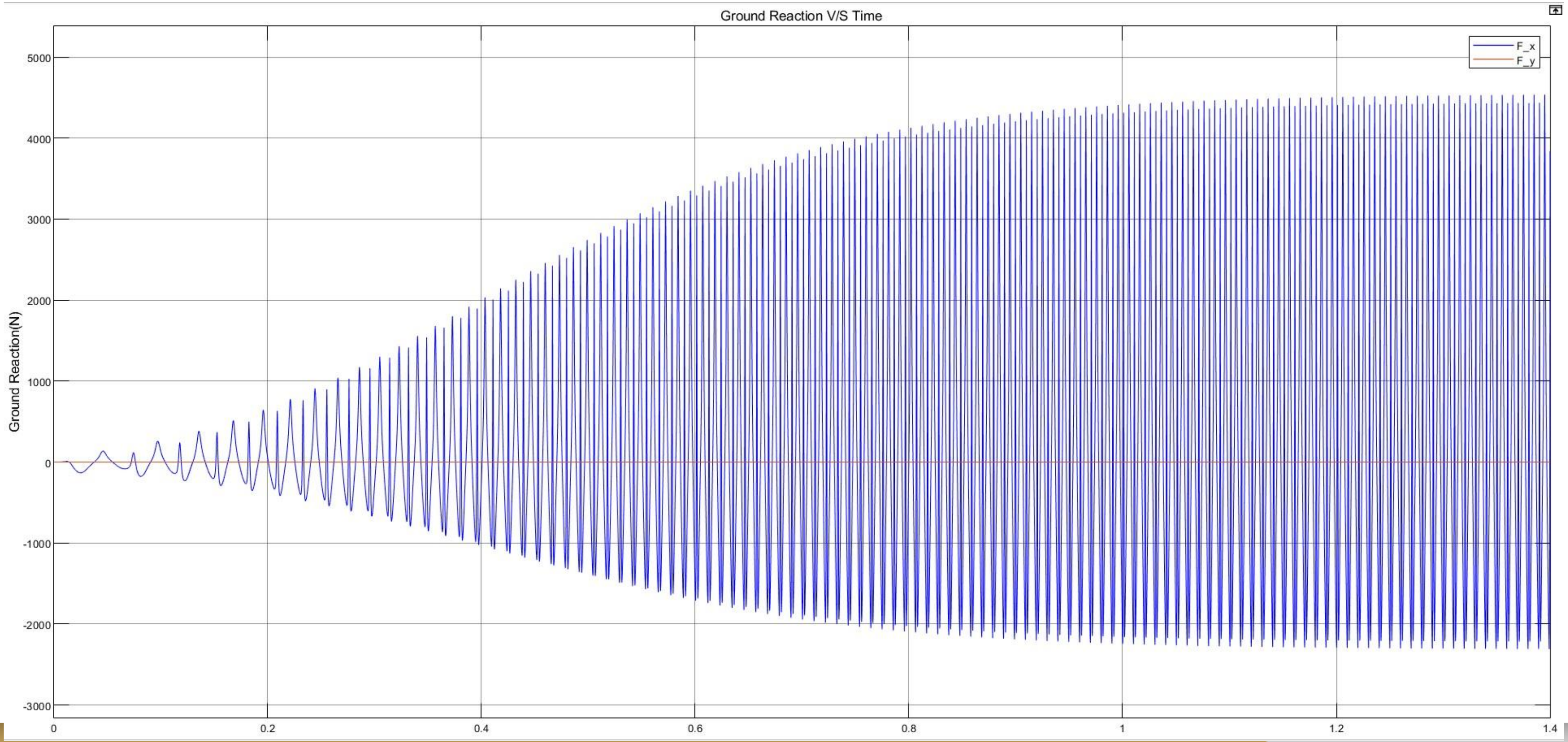
$$F_x = -Mr(\ddot{\theta}cos\theta - \dot{\theta}^2 sin\theta) - 2MrA_2(\ddot{\theta}sin2\theta + 2\dot{\theta}^2 cos2\theta)$$

Where $$M = m_B + m_4 = m_3\frac{a}{l} + m_4$$

- We see that Fx has two terms left:
  - **Primary Unbalance**: The first term of Fx (has frequency same as theta)
  - **Secondary Unbalance**: The second term of Fx (has frequency same as twice of theta)

With Balancing: Balanced Crank

- The graph of ground forces vs time is shown
- We can see that only Fx contributes, wheres Fy= 0 (Transverse force balance)



With Balancing: Reciprocatory Mass Balance

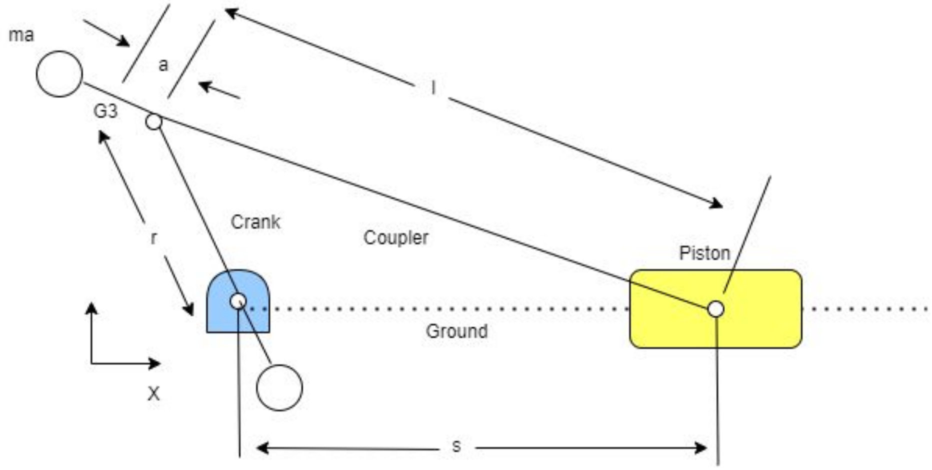
- To further improve our balancing, we adjust the connecting rod mass to act as reciprocating mass balance.

- Hence, we will try to minimize Fx by trying to **nullify M**.

$$M = 0$$

$$m_3 \frac{a}{l} + m_4 = 0$$

$$\Rightarrow \ a = -\frac{m_4}{m_3} l$$

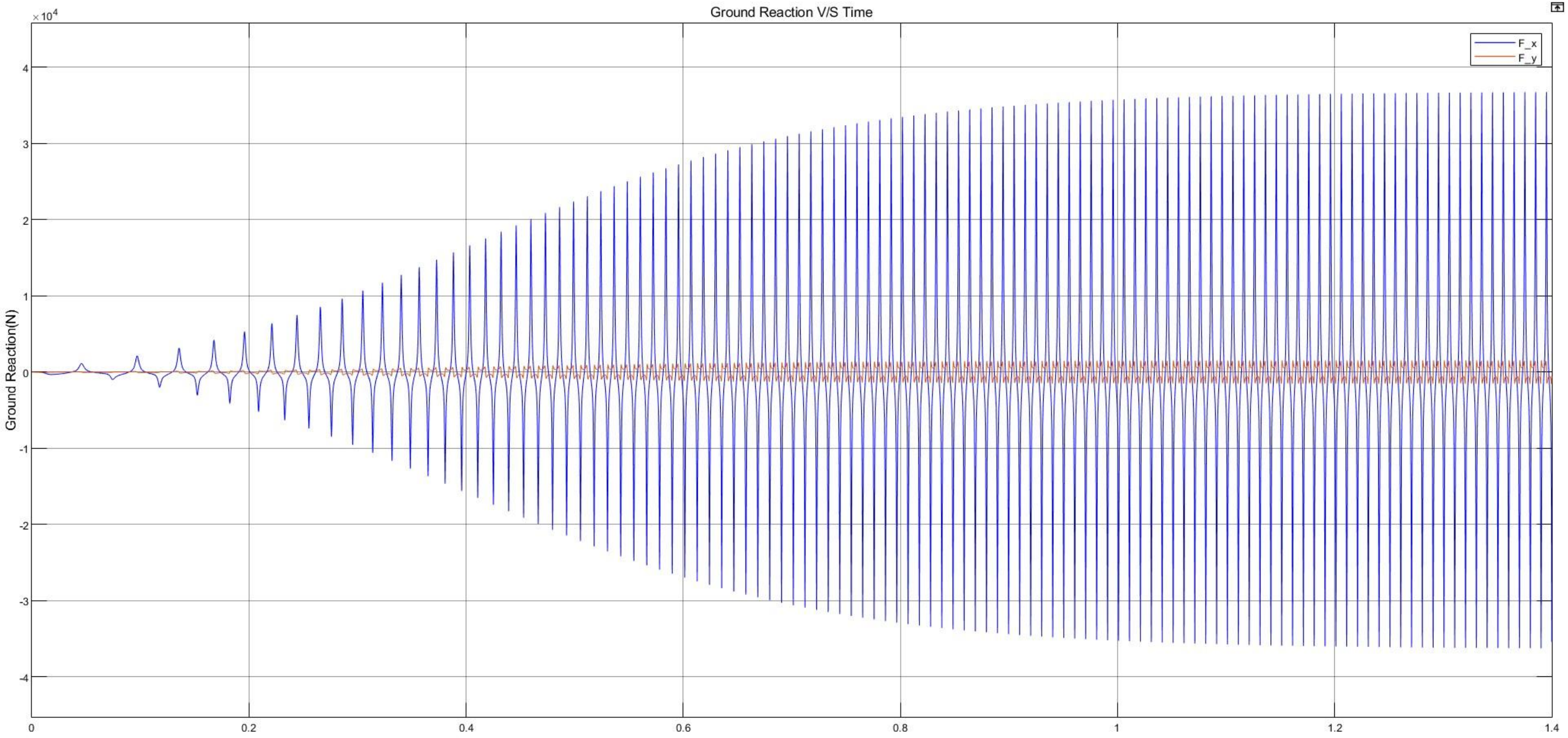


- Using this, we can have **force balance upto second order terms**

- But, this is a **faulty design** (as it is impractical)

With Balancing: Reciprocatory Mass Balance

- The graph of Ground forces (Fx and Fy) with time is shown:



With Balancing: Reciprocatory Mass Balance

A More Practical Approach

- We see that these two conditions cannot be satisfied simultaneously.

$$m_2\rho_2 + m_A r = 0 \qquad\qquad M = 0$$

- Hence, we try to **nullify the linear combination** of these two:

$$(m_2\rho_2 + m_A r) + \chi M r = 0$$

$$m_A = -m_2\frac{\rho_2}{r} - \chi(m_3\frac{a}{l} + m_4)$$

- Here, the value of **X lies between 0 and 1** and we can vary and adjust our value as per our needs.

With Balancing: Dynamic Moment Balance

- The **equation of dynamic moment** is as follows:

$$(I_{O_2} + m_A r^2)\ddot{\theta} - I'_{G_3}\ddot{\phi} = \tau + sN + r_p f$$

- If we try to balance the first term, we get:

$$a = \left(\frac{I_{O_3}}{m_3 r^2} + 1\right) l$$

- We see that a > l. This **condition is not possible**. Therefore, we cannot balance the first term.

- Hence, a **flywheel is used to minimize fluctuations in speed**. Hence, it minimizes angular acceleration, which in turn minimizes dynamic moments in an engine.

With Balancing: Dynamic Moment Balance

- Now, we will try to **nullify the second term**:

$$I'_{G_3} = I_{G_3} - m_3 a(l - a) = 0$$

- This can be achieved by **special construction**. We need the geometry to satisfy this condition for radius of gyration:

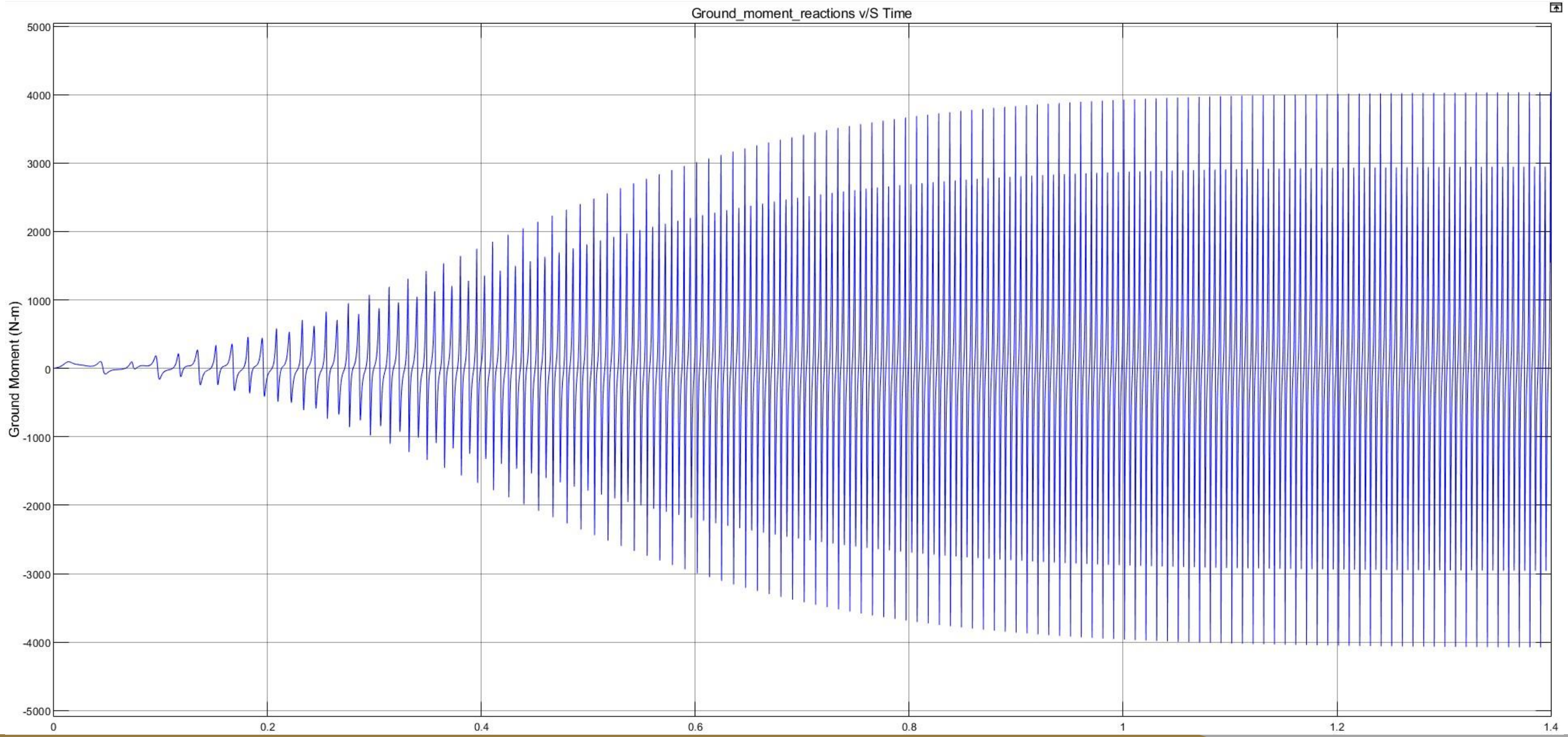
$$k_{G_3}^2 = ab = a(l - a)$$

- This is equivalent to the **Center of Percussion** of a body.

- Even with this, we see that dynamic moment of a body **can not be balanced completely**.

With Balancing: Dynamic Moment Balance

- The graph of Ground Moment with time is shown:

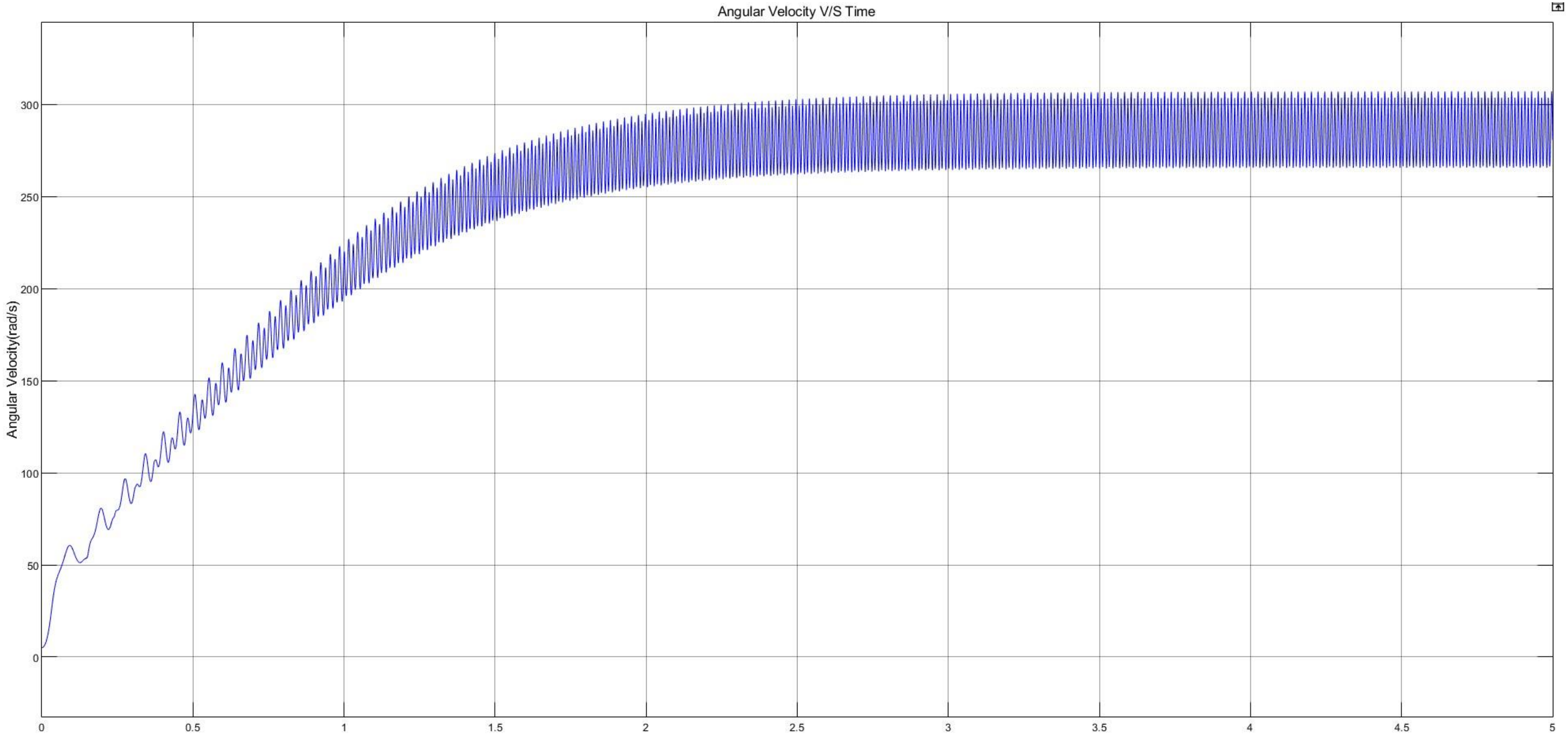


With Flywheel

- As shown in dynamic moment balancing, all fluctuations cannot be nullified. Hence, we use a flywheel.
- A flywheel is a mechanical device which uses the **conservation of angular momentum** to store rotational energy.
- Hence, a flywheel is used to **curb the fluctuation in $d\theta/dt$**.
- This helps in cancelling vibration inducing forces, which in turn increases the performance of the engine.

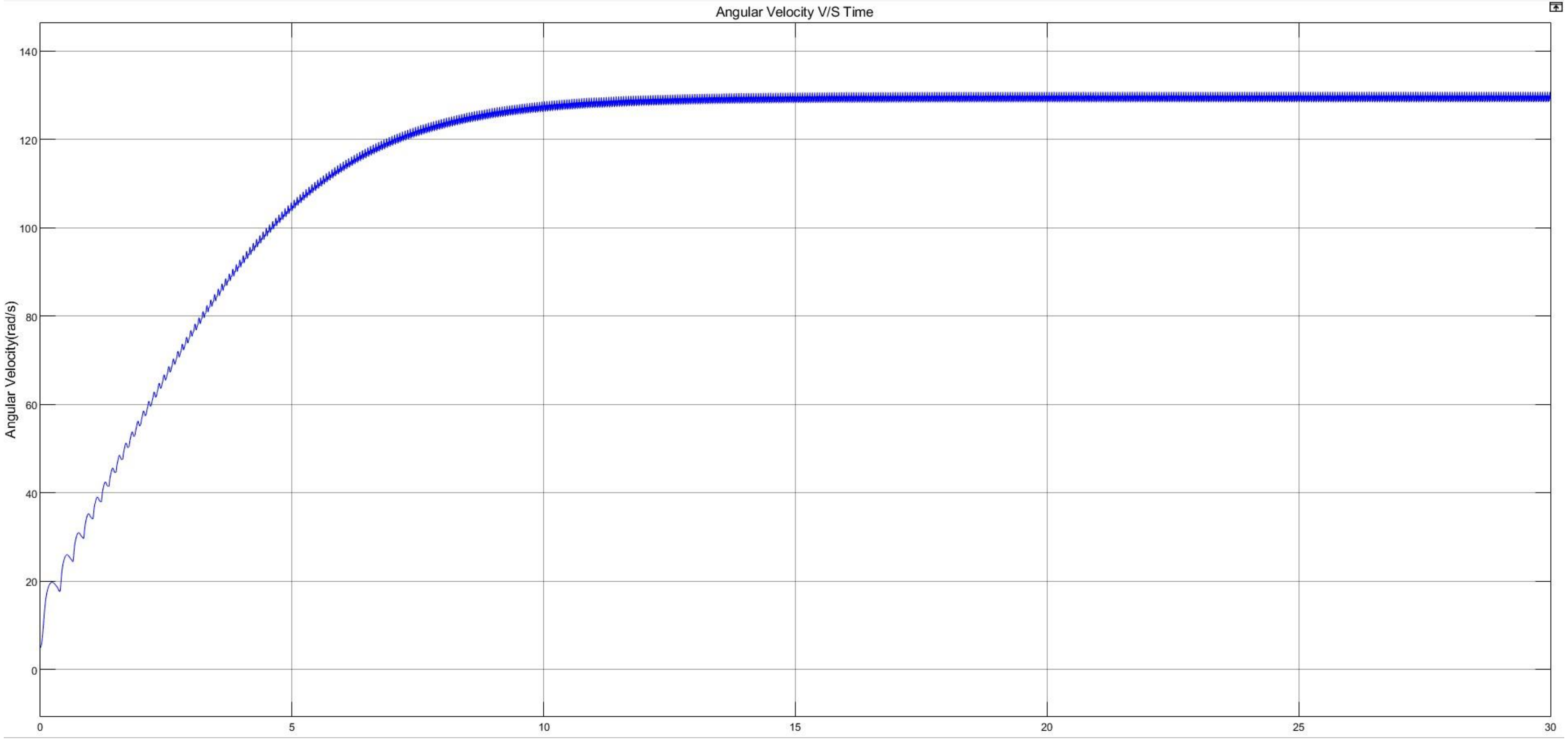
With Flywheel

- The graph of Angular velocity (dθ/dt) vs time is shown: **(Moment of Inertia of Flywheel = 0.1 kgm²)**



With Flywheel

- The graph of Angular velocity (dθ/dt) vs time is shown: **(Moment of Inertia of Flywheel = 1 kgm^2)**



Performance Analysis

Analysis Factor	Without Balancing	Balanced Crank	Reciprocatory Mass Balance	Dynamic Moment Balance	Flywheel (with MOI = 0.1)	Flywheel (with MOI = 1)
Ground Shaking Force Fx (kN)	Max = 41, Min = -41	Max = 4.5, Min = -2.3	Max = 35, Min = -35	NA	NA	NA
Ground Shaking Force Fy (kN)	Max = 1.6, Min = -1.6	0	Max = 2.5, Min = -2.5	NA	NA	NA
Ground Moment (Nm)	25000	NA	NA	4500	NA	NA
Angular Velocity of Crank (rad/sec)	737.5 +- 262.5	Same as without balancing	Same as without balancing	Same as without balancing	285 +- 25	130 +- 2
Stabilisation Time (sec)	1.1	Same as without balancing	Same as without balancing	Same as without balancing	3	12
Output Torque (Nm)	73.5 +- 26.5	Same as without balancing	Same as without balancing	Same as without balancing	28.5 +- 2.5	13 +- 0.2
Mean Output Power (kW)	54.2	Same as without balancing	Same as without balancing	Same as without balancing	8.1	1.7

Key Features:

Now we will look at some key features that we implemented in our project:

- We made a **geometrically accurate model** of an IC Engine in Fusion 360
- We considered **theoretically exact Pressure Variation** (Otto Cycle)
- Computed numerical solution for Dynamic and Kinematic equations **without making any assumptions** and **without neglecting any higher order terms**.
- Used Simulink software to **generate Computational Graph** for accurately plotting all parameters.
- We provided **initial impulse** to start the IC engine.
- We build a simulation model which **can handle angular acceleration at dead center configurations**
- **Computed and compared** effect of different methods of balancing for producing optimal results.

Suggestions:

Now let us look at some suggestions from our side, which can make our Analysis more effective and accurate:

- It is found experimentally that approximately **20% of the generated power is lost due to external factors** (friction, thermal losses, viscous drag, etc). We could take these factors into consideration for getting more accurate results.
- We have not accounted for features that change with temperature like **changes in dimensions due to thermal expansion, introduction of uneven forces due to thermal stresses**, and so on
- The **anomalies resulting due to wearing and fatigue of machine parts** is not accounted for.
- We have assumed the 3R1P chain to be ideal. In reality, we have **abnormalities, eccentricities, friction at joints, inhomogeneity, misalignment** and several other factors.
- The In-cylinder pressure variation depends on a lot of factors like **kind of fuel used, spark ignition timing, air-fuel ratio,** etc. These need to be accounted for.

Conclusion

- We carried out a thorough dynamic analysis of an Internal Combustion engine. We considered cases of **unbalance, balanced crank, reciprocating mass balance, dynamic moment balance** and the use of **flywheel**.

- From performance analysis, we observe that unbalanced engine has **higher fluctuation** in terms of **angular velocity**. Moreover, the **ground shaking forces and moments are very high**.

- When we **use balancing**, we discovered that **the fluctuations in angular velocity decreased** and the **ground shaking forces and moments decreased.**

- A better design is with the **use of flywheel**. We observed that as the **moment of inertia** of the flywheel **increased**, the **fluctuations** in the angular velocity **decreased**.

- The **stabilisation time is maximum** with the use of flywheel with **higher moment of inertia**.

References

- **Books :**
  - “Internal Combustion Engine Fundamentals” : John B Heywood
  - “Theory of Mechanisms and Machines” : Ghosh and Mallik
  - “Design of Machinery” : RL Norton

- **Research Papers:**
  - Dynamic Analysis of Single Cylinder Petrol Engine : Jatkar and Dhanwe
  - Experimental Investigation on the combustion process in a spark ignition optically accessible engine fueled with syngas : S D M Boggio

- **Websites and Others:**
  - Lecture notes of Prof. Anirvan DasGupta (Course: Dynamics of Machines (ME22004))
  - Nptel - Mechanism and Robot Kinematics - Prof Anirvan DasGupta : https://youtube.com/playlist?list=PLSGws_74K01-glG85g16JD0I5eNdPmRP9
  - Engine Balancing: https://en.wikipedia.org/wiki/Engine_balance
  - Modelling of IC Engine : https://youtu.be/PuWRsVyU3PM
  - Simulink Differential Equation Model : www.mathworks.com/help/simulink/ug/model-a-differential-algebraic-equation.html

Link to Repository

- All the details and materials used in our project (including Fusion 360 simulation, Simulink equations, Formulas used, etc) can be found in the repository link below.

- **Github Link:** https://github.com/Sohanpatnaik106/Internal-Combustion-Engine-Analysis

We, (Pratyush Pande, Sohan Patnaik and Souvik Basak), would like to express our immense gratitude towards our Dynamics of Machines Professor, Dr. Anirvan DasGupta, and his Teaching Assistants, for their precious guidance and support throughout this course.