Day 1

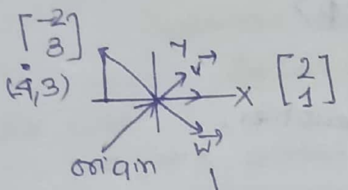
Quantum Study Plan

Escence of Linear Algebra

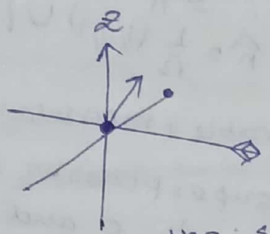
" The introduction of numbers ~~ate~~ as coordinates is an act of violence."

— Hermann Weyl

Vector
- physics → magnitude, direction
- CS student → ordered list of numbers
- Mathematician → multiplying + ~~no~~ addition operations.



3-Dimension



$\begin{bmatrix}2\\1\\3\end{bmatrix}$, $\begin{bmatrix}x_1\\y_1\end{bmatrix} + \begin{bmatrix}x_2\\y_2\end{bmatrix} = \begin{bmatrix}x_1+x_2\\y_1+y_2\end{bmatrix}$

$\frac{1}{3}\vec{v}$ ~~str~~ scaling : scalars

Conclusion

a language to describe space and manipulation of space using numbers that can be crunched and run through a computer

Qubits & Superposition

Quantum bit = basic unit of quantum information
- superposition
- complex vector math

classical bit: off 0 or 1 → freely charged capacitor

Quantum bit: vector made of a combination of 0 and 1 state

Day 2

Classical vs Quantum bits and their logic gates

classical → electrically controlled logic gates

A —▷o— Not A (Inverter)

Q- quantum mechanical state $|\psi\rangle = [1\ 0]$ state-space

ket

$|\psi\rangle = a|0\rangle + b|1\rangle$

Linear combination of Quantum state

$= \begin{bmatrix}a\\b\end{bmatrix}$ → complex number

total probability $= |a|^2 + |b|^2 = 1$

Normalization

Quantum dot
Trapped ion
Superconducting transmon

$|1\rangle$
$|0\rangle$

$\Delta E = E_1 - E_0$

E_1 Excited state

E_0 ground state

Pauli-X gate

$|1\rangle$ —[X]— $|0\rangle$

Qubit as quantum superposition state

Matrix representation of qubits and gates

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\rightarrow$ 2-D vectors

$\rightarrow H^2 = HH = HH^+ = I_{2\times 2}$

$|\psi\rangle = a|0\rangle + b|1\rangle = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix} = a\begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

$\hookrightarrow$ span the 2D Hilbert space.

Quantum logic gates

Quantum gate

I/P Qubits $|\psi\rangle \rightarrow \boxed{U} \rightarrow |\phi\rangle$ O/P Qubit

$\hookrightarrow$ 2×2 matrices

Pauli X gate

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$

Hadamard gate

$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(X+Z)$

rotation angle $= \pi$ $|+\rangle$ $|-\rangle$

$\hat{n} = \frac{1}{\sqrt{2}}(1,0,1)$ $U|\psi\rangle = |\phi\rangle$

$X|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$ $\rightarrow$ observable + measurable quantities.

superposition state with 0 and 1

Ensures real Eigen values.

$U = U^+$

$\hookrightarrow$ Hermitian = Transpose complex conjugate

$H|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} a+b \\ a-b \end{bmatrix}$

$H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \equiv |+\rangle$

$H|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \equiv |-\rangle$

Quantum mechanics $\leftrightarrow$ Quantum computing

Unitary

$UU^+ = I$

Matrix multiplication = identity matrix

$\downarrow$

conservation of probability

Geometrical Bloch Sphere Representation

$|\psi\rangle = a|0\rangle + b|1\rangle$

$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$

Visualizing the action of quantum gates on qubits with Bloch sphere.

Pauli x gate

$\hookrightarrow$ Spin 1/2 rotation operator

$\hat{D}(\hat{n},\phi) = \cos\left(\frac{\phi}{2}\right)I - i\sin\left(\frac{\phi}{2}\right)\vec{\sigma}.\hat{n}$

Rotation of π around the x axis

$\hat{D}(\hat{x},\phi=\pi) = \cos\left(\frac{\pi}{2}\right)I - i\sin\left(\frac{\pi}{2}\right)\vec{\sigma}\,\vec{x}$

$= 0 - i(1)\sigma_x$

$= -i\sigma_x$

$= \sigma_x$ ignore global phase

Pauli Y and Z gates

Circuit representation

Matrix Representation

$|0\rangle - \boxed{Y} - i|1\rangle$ $Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$

$|+\rangle - \boxed{Z} - |-\rangle$ $Z|+\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

$= \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = |-\rangle$

Day 3

Quantum Computing

Tensor Product and Multi-Qubit states

$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ → 2-D Hilbert space

orthogonal (mutually) state ↳ + normalized: inner product = 1

$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

$$\begin{bmatrix}a_0\\a_1\end{bmatrix} \otimes \begin{bmatrix}b_0\\b_1\end{bmatrix} = \begin{bmatrix}a_0\begin{bmatrix}b_0\\b_1\end{bmatrix}\\a_1\begin{bmatrix}b_0\\b_1\end{bmatrix}\end{bmatrix} \rightarrow \text{col. vec. with 4 entries} = \begin{bmatrix}a_0b_0\\a_0b_1\\a_1b_0\\a_1b_1\end{bmatrix}$$

Tensor product

1. $|00\rangle = |0\rangle|0\rangle = |0\rangle \otimes |0\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$
2. $|01\rangle = |0\rangle|1\rangle = |0\rangle \otimes |1\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$
3. $|10\rangle = |1\rangle|0\rangle = |1\rangle \otimes |0\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}$
4. $|11\rangle = |1\rangle|1\rangle = |1\rangle \otimes |1\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

Four computational Basis states

$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

Any terms of qubit states on the computational basis

↳ linear combination

Quantum Entanglement

particles (2) in lab

opp. spins

spin points up / down (not faster than light communication)

Bell states

↳ A pair of maximally-entangled qubits

co-related / anti

$|0\rangle$ —H— •

$|0\rangle$ ——— ⊕ $\Big\} \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Quantum teleportation

Quantum Nonlocality

physics — consciousness

quiet state of

altered the structure

photons (twin)

↳ can move off ~~from~~ into a distance

change polarization of the other by grabbing one } entangled with one another.

Day 4

$f: \{0,1\} \xrightarrow{\text{Black box}} \{0,1\}$

constant / balanced? → $f(0) = f(1)$

	x	f(x)	Not gate	
constant zero →	0	0	0	1
	1	0	1	0
constant one	0	1		
	1	1.		

classical computers need 2 queries!

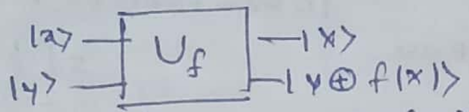
∴ f(0) and f(1) both need to be calculated.

Quantum computers: 1 query of func!

Create reversible classical gates

↓

create quantum operations / functions (Quantum oracles)



$|x\rangle|y\rangle \xrightarrow{U_f} |x\rangle|y \oplus f(x)\rangle$

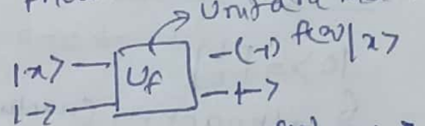
$\xrightarrow{\quad} = ~~\ast~~ \mapsto$ in quantum oracle

input $y=0$

$|x\rangle|0\rangle \xrightarrow{U_f} |x\rangle f(x)\rangle$

$0 \oplus 0 = 0,\ 0 \oplus 1 = 1$

phase oracle



$|x\rangle|-\rangle \longrightarrow (-1)^{f(x)}|x\rangle|-\rangle$

$= \begin{cases} |x\rangle|-\rangle & f(x)=0 \\ \frac{1}{\sqrt{2}}~~(|x\rangle|0\rangle)~~ \\ -|x\rangle|-\rangle & =1 \end{cases}$

$= (-1)^{f(x)}|x\rangle|-\rangle$

Deutsch's Algorithm

$\boxed{\text{if } f(0) = f(1)}$

$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad f(0)=0,\ f(1)=0$

$|\psi_3\rangle = -\frac{1}{\sqrt{2}}(+|0\rangle + |1\rangle)$ $\quad f(0)=1,\ f(1)=1$

$\boxed{|\psi_3\rangle = \pm\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}$

$f(0) \neq f(1)$

$\boxed{|\psi_3\rangle = \pm\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)}$

measure 0, func = constant
measure 1, func = balanced

Grover's Algorithm [1996] → speed up on quantum computers.

unstr. database search algorithm

↓

IoT: bigger

crypto [Hadamard transform]

Day 5

Quantum fourier transform → frequency

destructive system ⟲ constructive

↑ interference on the components of superposn

↓ Shor's algorithm ↳ period finding.

$|\psi_{in}\rangle = \sum_{k=0}^{N-1} a_k|k\rangle$

$|\psi_{out}\rangle = \left(\frac{1}{\sqrt{N}}\sum_{l=0}^{N-1}\sum_{k=0}^{N-1} e^{\frac{i2\pi k}{N}}|l \times k|\right)|\psi_{in}\rangle$

More qubits = more unitary gates

Day 6

Quantum Encryption → Method: randomness of quantum mechanics (Unbreakable)

agree on a key

↳ none can eavesdrop on it w/o being noticed

Quantum teleportation

Some particles in a particular arrangement → state changes.

↓

corresponds you.

$\boxed{\text{No cloning theorem} \rightarrow \text{consumed}}$

Scanned with OKEN Scanner

High dimensional feature spaces (Quantum circuits)

IBM 2021

↓

structured data on machine learning algo (classification)

↳ diff data point (col. matrix)

SVM →