

# **Interleave Division Multiple Access**

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# Outline

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- Introduction
  - IDMA
- Chip-by-chip multiuser detection
  - Analysis and optimization
- IDM space-time coding and IDM coded modulation
  - Conclusions

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# Background

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- Low-rate coded systems: Viterbi and Verdu
- Iterative detectors (1998/1999): Moher, Reed, Schlegel, Alexander, Wang and Poor
- TCMA (2002) Brannstrom, Aulin and Rasmussen
- Graph-code based multiple access (2001): McEliece
- Chip-interleaved CDMA (2002): Mahavadevappa and Proakis
- CDMA power control (2003/2004): Verdu, Shaimai, Caire and Muller

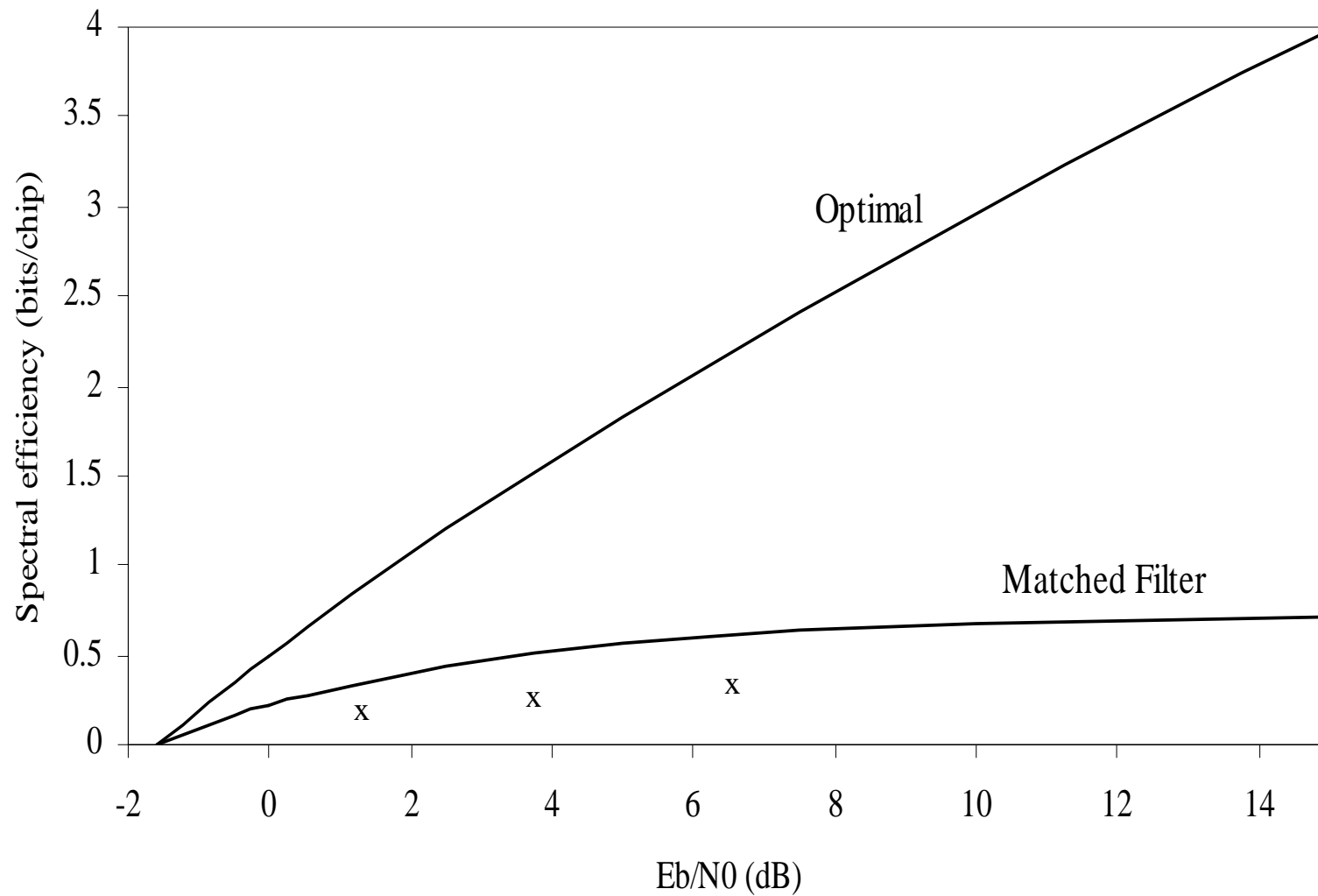
# Some Requirements for Future Wireless Systems

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- low receiver cost
- de-centralized (i.e., asynchronous) control,
- simple treatment of ISI,
- cross-cell interference mitigation,
- diversity against fading,
- power efficiency (long battery life),
- multi-media services (e.g., mixed voice and IP),
- high user number,
- high throughput and high spectral efficiency,

FDMA ?  
TDMA ?  
CDMA ?  
OFDMA ?

# CDMA Spectrum Efficiency (per Dimension)



# Outline

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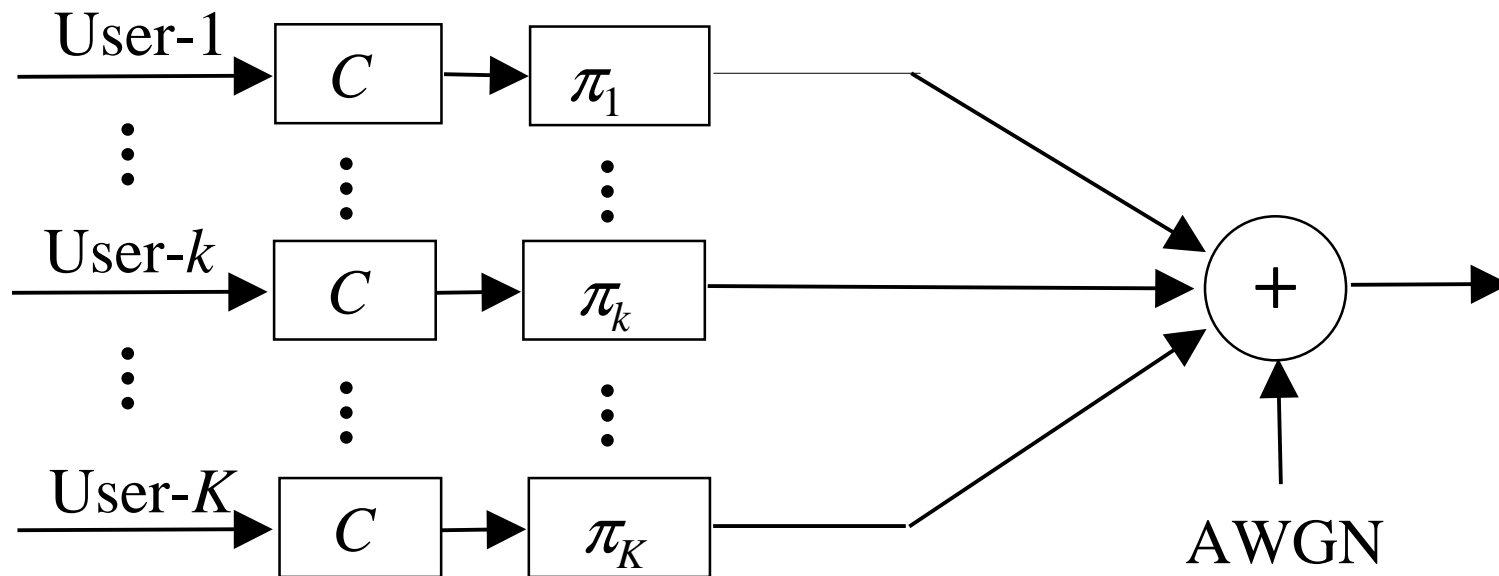
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# Interleave Division Multiple Access (IDMA)

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Key: The interleavers  $\pi_1, \dots, \pi_K$  must be user-specific.

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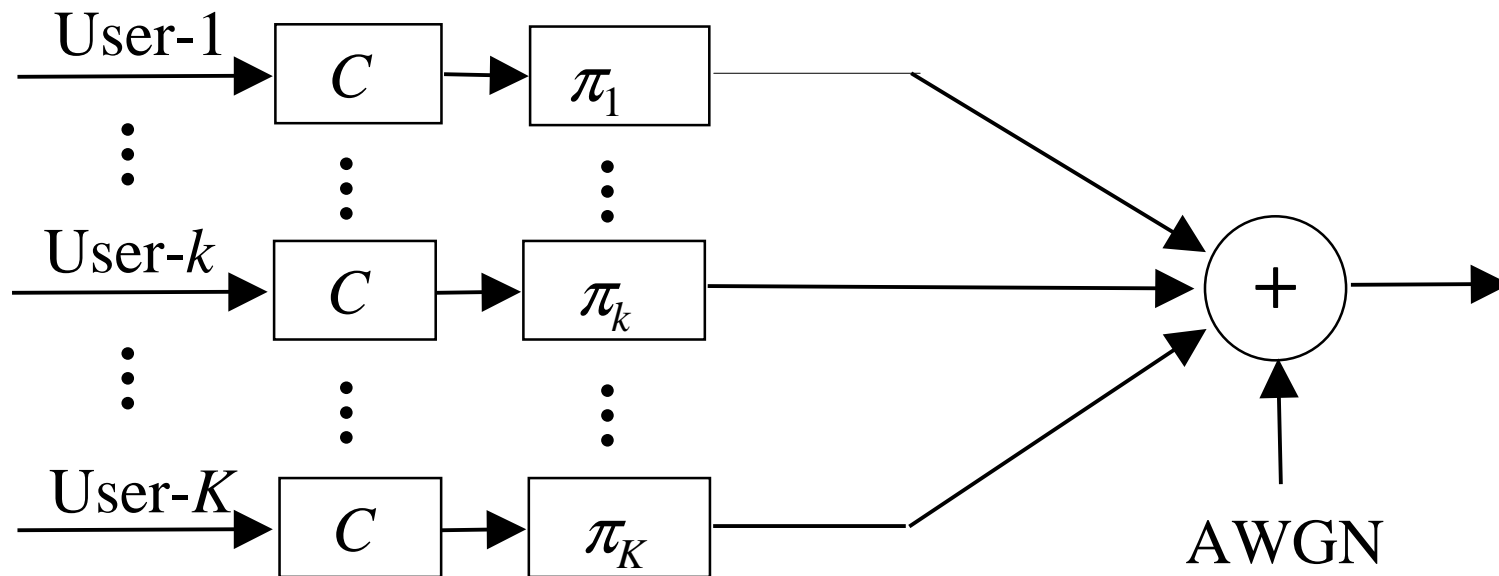
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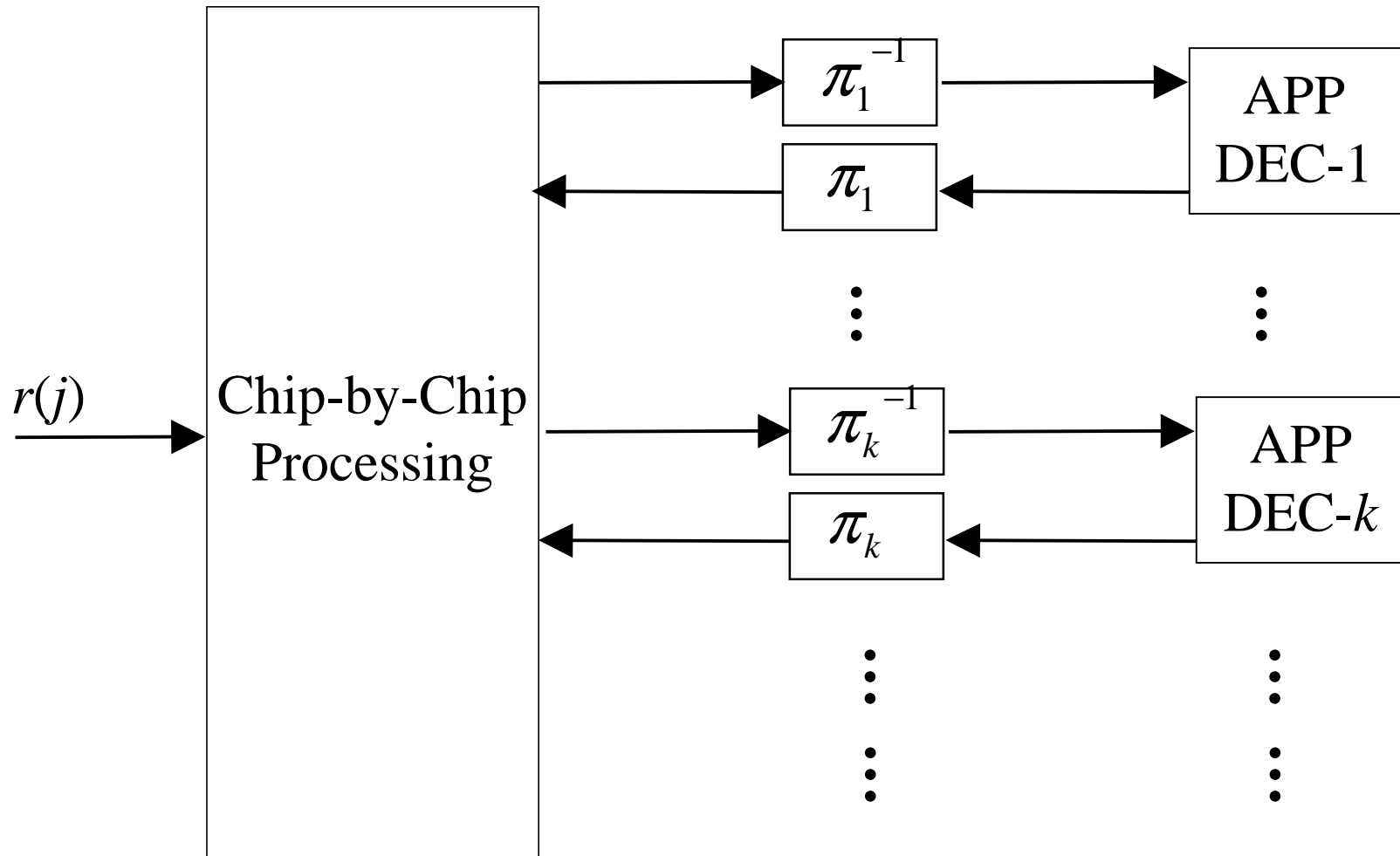
# Interleave Division Multiple Access (IDMA)

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Key: The interleavers  $\pi_1, \dots, \pi_K$  must be user-specific.

# Chip-by-Chip Multiuser Detection



## Chip-by-Chip Detection

Gaussian

Step 1. Chip-level path model:  $r(j) = \sum_{k=1}^K h_k x_k(j) + n(j)$

Step 2. Gaussian approximation:  $r(j) = h_k x_k(j) + \zeta_k(j)$

Step 3. Estimation: 
$$e(x_k(j)) = \log \frac{\Pr(x_k(j) = +1)}{\Pr(x_k(j) = -1)}$$
$$= \log \frac{\exp\left(-\frac{(r(j) - E(\zeta_k(j)) - h_k)^2}{2\text{Var}(\zeta_k(j))}\right)}{\exp\left(-\frac{(r(j) - E(\zeta_k(j)) + h_k)^2}{2\text{Var}(\zeta_k(j))}\right)}$$
$$= \frac{2h_k}{\text{Var}(\zeta_k(j))} \cdot (r(j) - E(\zeta_k(j)))$$

## The Single-Path Chip-by-Chip Detection Algorithm

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$$\text{Step 1. } E(r(j)) = \sum_{k=1}^K h_k E(x_k(j)) \quad \text{Var}(r(j)) = \sum_{k=1}^K |h_k|^2 \text{Var}(x_k(j))$$

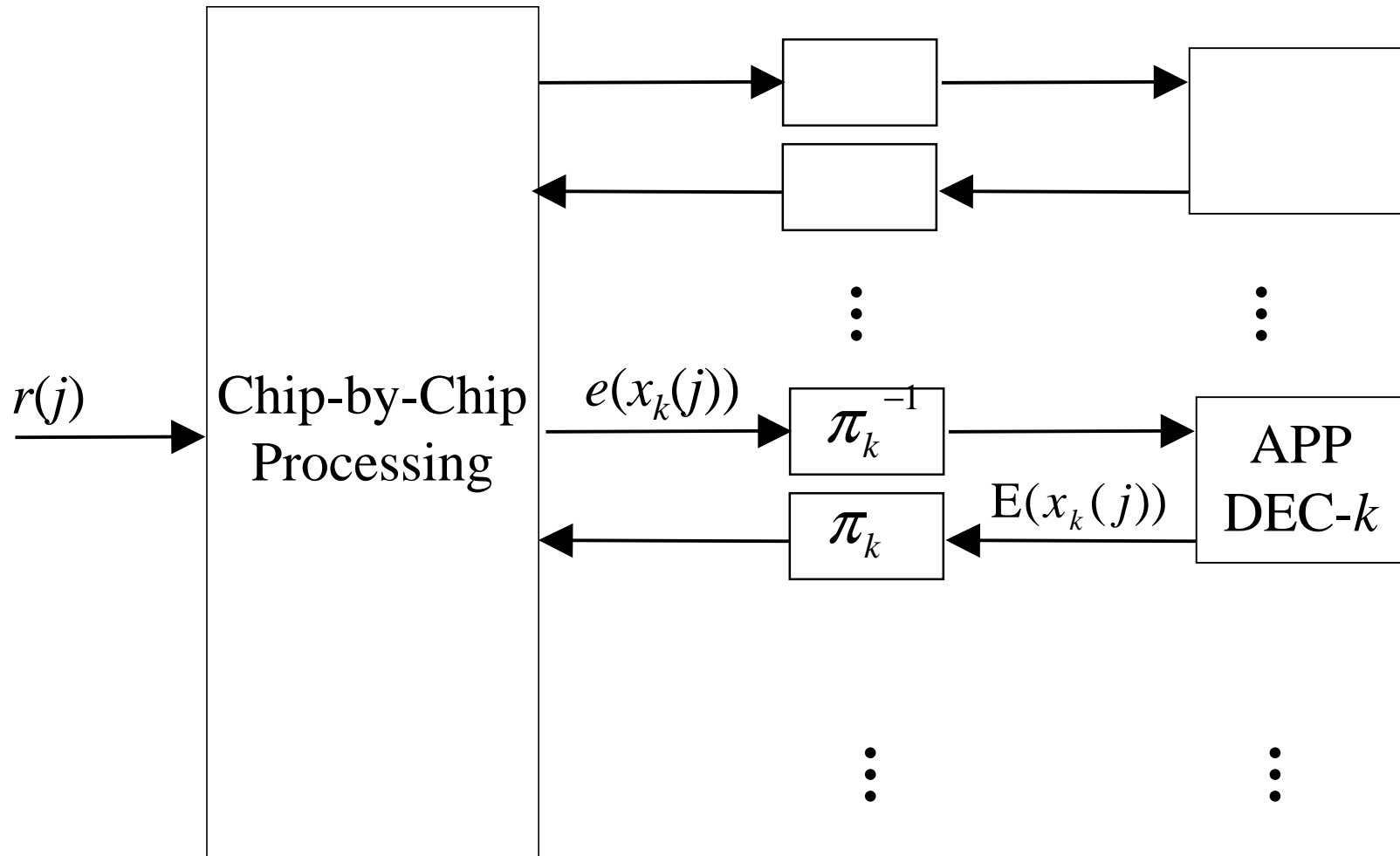
$$\text{Step 2. } E(\zeta_k(j)) = E(r(j)) - h_k E(x_k(j)) \quad \text{Var}(\zeta_k(j)) = \text{Var}(r(j)) - |h_k|^2 \text{Var}(x_k(j))$$

$$\text{Step 3. } e(x_k(j)) = \frac{2h_k}{\text{Var}(\zeta_k(j))} \cdot (r(j) - E(\zeta_k(j)))$$

Notes:

- (1) This is an extremely simplified version of Wang-Poor Algorithm.
- (2) No matrix operations.

# Chip-by-Chip Multiuser Detection Again



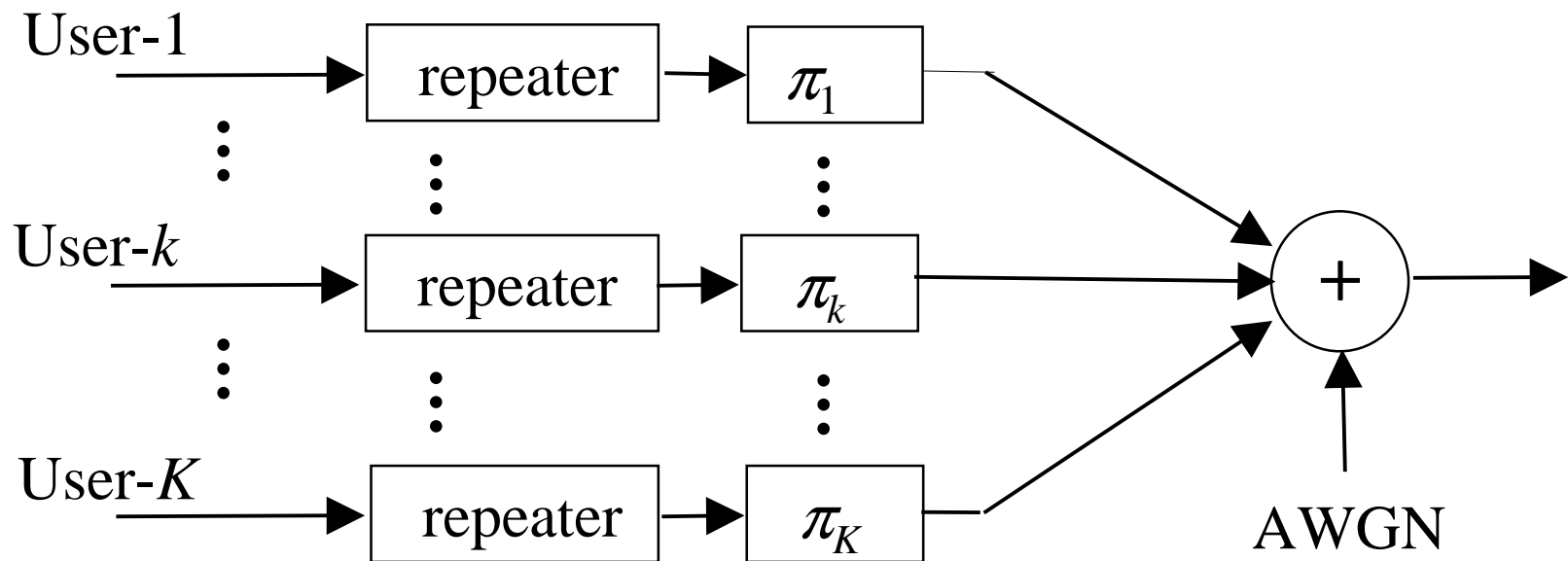
# Complexity

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- 6 additions and 6 multiplications **per chip** per iteration **per user**.
- Complexity (per user) is independent of user number  $K$  .

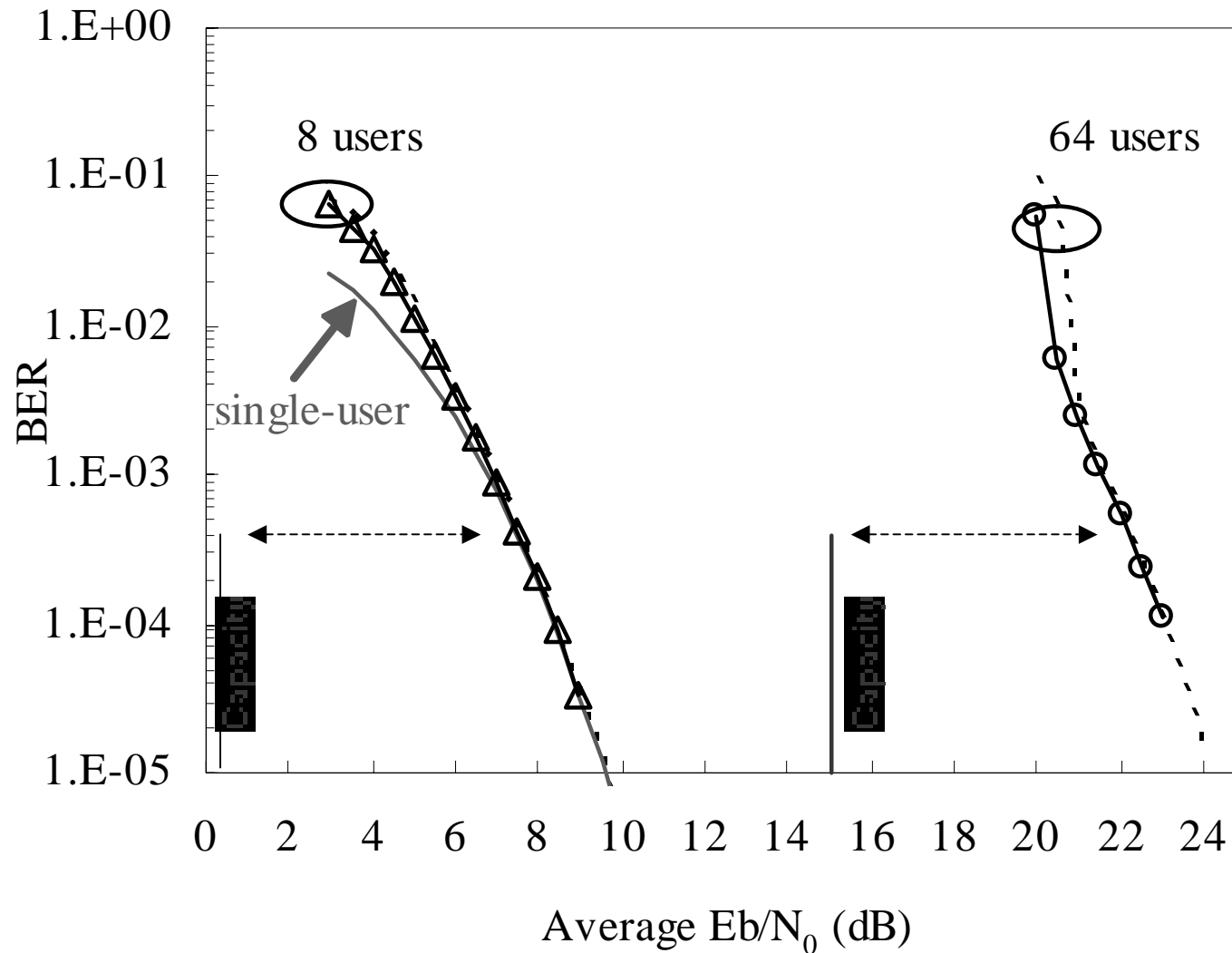
**Comparison:** To achieve good performance, the cost for MMSE CDMA multi-user detection is  $O(K^2)$  due to matrix operations.

# IDMA with Repetition Coding

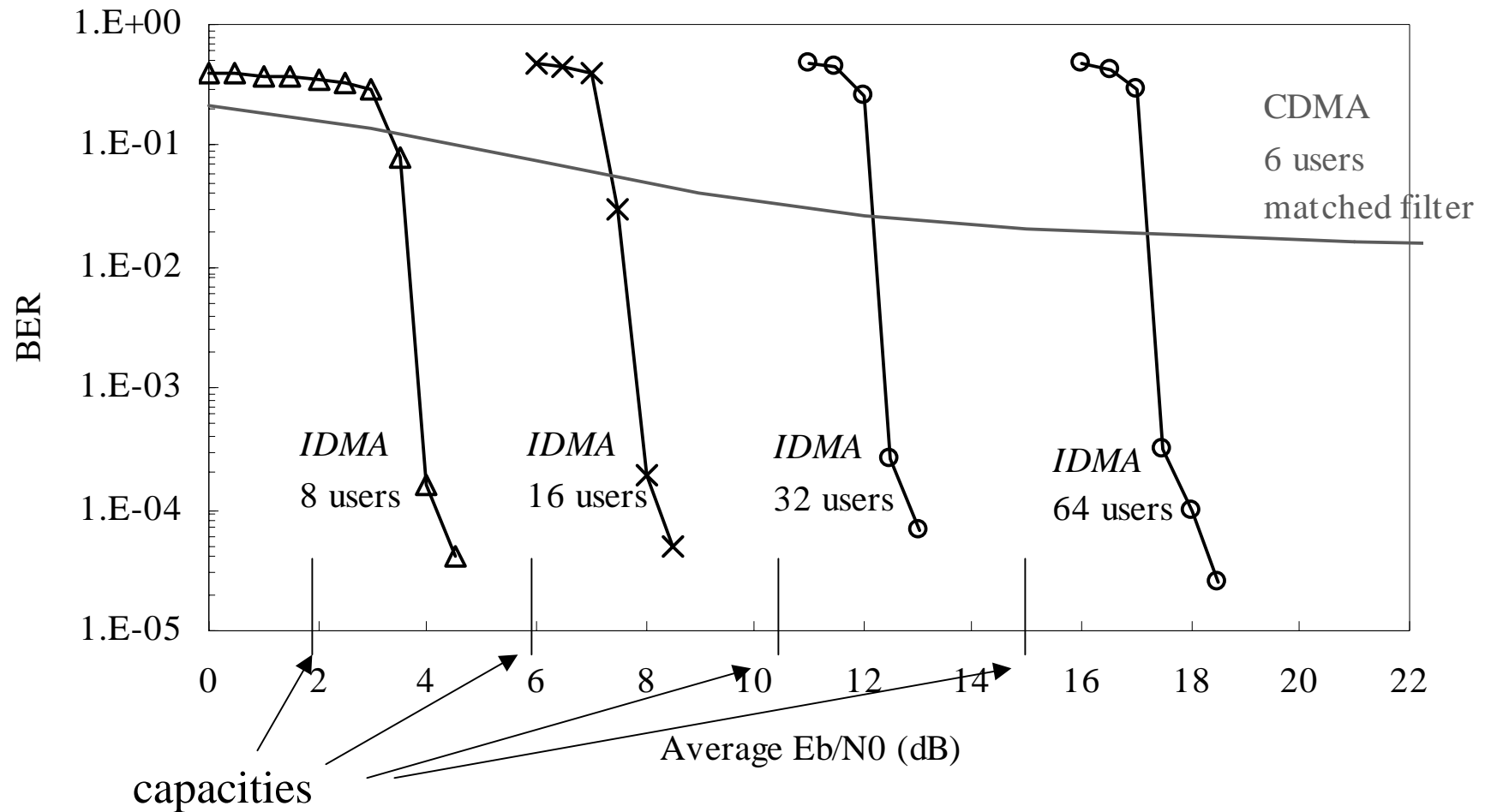




# Un-coded IDMA (with rate-1/8 repetition coding)



# Rate 1/8 Convolutional-Repeat Coded IDMA



# Multuser Detection in Multipath Channels

Step 1. Chip-level path model

$$r(j) = \sum_{l=0}^{L-1} \sum_{k=1}^K h_{k,l} x_k(j-l) + n(j)$$

Step 2. Gaussian approximation

$$r(j) = h_{k,l} x_k(j-l) + \zeta_{k,l}(j)$$

Step 3. Estimation:

$$e(x_k(j-l))_l = \frac{2 \cdot h_{k,l}}{\text{Var}(\zeta_{k,l}(j))} \cdot (r(j) - \text{E}(\zeta_{k,l}(j)))$$

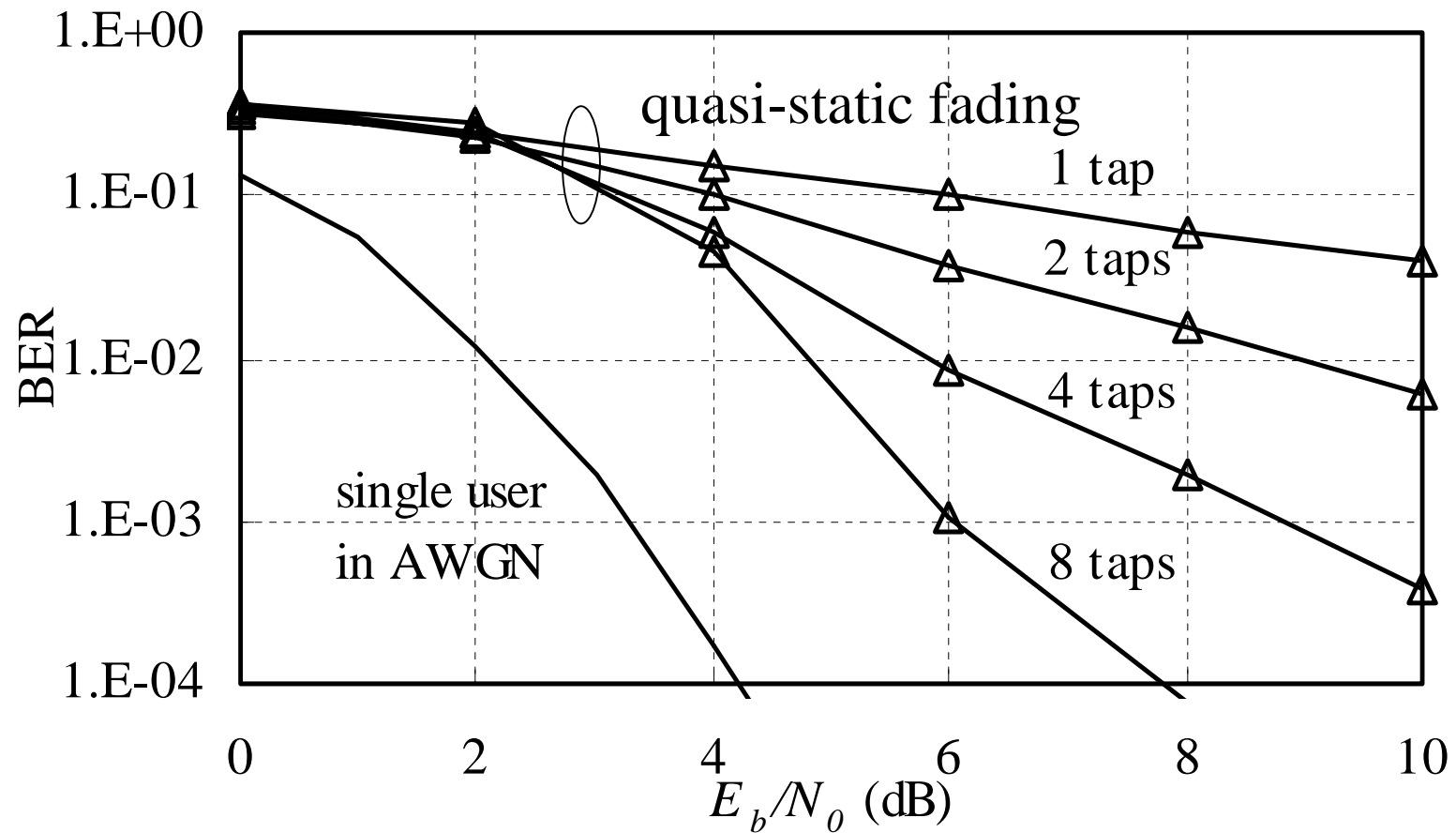
Step 4. Rake combining:

$$e(x_k(j)) = \sum_{l=0}^{L-1} e(x_k(j))_l$$

Note: Still no matrix operations here.

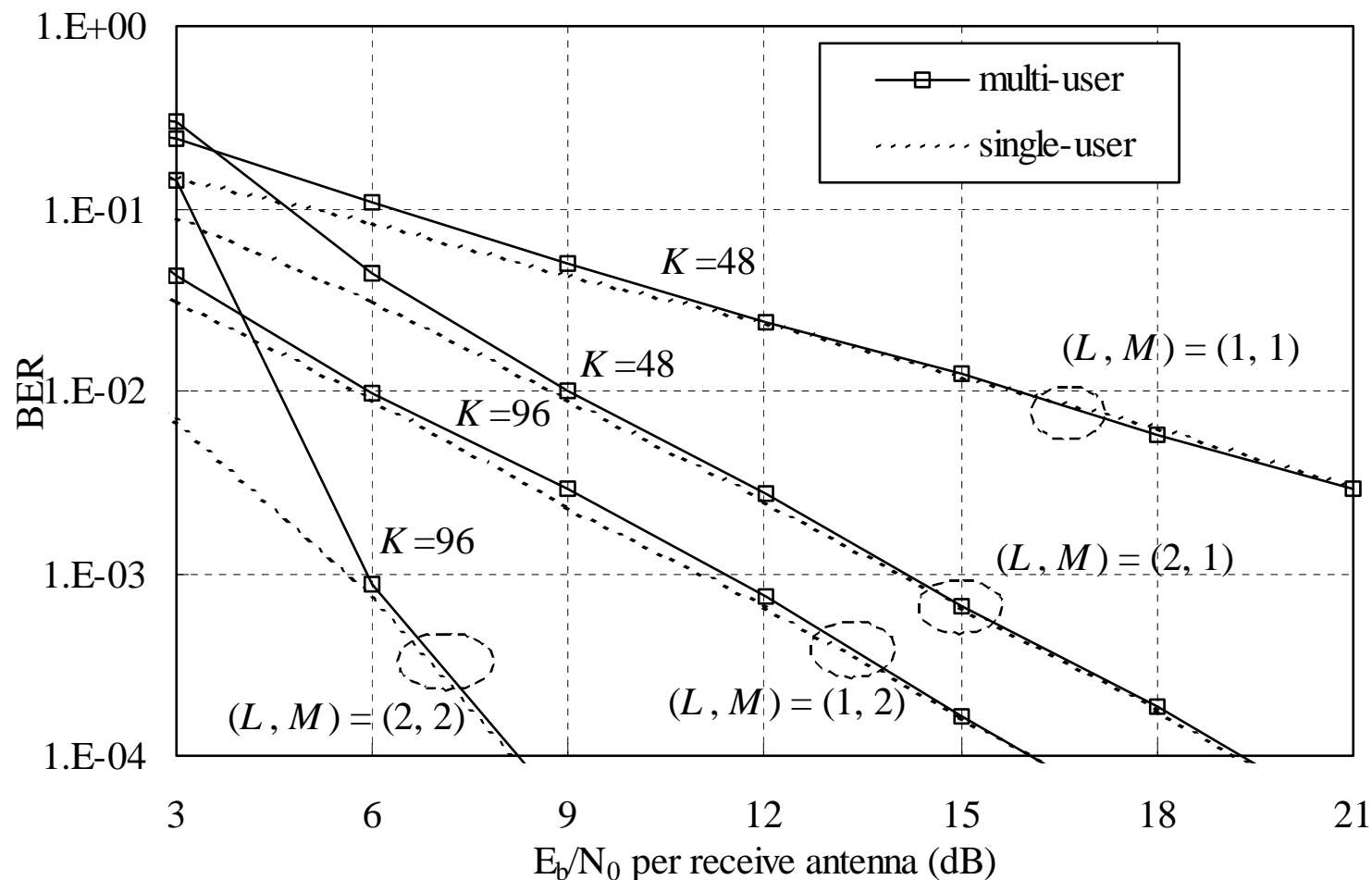
# Rake Detector in Multipath Channels

(rate 1/2 convolutional & length-8 repetition, 32 users)



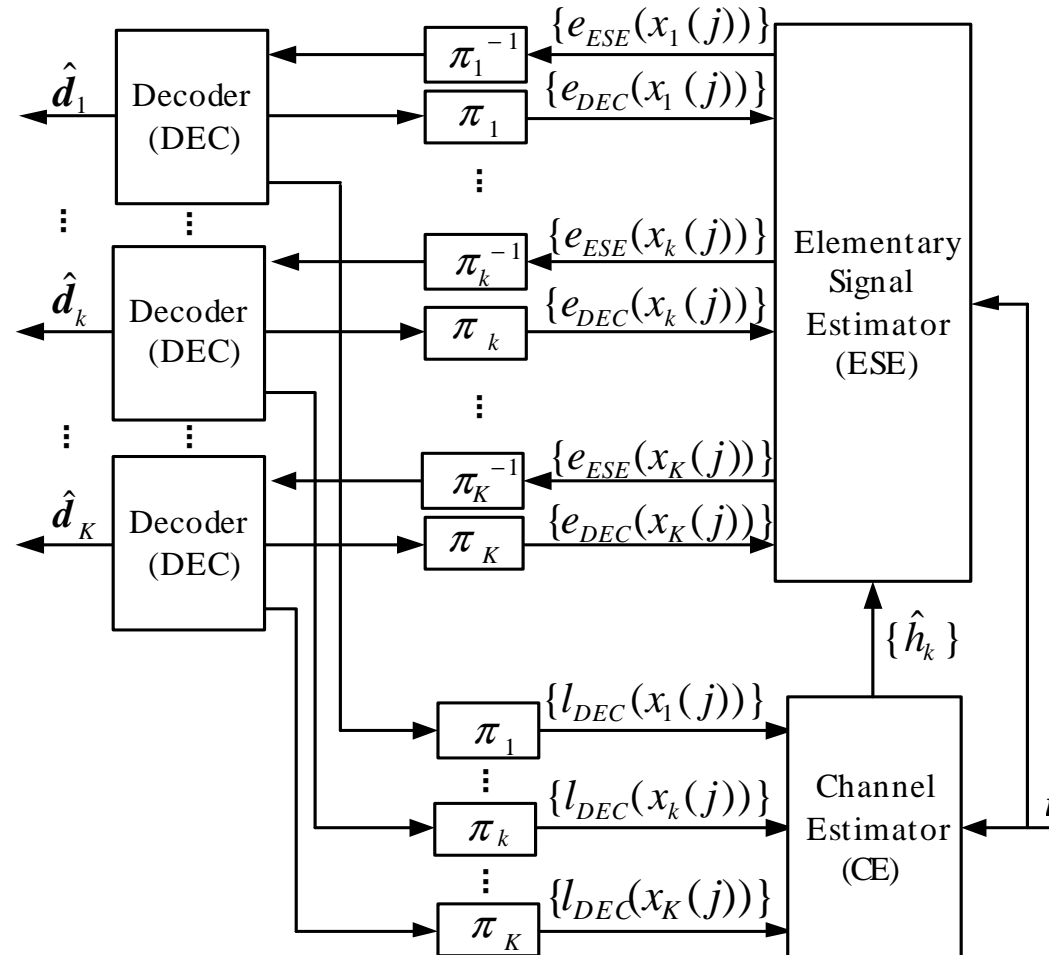
# Multipath Performance

(rate-1/2 convolutional & length-8 repetition)

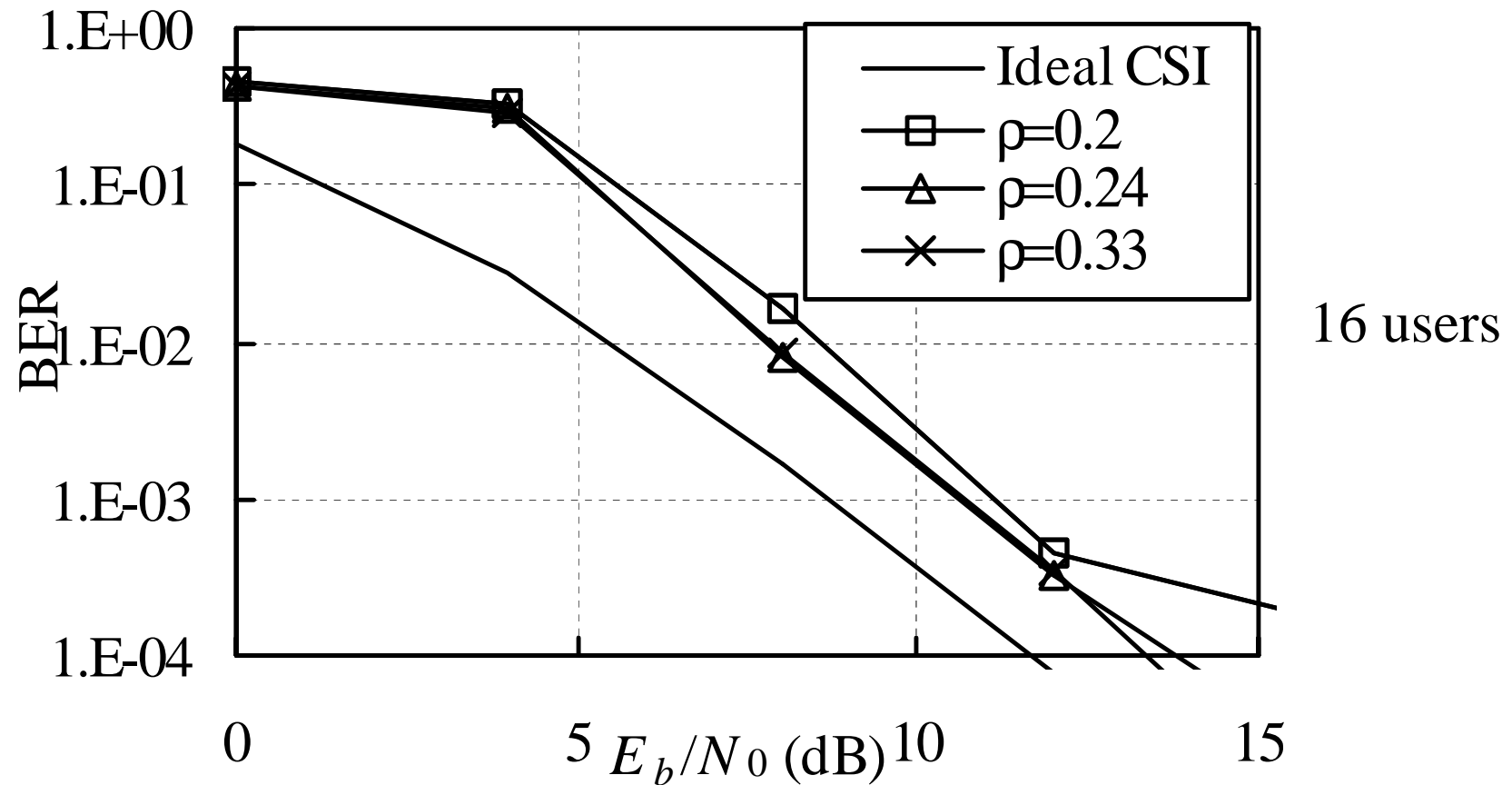


$L$ = the number of taps.  $M$  = the number of receive antennas.  $K$ =the number of users

# Chip-by-Chip Joint Channel Estimation and Multi-User Detection



# Performance with Joint Channel Estimation and Multi-user Detection



$E_b$  includes the pilot overhead.

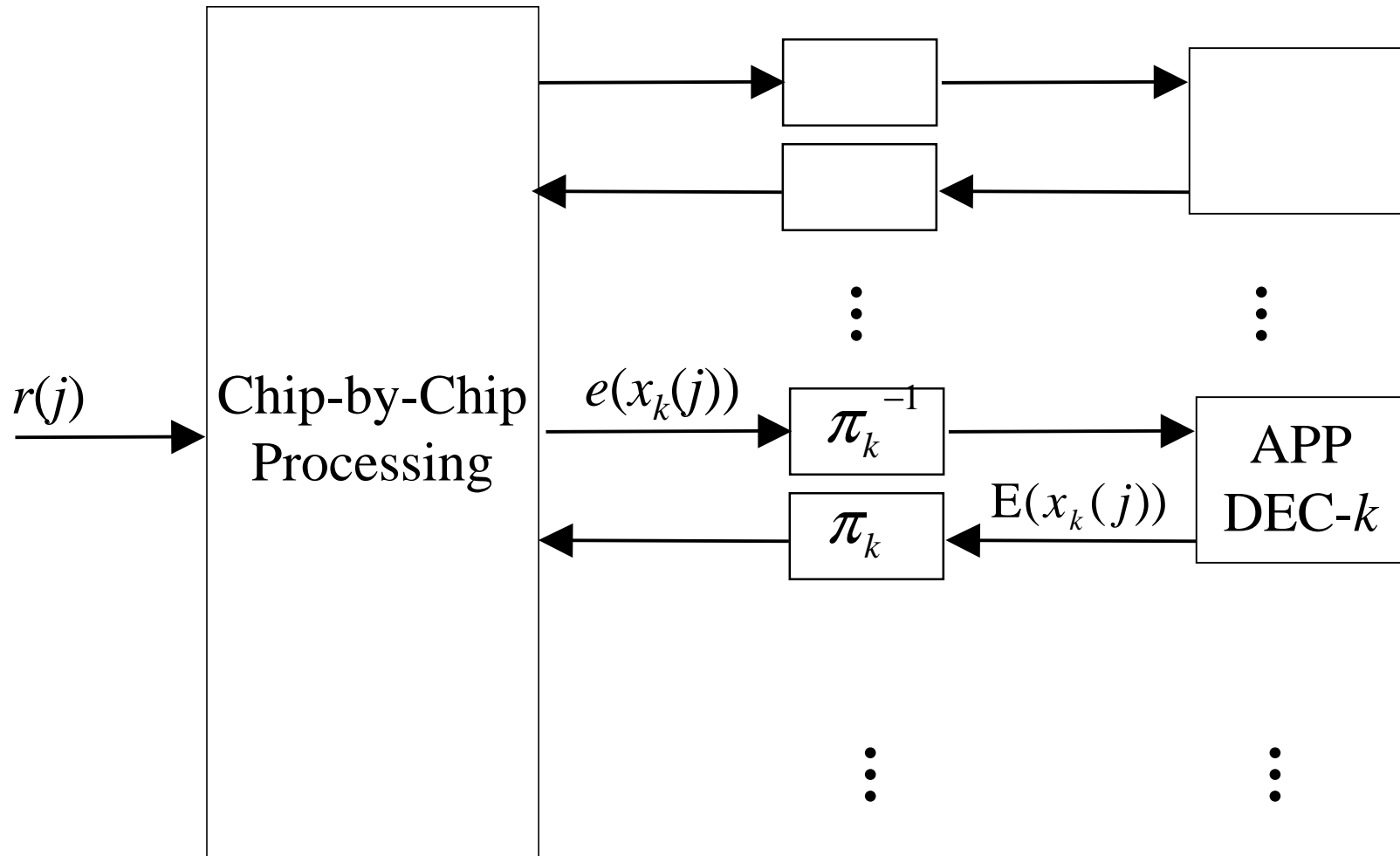
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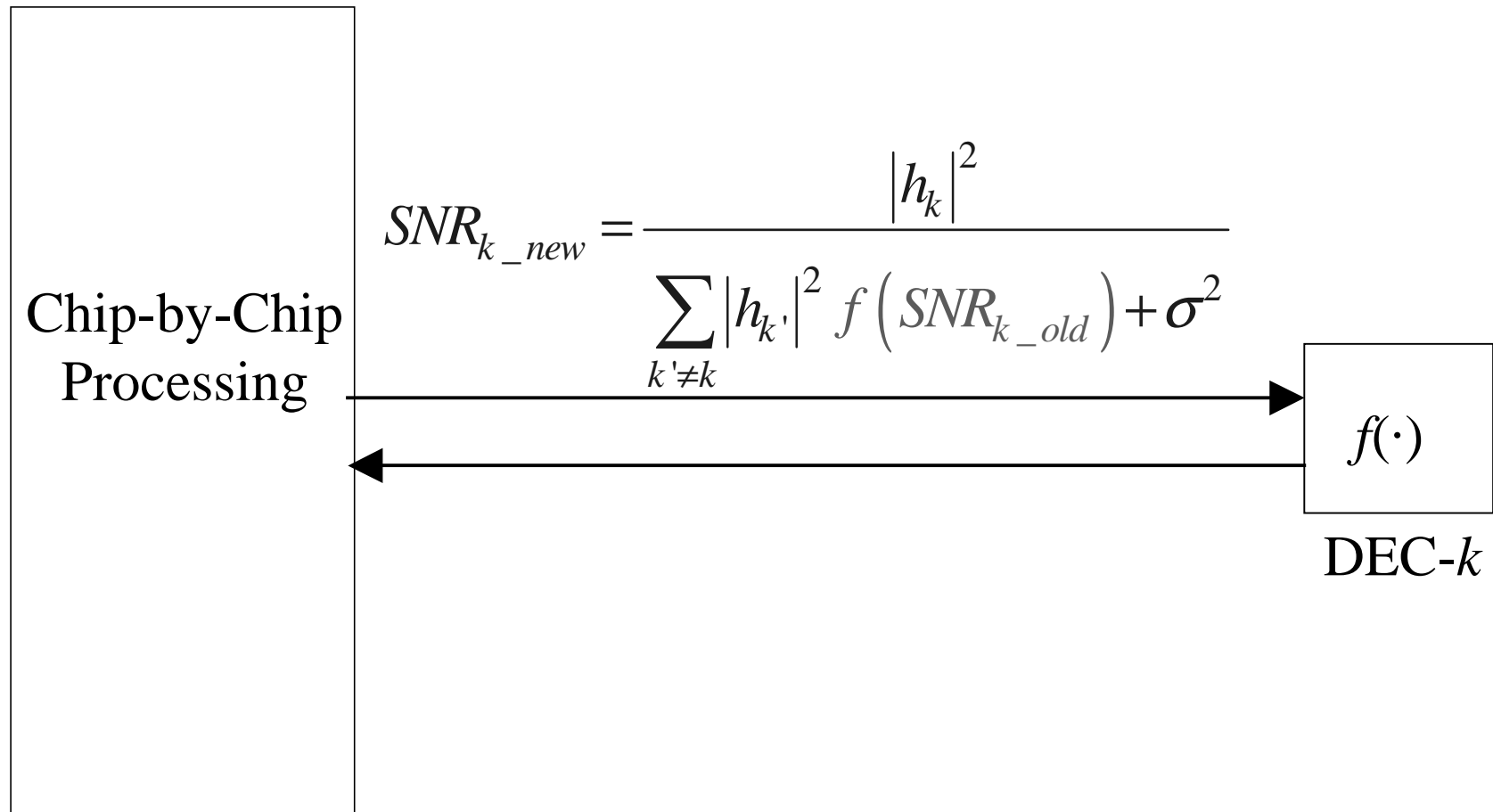
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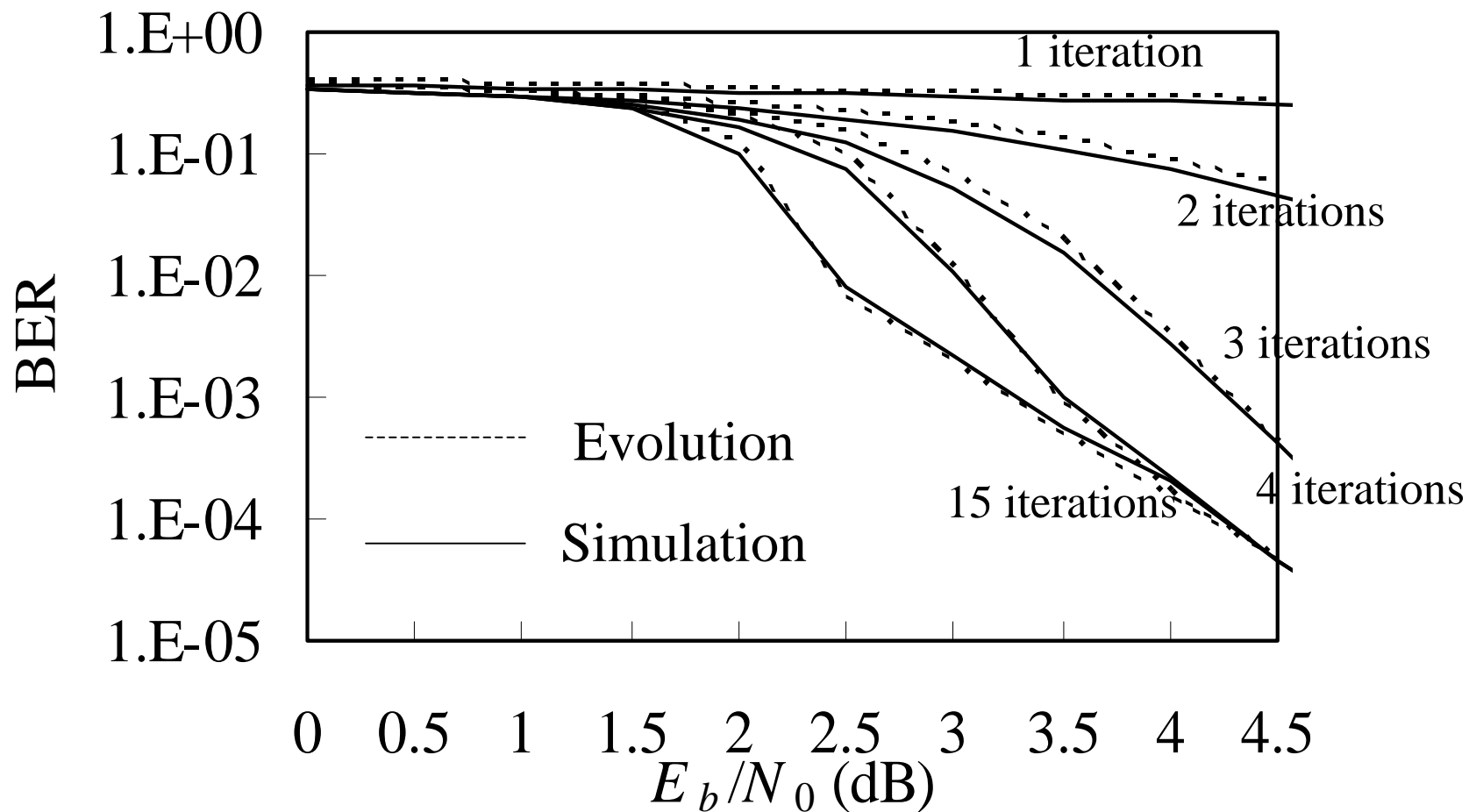
# Chip-by-Chip Multiuser Detection Again



# SNR Evolution in the Chip-by-Chip Algorithm



# Number of Iterations Required by IDMA (24 users, 1/2 convolutional + 1/8 repetition coding)



# Power Allocation for Non-ideal Coding

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## Optimization:

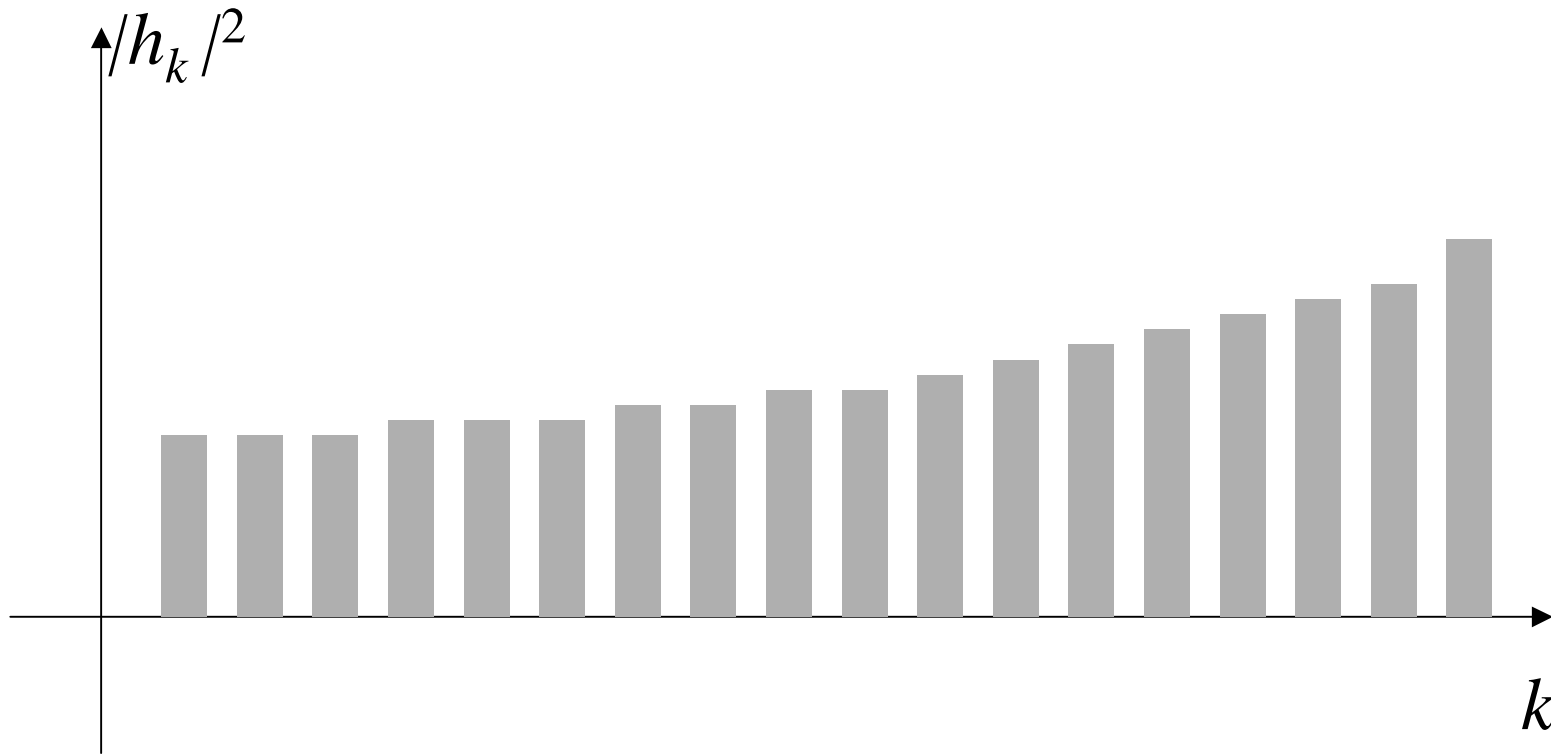
Find  $\{h_k\}$  to maximize  $\{SNR_k\}$  after certain iterations.

$$SNR_{k\_new} = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 f(SNR_{k\_old}) + \sigma^2}$$

**Constraint:**  $\sum |h_k|^2 = \text{fixed}$

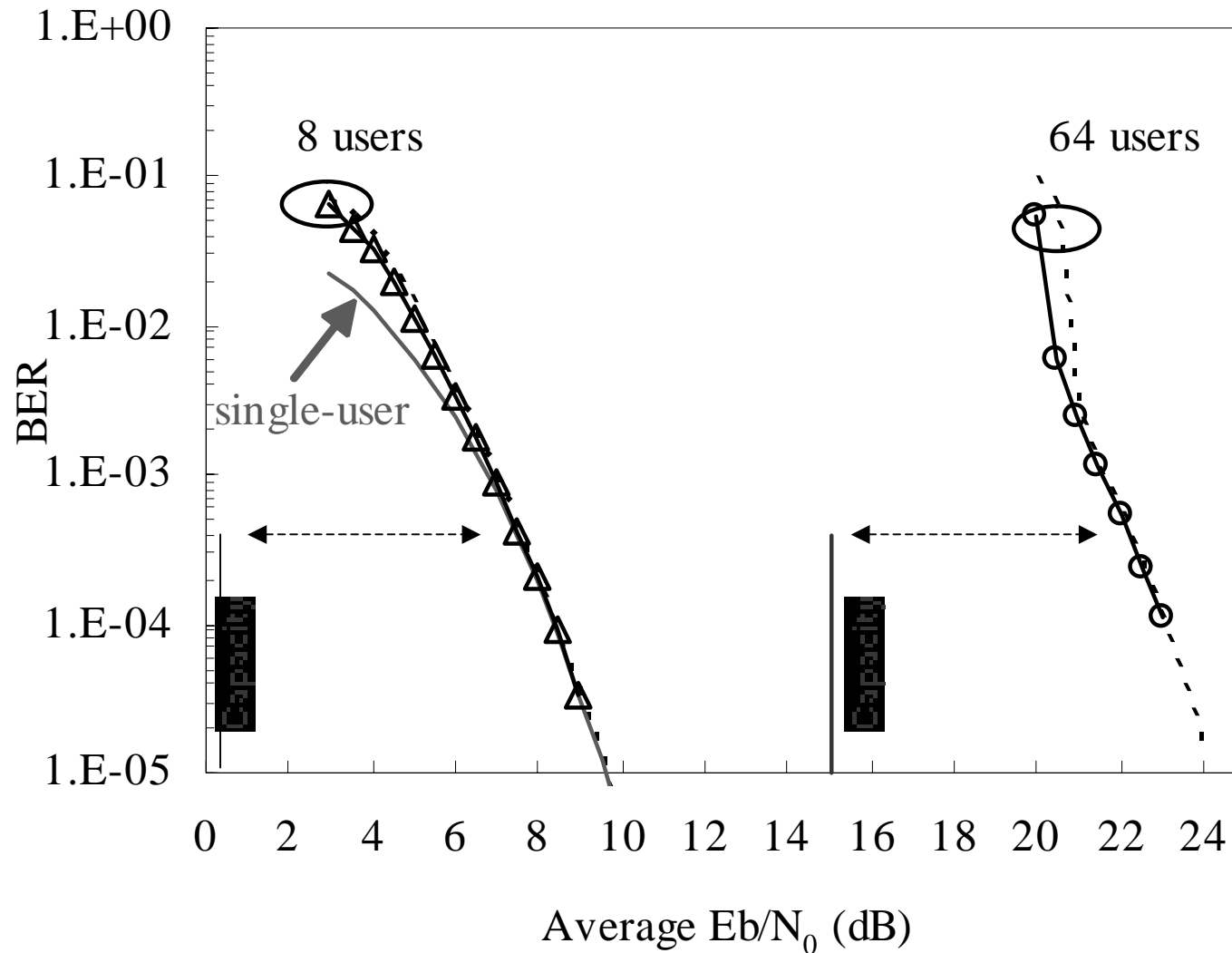
# Power Allocation for Different Users

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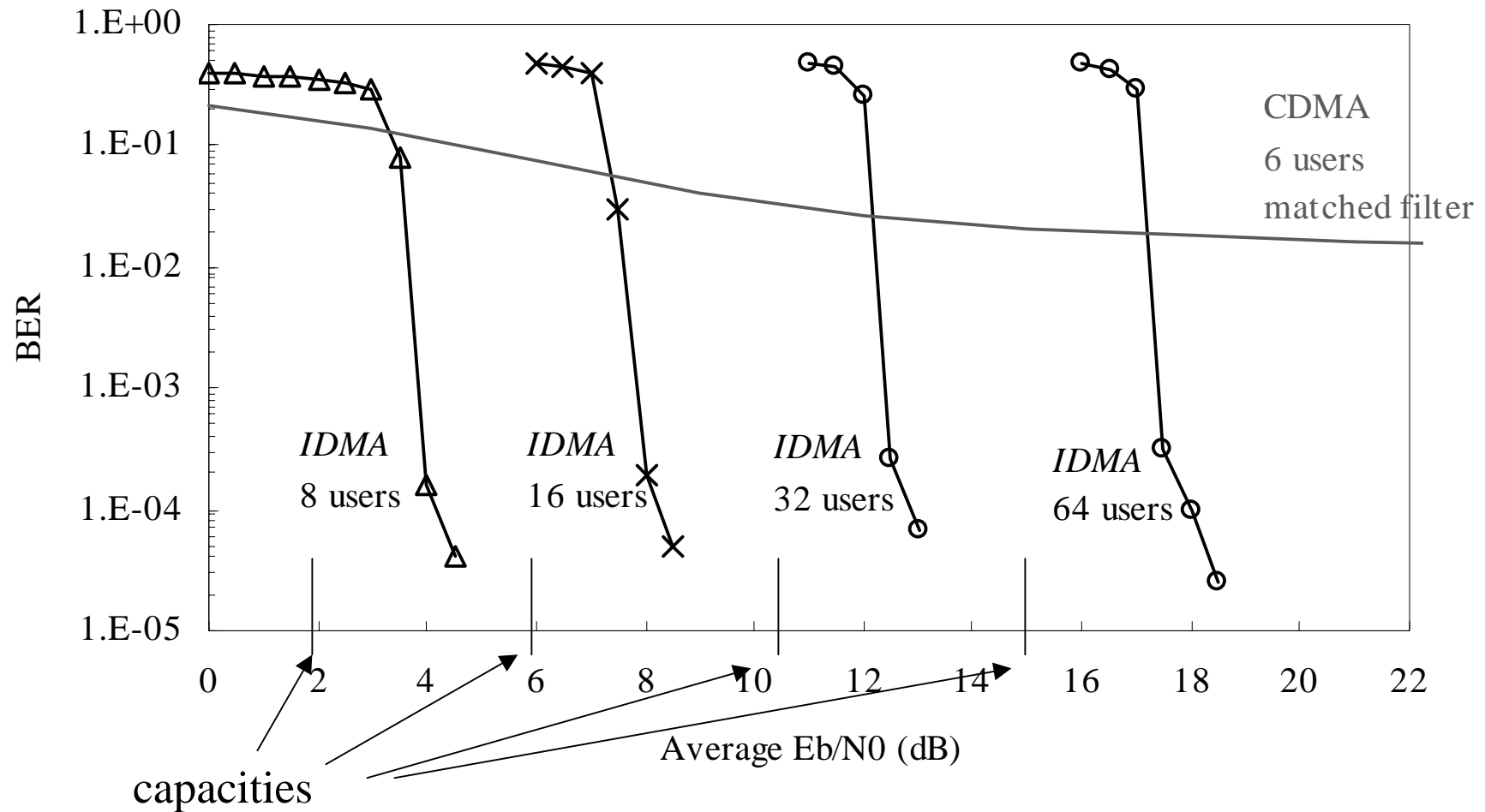


# Un-coded IDMA

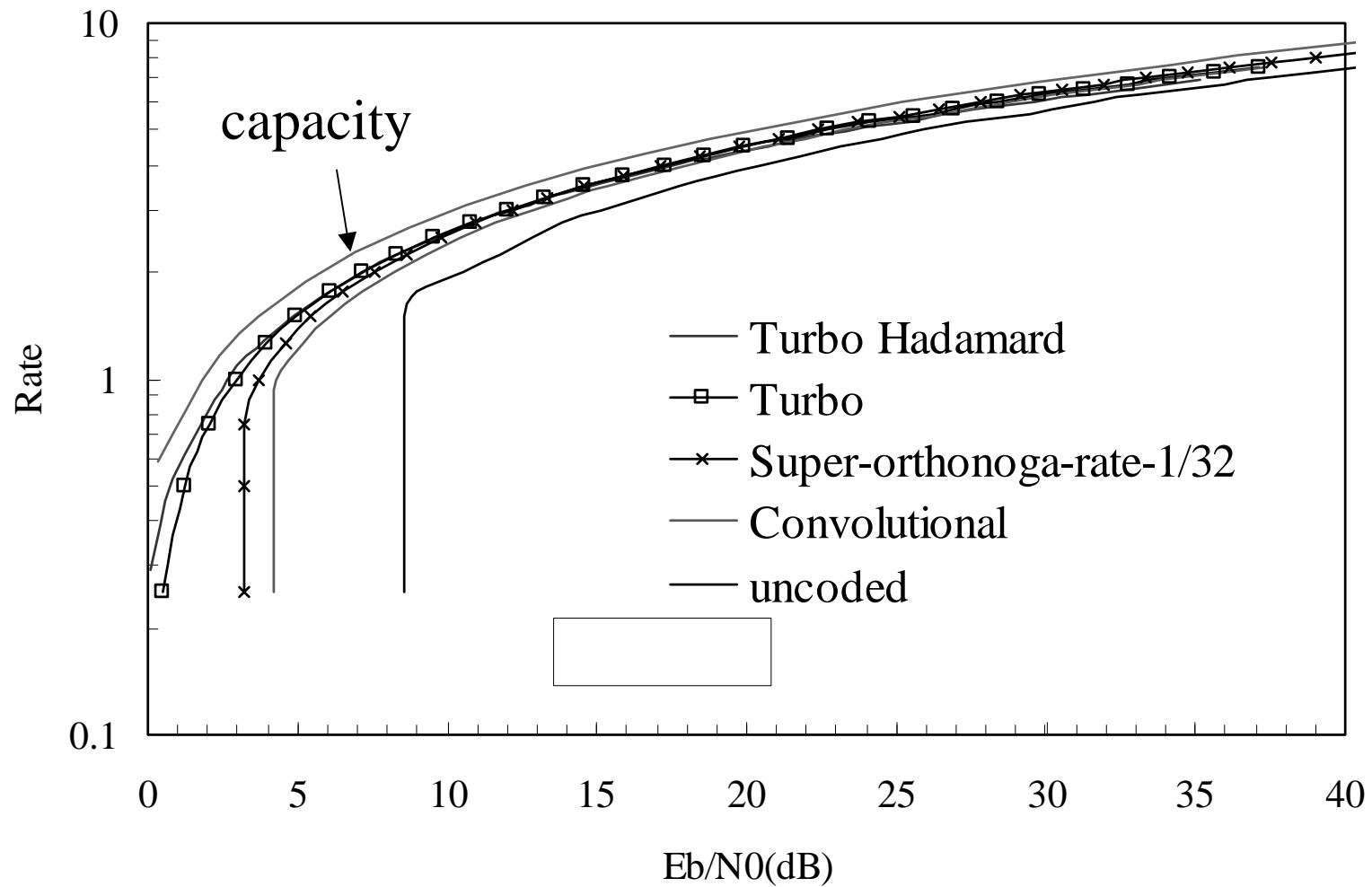
(with rate-1/8 repetition coding)



# Rate 1/8 Convolutional-Repeat Coded IDMA



# Impact of FEC Coding on IDMA





# Spectral Efficiency

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1/8 repeating with 64 users, spectral efficiency = 8bits/chip.

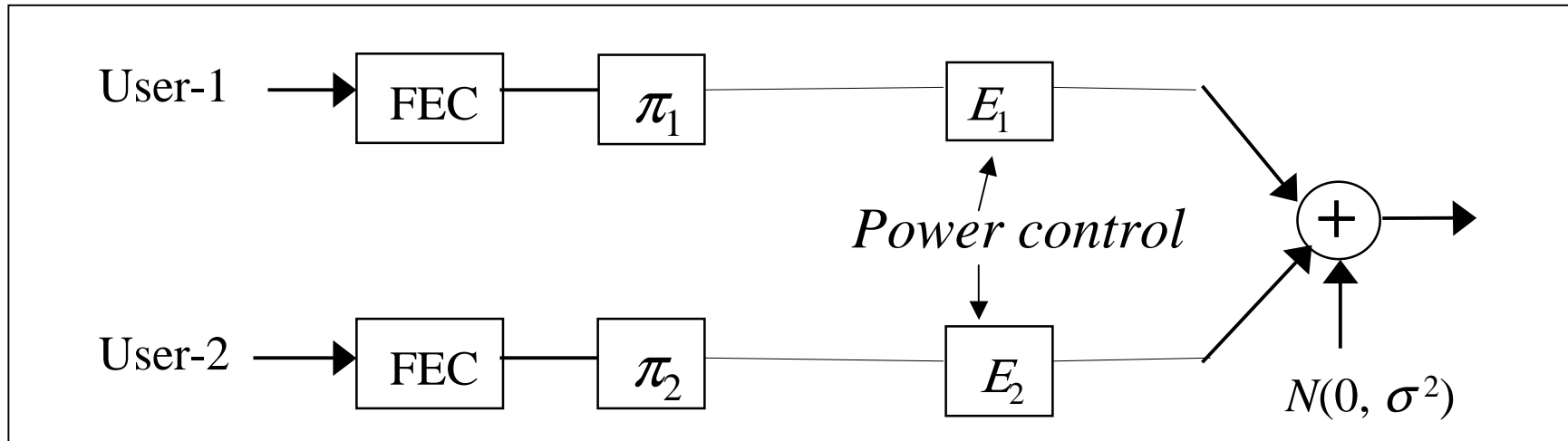
Equivalent to single-user 256-QAM.

Comparison: IS-95 CDMA efficiency ?

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**with ideal coding**

# with ideal coding



Achieving overall capacity

Onion-peeling capacity

$$C = \log\left(1 + \frac{E_1 + E_2}{\sigma^2}\right) = \log\left(1 + \frac{E_1}{\sigma^2}\right) + \log\left(1 + \frac{E_2}{\sigma^2 + E_1}\right)$$

Single-user capacity

# with ideal coding

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$$\begin{aligned} & \log\left(1 + \frac{E_1 + E_2 + E_3}{\sigma^2}\right) \\ &= \log\left(1 + \frac{E_3}{\sigma^2 + E_1 + E_2}\right) + \log\left(1 + \frac{E_2}{\sigma^2 + E_1}\right) + \log\left(1 + \frac{E_1}{\sigma^2}\right) \end{aligned}$$

We can achieve multi-user capacity provided that an ideal code is used for every user.

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- multi-media services (e.g., mixed voice and IP),
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IDMA

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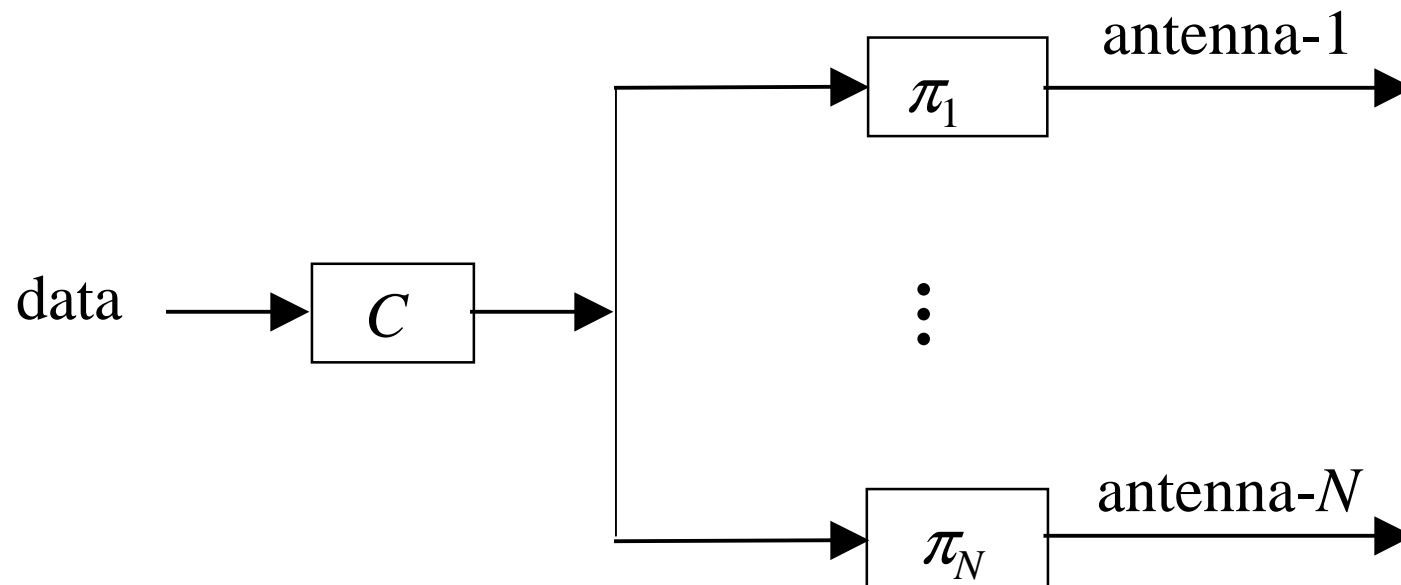
# **Application 1:**

## **IDM Space-Time Coding**



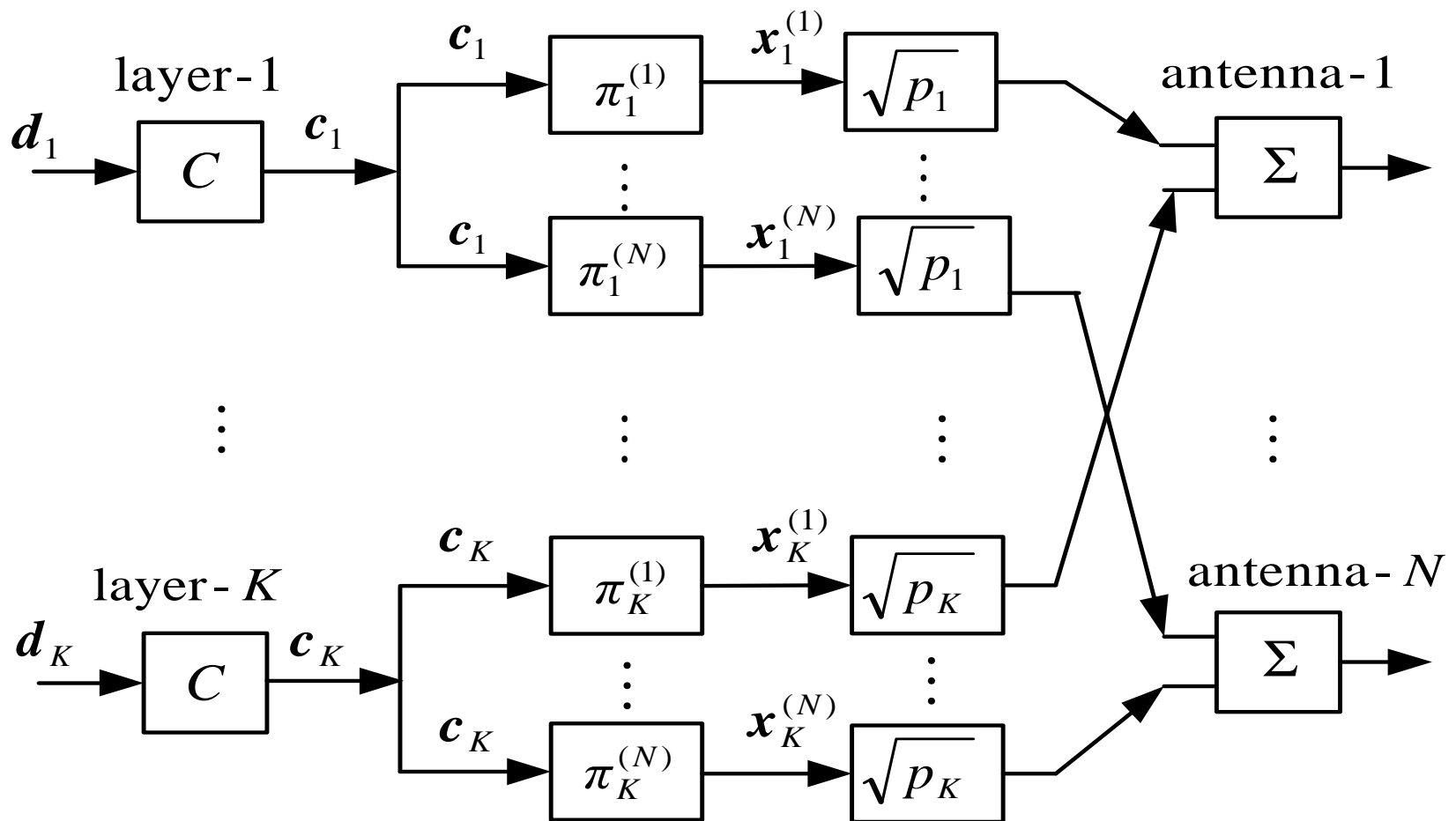
# IDM Space-Time Coding

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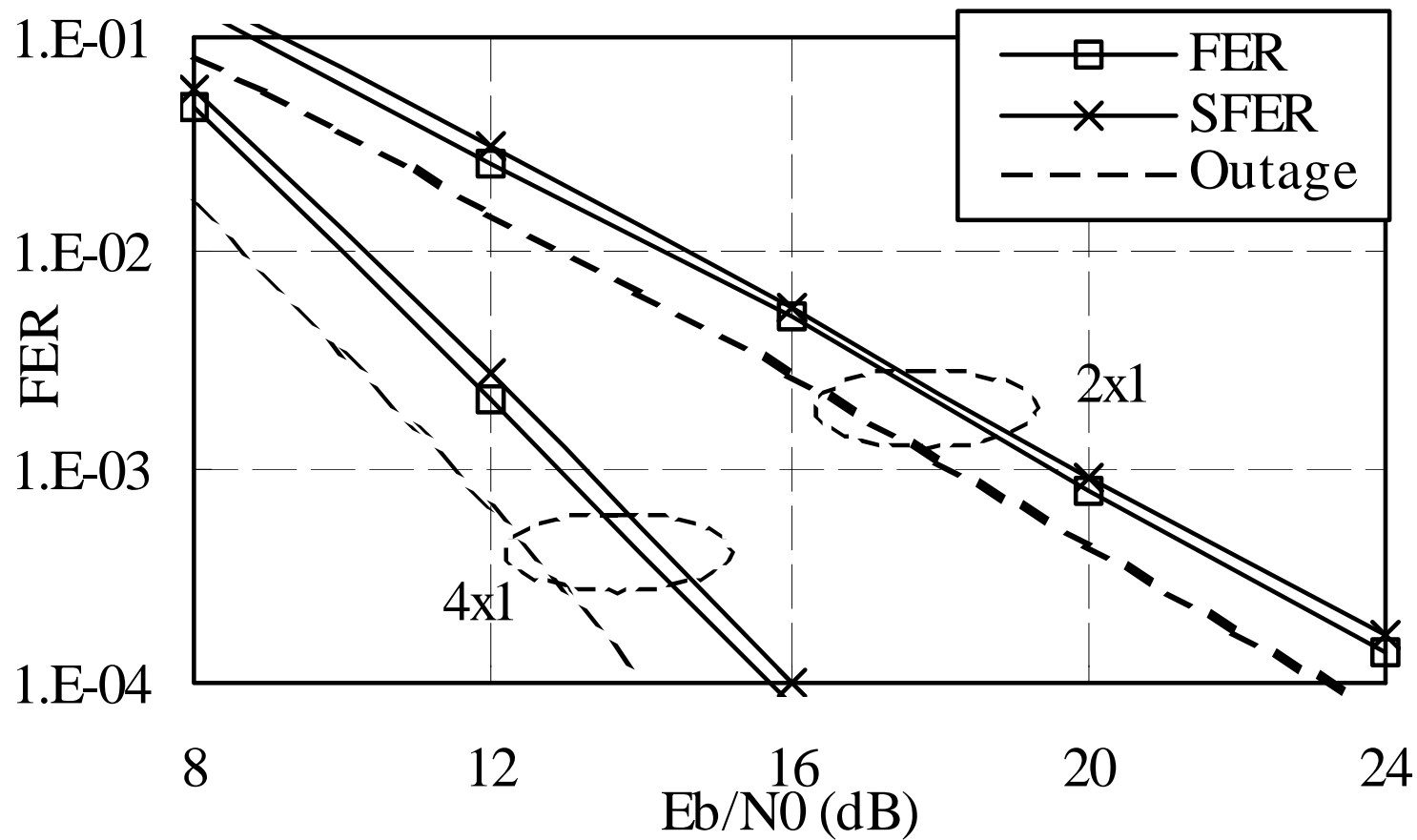


The interleavers  $\pi_1, \dots, \pi_N$  are randomly chosen.

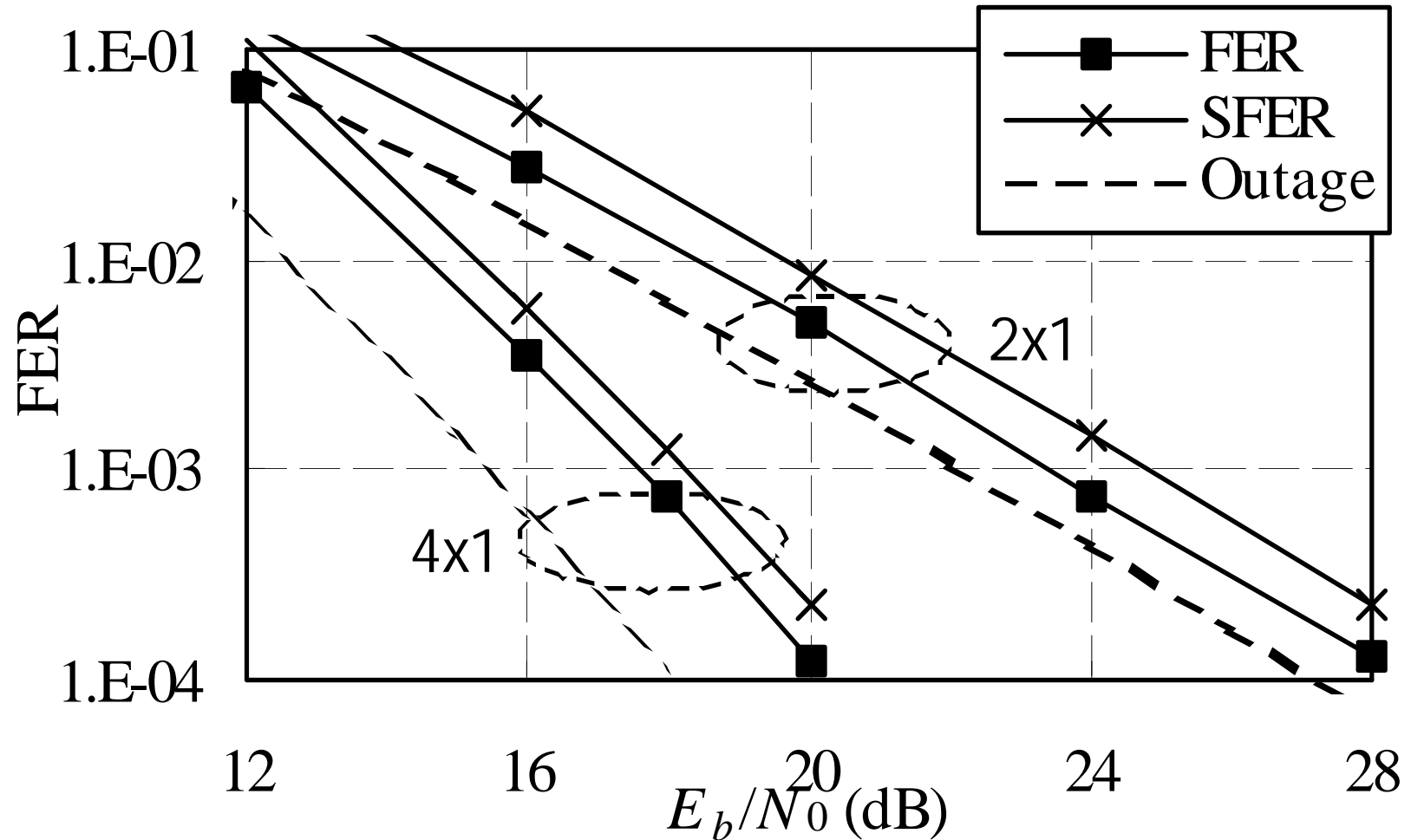
# Multi Layer IDM Space-Time Coding



# Performance of IDM Space-Time Codes (overall rate $R = 2$ bits/symbol)



# Performance of IDM Space-Time Codes (overall rate $R = 4$ bits/symbol)



# Performance Analysis of Space-Time Codes

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For performance analysis of space-time codes, we have to consider all possible fading coefficients  $\{h_n\}$ .

This is usually very difficult, involving multi-dimensional integration over the distribution of  $\{h_n\}$ .

# Performance Bounds of IDM Space-Time Codes

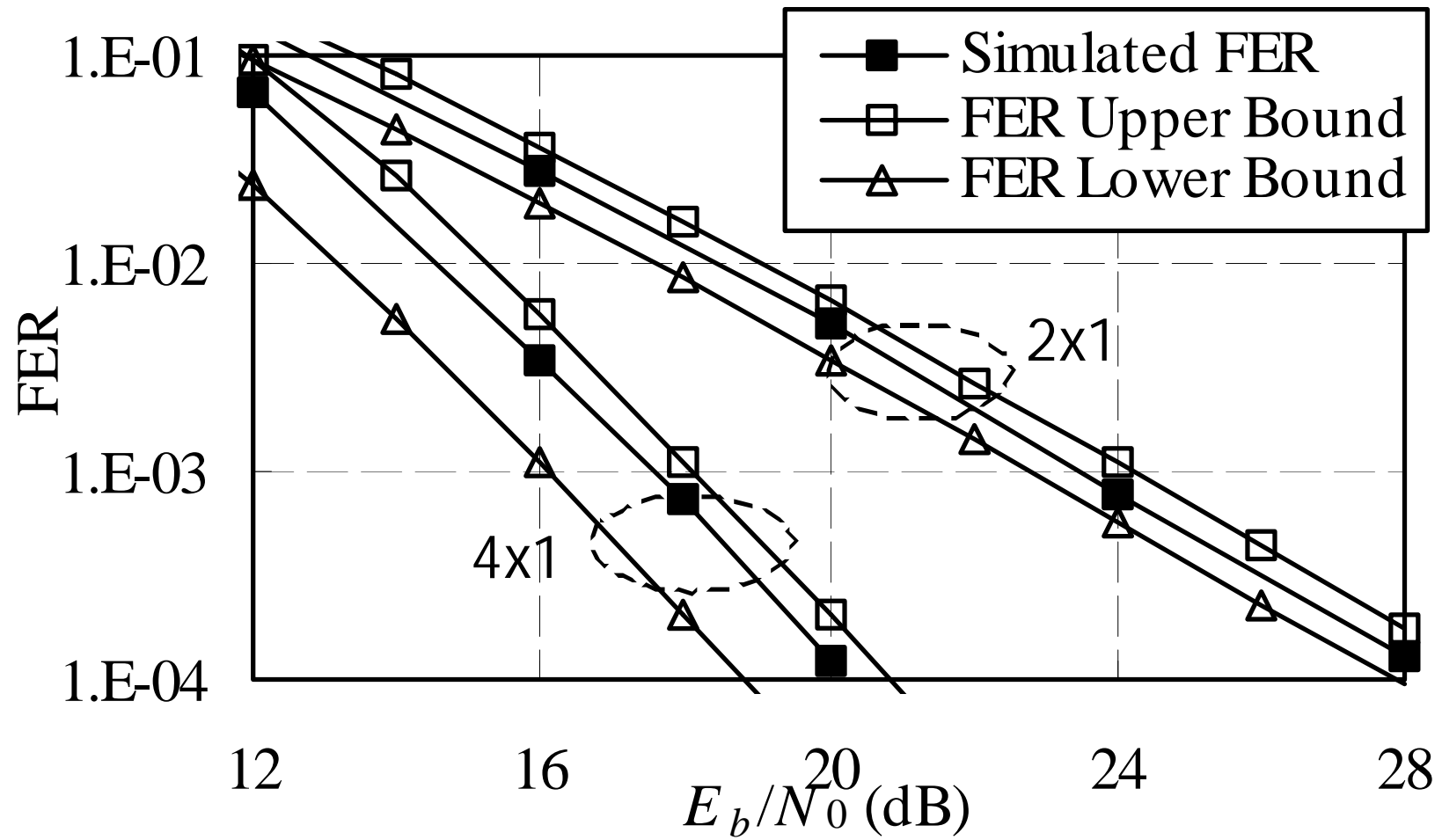
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**Theorem 1:** Worst performance at:  $h_1 = h_2 = \dots = h_N$

**Theorem 2:** Best performance at:  $h_1 = 1,$   
 $h_2 = \dots = h_N = 0$

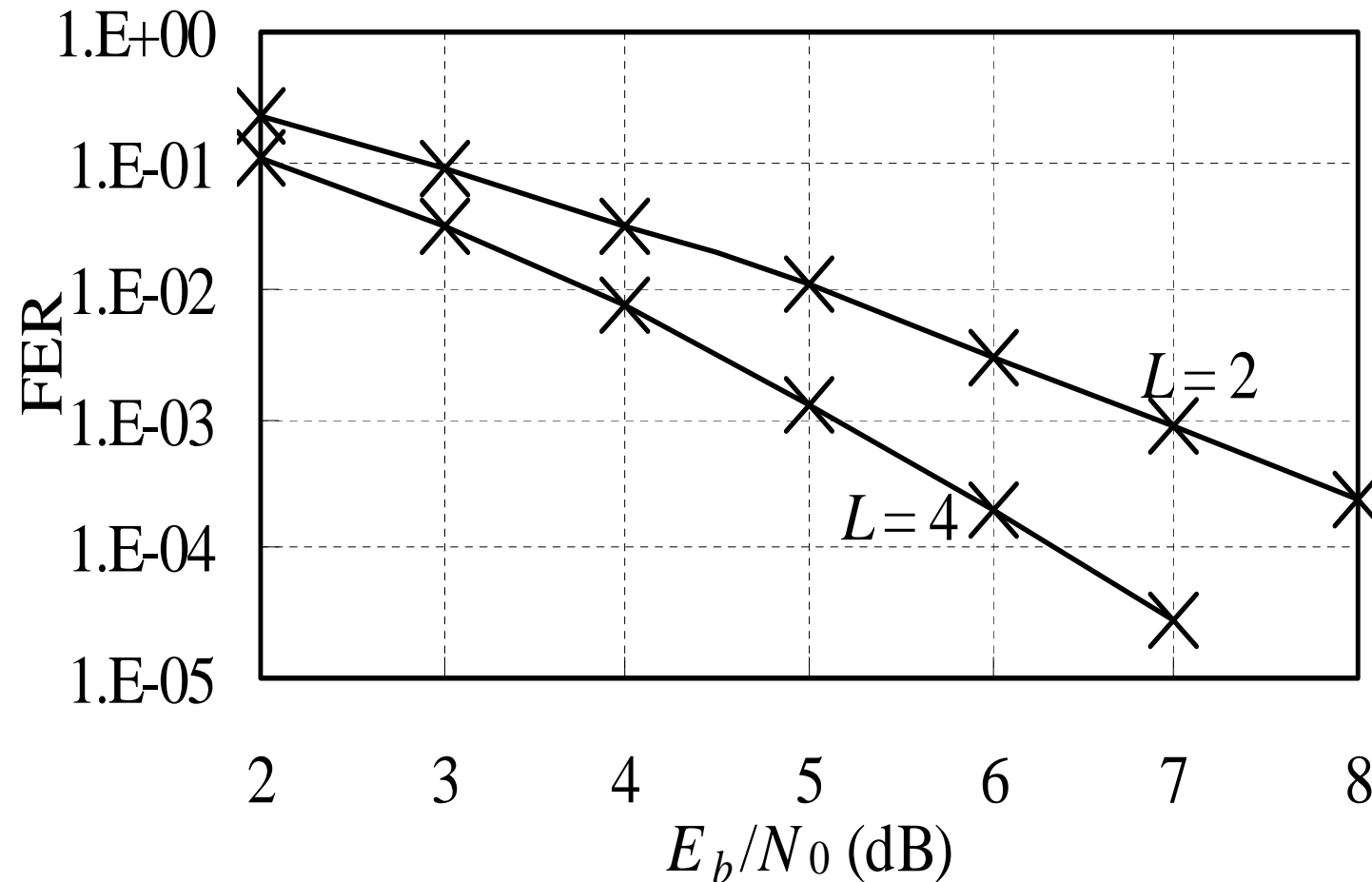
# Performance Bounds

(overall rate  $R = 4$  bits/symbol)



# Performance in Multi-Path Channels

( $R = 2$  bits/symbol,  $2 \times 2$  system)



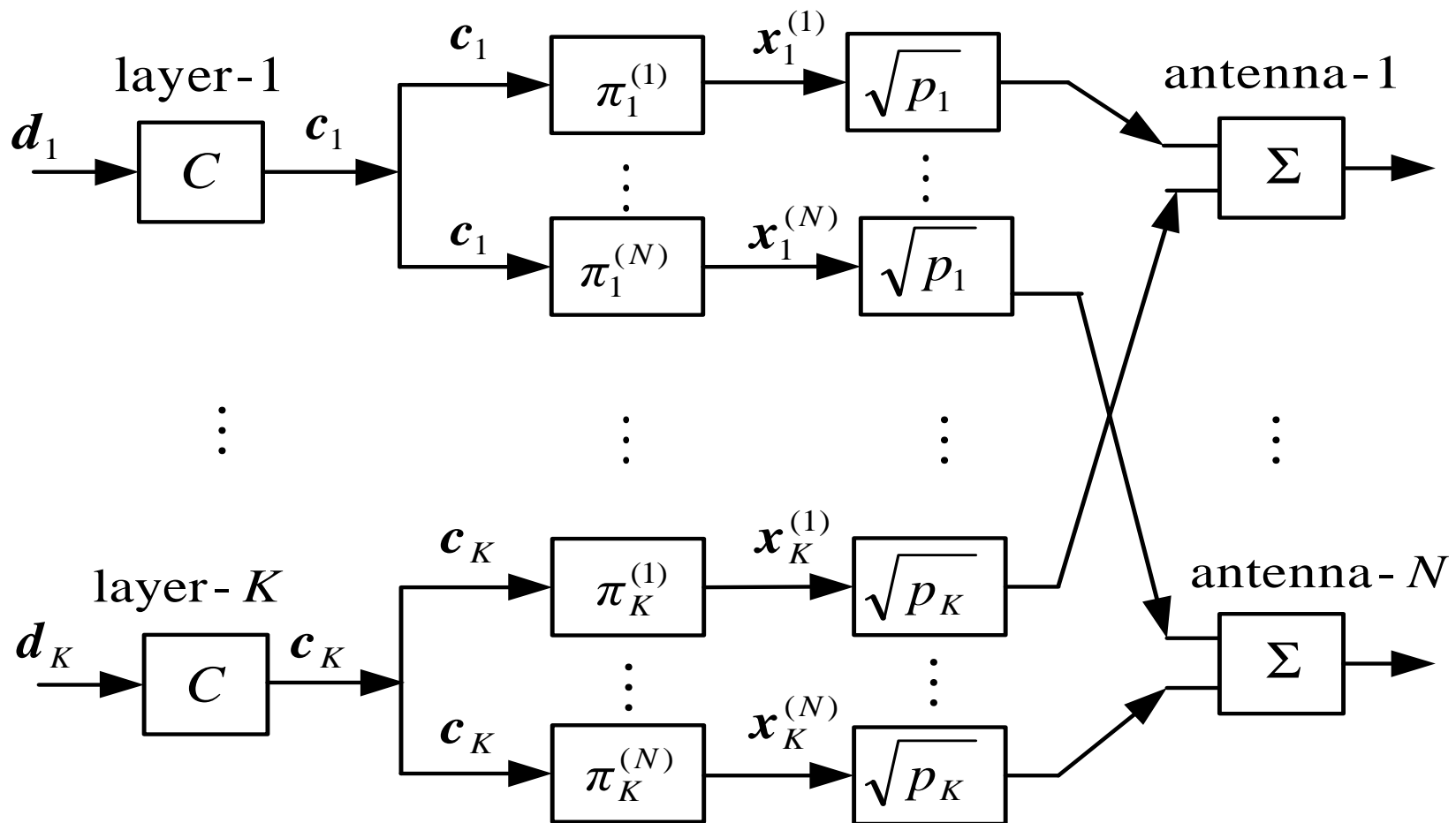


# The Capacity Achieving Property

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An IDM-ST code can achieve capacity  
if  $C$  is low-rate and achieves capacity in AWGN.

# Multi Layer IDM Space-Time Coding



# Summary: Properties of IDM ST Codes

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Conceptually simple.

Potentially capacity achieving.

Low decoding complexity.

Multi-path resolution.

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# **Application 2:**

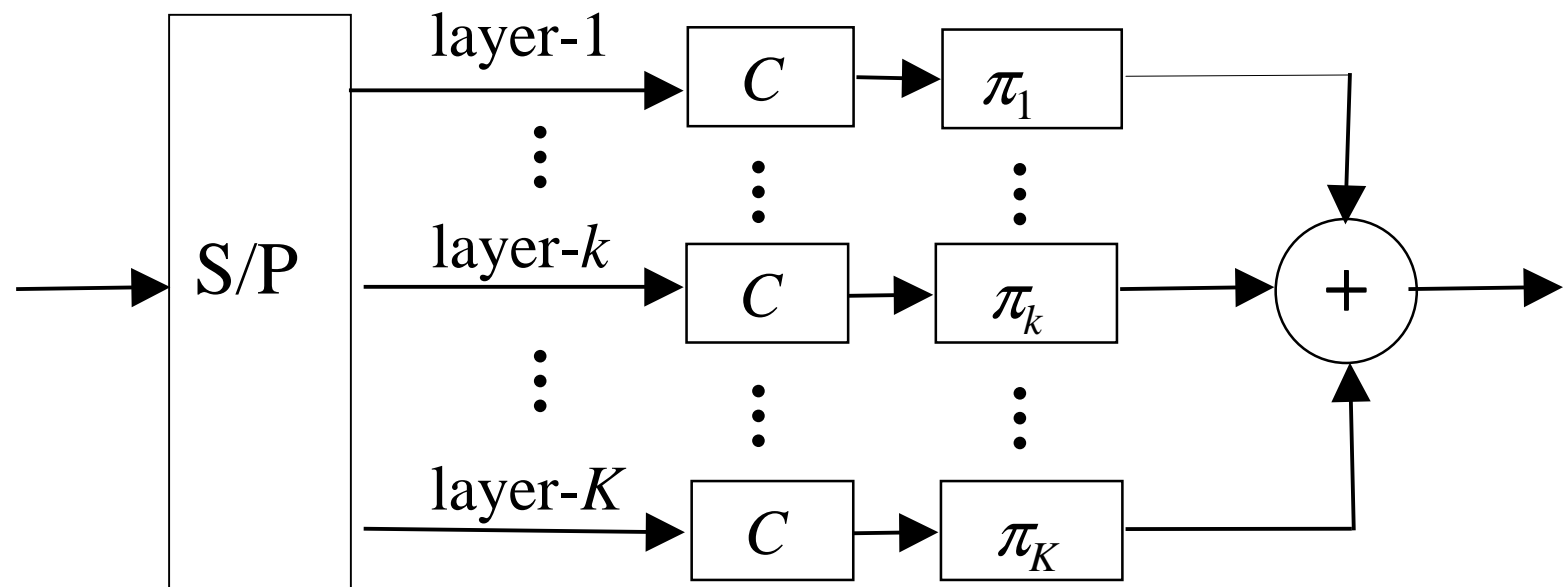
## **IDM Coded Modulation**

# IDM Coded Modulation

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- Sigma mapping: Duan Rimoldi and Urbanke.
- Multi-level codes: Imai and Hirakawa

# IDM Coded Modulation

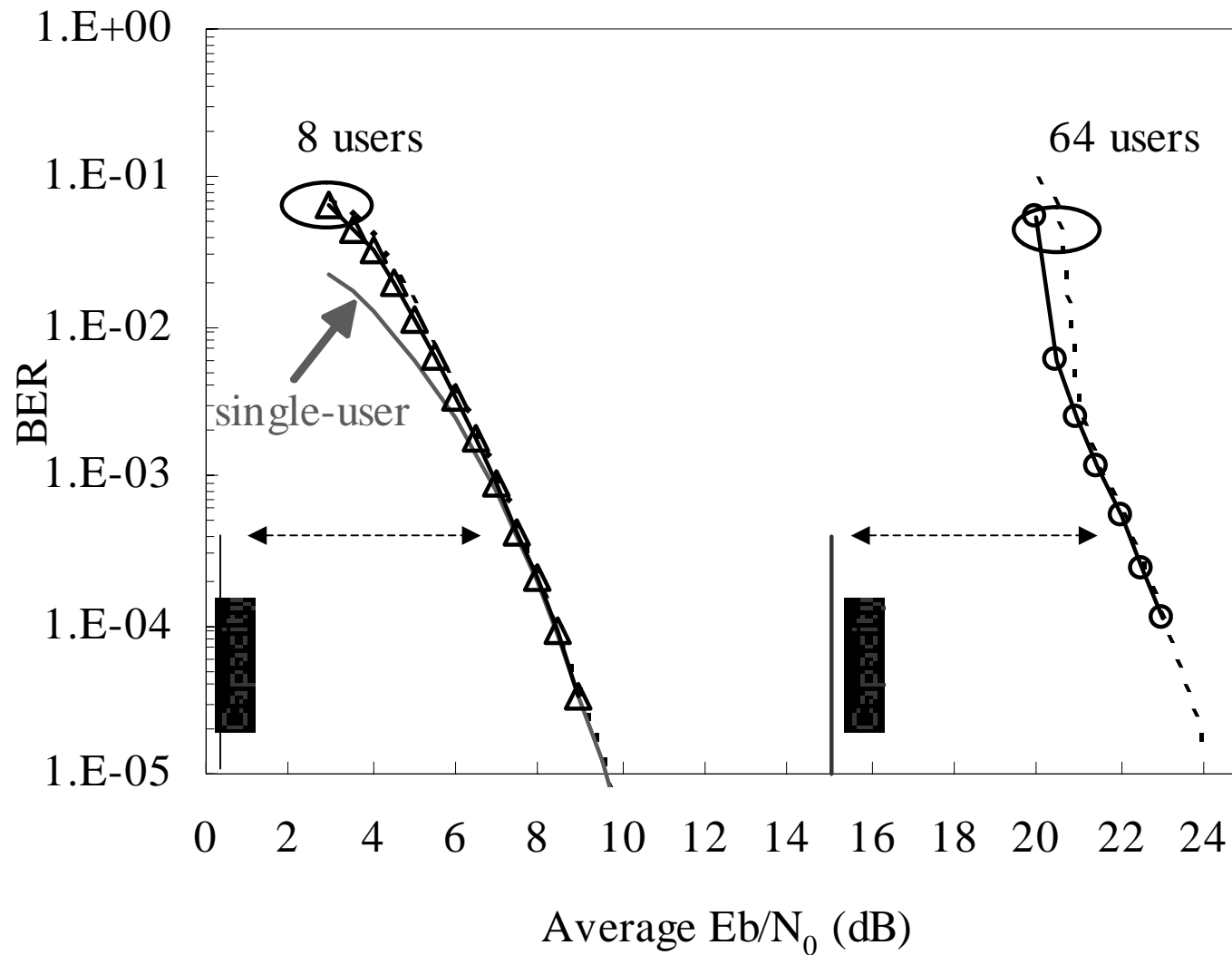


# Advantages of IDM Coded Modulation

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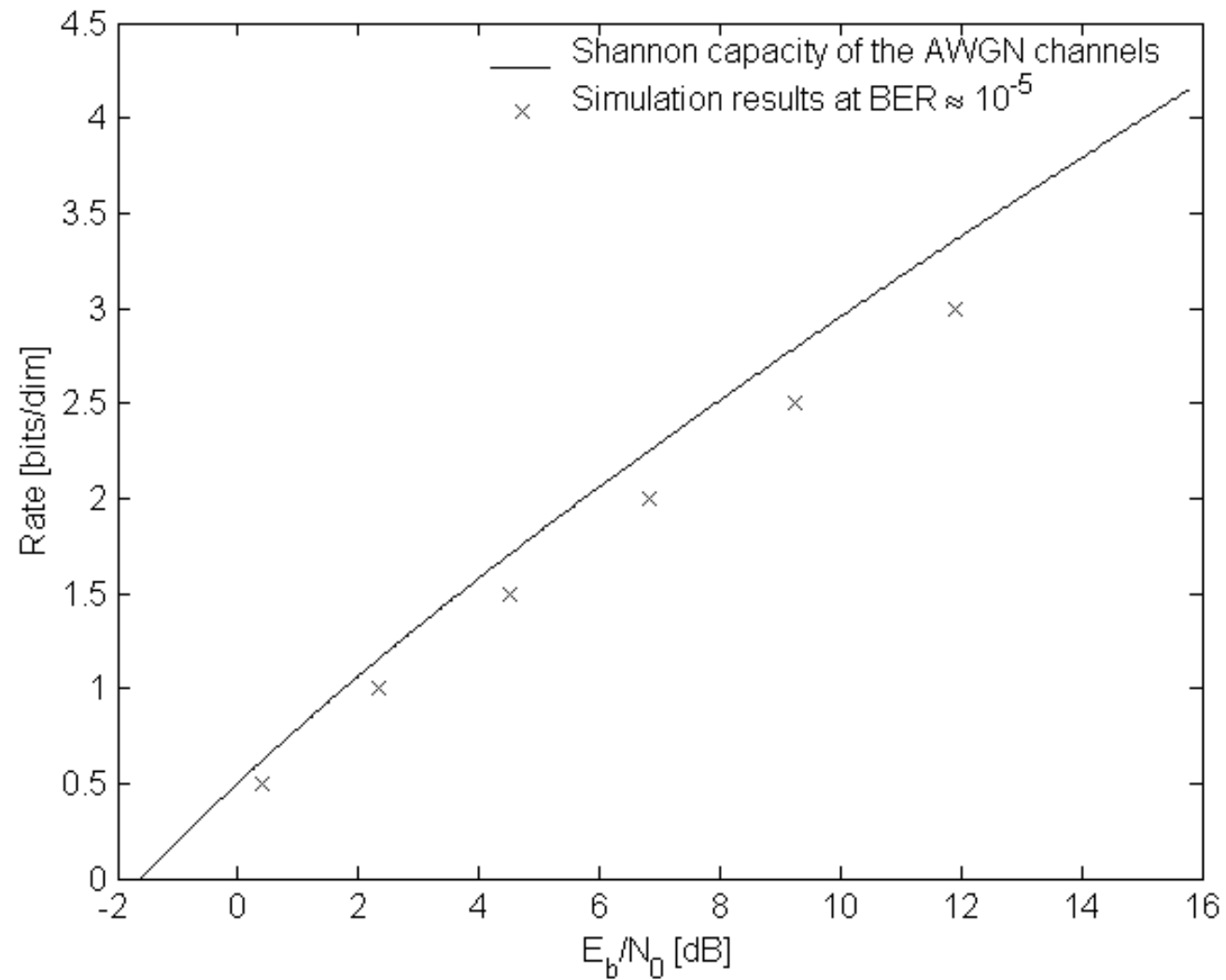
- Simplicity
- Flexibility
- High performance
- Low-decoding cost
- Easy treatments for ISI

# Rate-1/8-Repeating IDMA





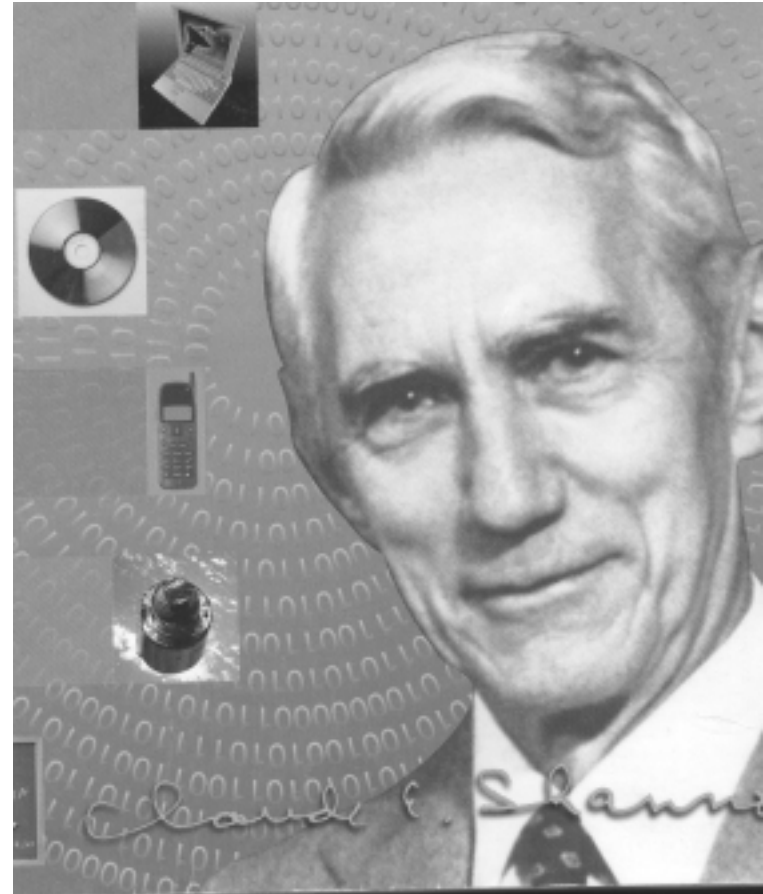
# Performance of IDM Coded Modulation (per real dimension)



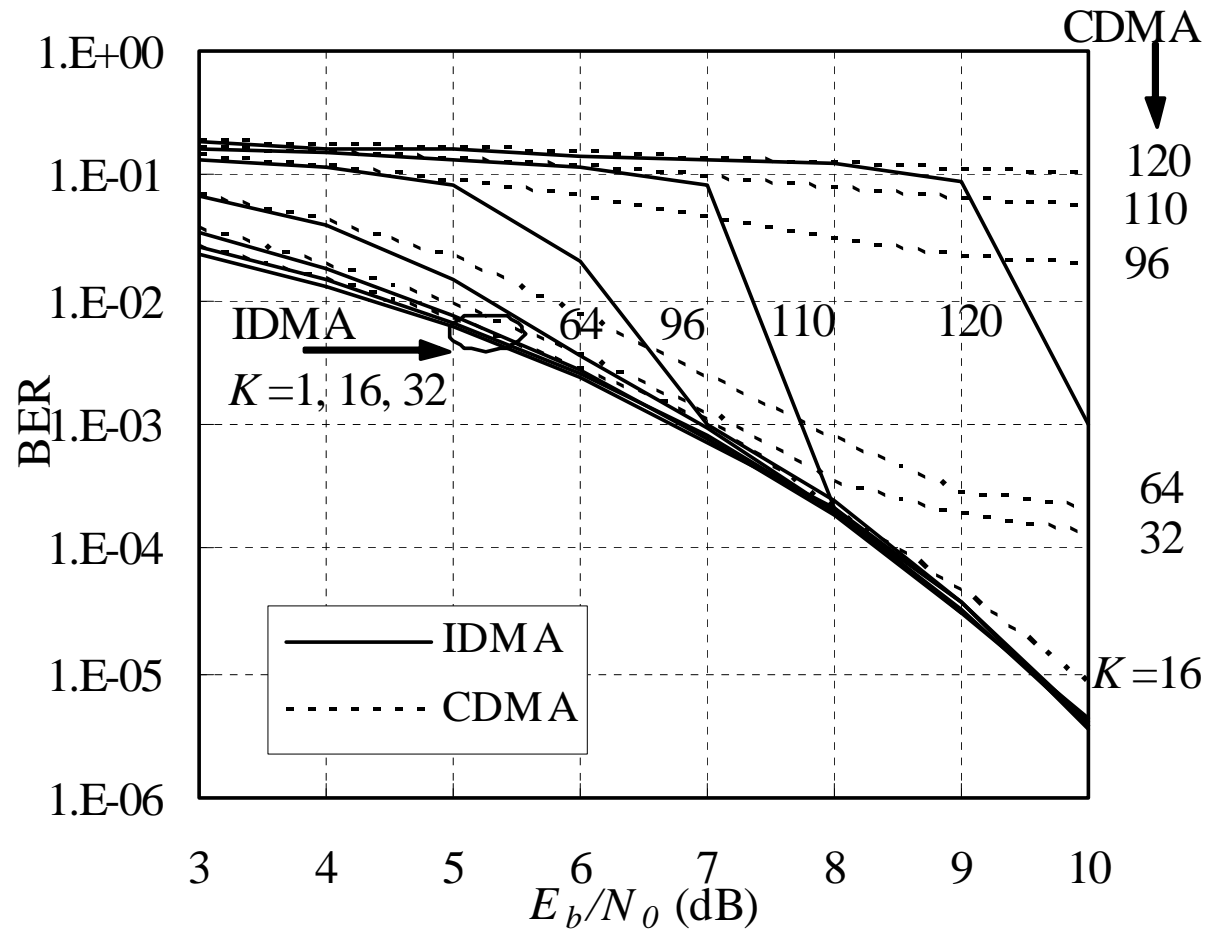
# Conclusions Again

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What makes IDMA work?  
Randomness.



## A Comparison between Un-coded IDMA and CDMA



# For Details

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<http://www.ee.cityu.edu.hk/~liping/research/>

## Chip-by-Chip Detection

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Step 1. Chip-level path model:  $r(j) = \sum_{k=1}^K h_k x_k(j) + n(j)$

Step 2. Gaussian approximation:  $r(j) = h_k x_k(j) + \zeta_k(j)$

Gaussian

Step 3. Estimation:

$$e(x_k(j)) = \frac{2h_k}{\text{Var}(\zeta_k(j))} \cdot (r(j) - E(\zeta_k(j)))$$

For a chip, not much can be done. It must be simple.

---

# Analysis of the Chip-by-Chip Algorithm

$$\begin{aligned}
 e(x_k(j)) &= \frac{2h_k}{\text{Var}(\zeta_k(j))} \cdot (r(j) - E(\zeta_k(j))) \\
 &= \frac{2h_k}{\text{Var}(\zeta_k(j))} \cdot (\underbrace{h_k x_k(j)}_{\text{signal}} + \underbrace{\zeta_k(j) - E(\zeta_k(j))}_{\text{noise}})
 \end{aligned}$$

$$SNR_k = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 \text{Var}(x_{k'}(j)) + \sigma^2}$$