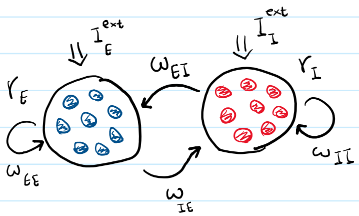
Wilson-Cowan Model (Review) → 2D Dynamical Systems



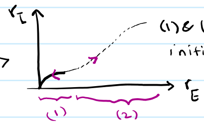
time constant → Population-Averaged neuronal rate

1) $\tau_E \frac{dr_E}{dt} = -r_E + F_E(w_{EE} r_E - w_{EI} r_I + I_E^{ext})$

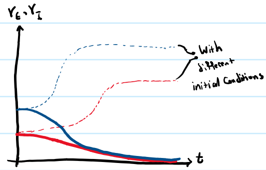
transfer function → sigmoid → $\frac{1}{1+e^{-(a-\theta)}}$; gain, threshold

2) $\tau_I \frac{dr_I}{dt} = -r_I + F_I(w_{IE} r_E - w_{II} r_I + I_I^{ext})$

Phase plane: activity of the two populations against each other ⇒

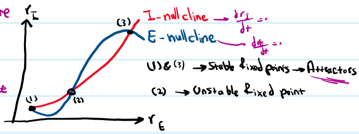


(1) & (2) have different initial Conditions.



E-nullcline: $\frac{dr_E}{dt} = 0$ $\xrightarrow{2D}$ a set of points in the phase plane where the excitatory population does not change.

I-nullcline: $\frac{dr_I}{dt} = 0$ ↳ a set of points in the phase plane where the inhibitory population does not change.



Computing fixed points:

$$\begin{cases} \frac{dr_E}{dt} = \frac{1}{\tau_E}\left[-r_E + F_E(w_{EE} r_E - w_{EI} r_I + I_E^{ext})\right] = G_E(r_E, r_I) \\ \frac{dr_I}{dt} = \frac{1}{\tau_I}\left[-r_I + F_E(w_{IE} r_E - w_{II} r_I + I_I^{ext})\right] = G_I(r_E, r_I) \end{cases}$$

$$\underbrace{J}_{\text{Jacobian Matrix}} = \left.\begin{pmatrix} \frac{\partial G_E}{\partial r_E} & \frac{\partial G_E}{\partial r_I} \\ \frac{\partial G_I}{\partial r_E} & \frac{\partial G_I}{\partial r_I} \end{pmatrix}\right|_{\underbrace{(r_E^*, r_I^*)}_{\text{Fixed Points}}} \Rightarrow \text{Eigenvalues } \lambda_1, \lambda_2$$

- Real eigenvalues
  - $\lambda_1 < 0$, $\lambda_2 < 0$ → Stable fixed point (node)
  - $\lambda_1 > 0$, $\lambda_2 > 0$ → Unstable fixed point (node)
  - $\lambda_1 > 0$, $\lambda_2 < 0$ → Unstable fixed point (saddle point)
- Imaginary eigenvalues
  - negative real part → Stable fixed point
  - positive real part → Unstable fixed point

Epileptic Seizures

Epilepsy → State transition

→ Neural Mass Model or Neural field Models (ODE) $\xrightarrow{\text{Data Analysis}}$ EEG ⟹ Data-Driven Modeling → Probabilistic models
→ ML

Neuro science $\xrightarrow{\text{exp}}$ 1) Synaptic transmition
2) Ion channels → $[Na^+]$
→ $[k^+]$
→ ** $[Ca^{2+}]$ **
→ DDM → Drift - Diffusion Models

μ_i → μ → $\in (0,1)$

3) Background neuronal activity → Glia

Epilepsy Cycle / State : 1) Intericetal * 2) preictal * 3) Ictal 4) postictal

→ Discrete states : Let $X_t \in \{$ Intericetal, Preictal, Ictal, Postictal $\}$ → Markov chains

HMM → Hidden Markov Models

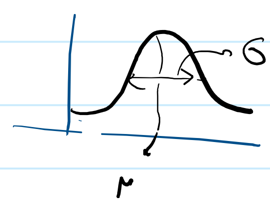
Point processe

x → a random variable ⟹ $x_i \in X$, $X = \{x_i \mid i \in \{1, \dots, N\}\}$ → $p(X = x_i) = p_i$ → * $\sum_{i=1}^{\sim} p_i = 1$

Continuous → $p(a \le X \le b) = \int_a^b \boxed{f(x)}\, dx$ → $G(x, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Mean ~~Avrege~~ $\bar{x}$

(σ, μ)



$x_i \in X$, $y_i \in Y$
→ $p(x_i, y_i)$

Marginal probability → $p(x) = \sum_y p(x, y)$ or $p(x) = \int p(x, y)\, dy$

*Bayes' Theorem: Conditional probability

$$\underbrace{p(\theta \mid \gamma)}_{\text{Posterior}} = \frac{\overbrace{p(\gamma \mid \theta)}^{\text{likelihood}} \cdot \overbrace{p(\theta)}^{\text{Prior}}}{\underbrace{p(\gamma)}_{\text{Evidence}}}$$

Bayesian Inference

θ : latent parameter e.g. → transition probability

γ : EEG data

* Bayesian HMM:
  - State: $X_t \in \{ \text{—}, \text{–}, \text{–}, \text{–} \}$
  - $O_t \equiv$ EEG-related parameters

prior
- → A
- → Emission parameter → $(\sigma, \mu) \equiv \Theta$

$$* \; p(X_{1:t}, O_{1:t}, A, \Theta) = p(A)\, p(\Theta) \cdot p(X_1) \cdot \prod_{t=2}^{T} p(X_t \mid X_{t-1}, A) \cdot \prod_{t=1}^{T} p(O_t \mid X_t, \Theta) \; *$$

1) Interictal 2) preictal 3) Ictal 4) postictal

- → $t \in \underbrace{T}_{\text{پیوسته}}$, seizures → گسسته } → Point process
- → $S_t \in \{ S_{1,t}, S_{2,t}, S_{3,t}, S_{4,t} \}$, $t \in T$
  - $S_{t+1} \in \{ S_{1,t+1}, S_{2,t+1}, \ldots \}$ } → Markov chain

Definition of Point process:

$\begin{cases} N(t) \\ t \end{cases} \rightarrow (\Omega, F, \mathbb{P})$ → $N(t) = \sum_{i=1}^{\infty} \mathbb{1}_{\{T_i \le t\}}$