Computational Neuroscience

Session 6: ***Modeling Epilepsy (1)***

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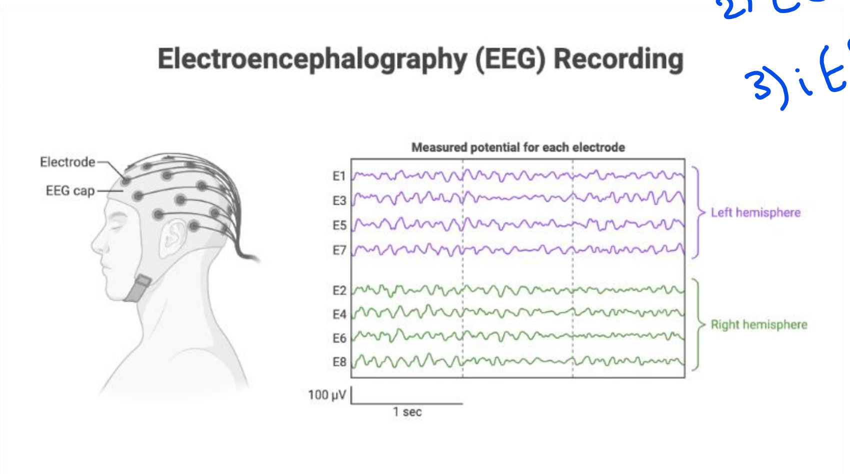
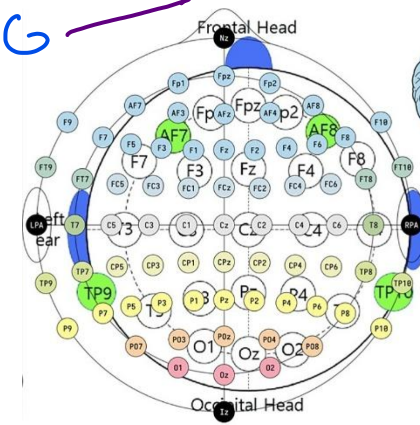
Electroencephalography (EEG)

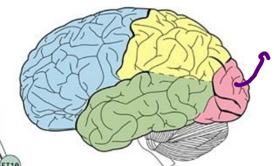
~~1) EEG~~ → غير تدخلي

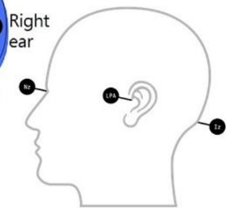
2) ECoG → تدخلي

3) iEEG →









Electroencephalography (EEG)

Jump

- EEG measures voltage fluctuations at the scalp resulting primarily from postsynaptic potentials (PSPs) in cortical pyramidal neurons.
- EEG signals are typically measured in microvolts and are classified into spectral bands with approximate frequency ranges and associated behavioral states.
- In epilepsy, interictal spikes and seizure discharges often exhibit dominant frequencies in the delta or theta ranges, and the amplitude can be much larger than during normal cognition.

Band (Hz) → Amplitude (μv)

Delta (0.5-4) → $\boxed{\text{Deep sleep}}$ → 25-200

Beta (30) → $\boxed{\text{Alerted}}$ → 6-15-20

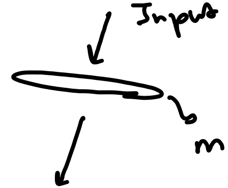
Wilson_Cowan

Network → E (Excitatory)

Network → I (Inhibitory)

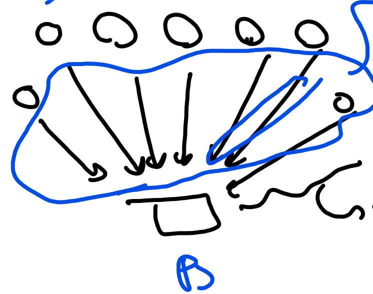
Feedforward / Recurrent

$$\sum_{j=1}^{N} m_j u_j = I$$



$$\tau_r \frac{dv}{dt} = -v + F(I^{ext}) = -v + F(m \cdot u)$$

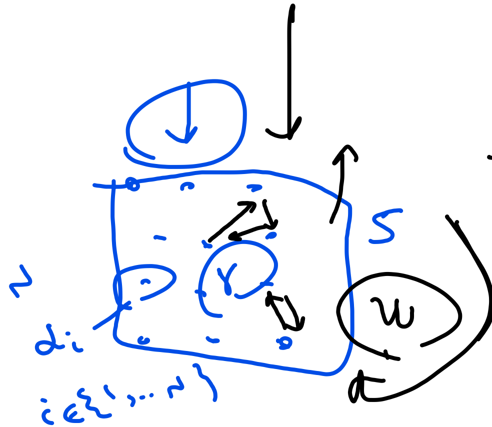
$$I^{ext} = \sum_{j=1}^{N} m_j u_j$$



m_i or μ_i

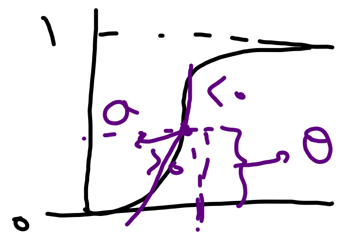
B

Recurrent



$$\tau \frac{dr}{dt} = -r + F\left(\boxed{w \cdot r} + I^{ext}\right)$$

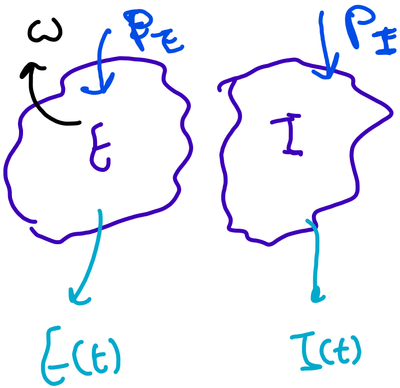
$F = ?$ → Transfer function → Sigmoidal → $F(I; a, \theta)$



$$= \frac{1}{1 + e^{(-a(I-\theta))}} - \frac{1}{1 + e^{a\theta}}$$

Gain

Wilson_Cowan



$$\begin{cases} \cancel{\tau_E} \dfrac{dE}{dt} = -E + F_E\left(w_{EE}E - w_{EI}I + P_E\right) \\ \cancel{\tau_I} \dfrac{dI}{dt} = -I + F_I\left(w_{IE}E - w_{II}I + P_I\right) \end{cases}$$

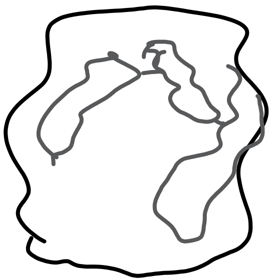
$$W = \begin{bmatrix} w_{EE} & w_{EI} \\ w_{IE} & w_{II} \end{bmatrix} \cdot \begin{bmatrix} E \\ I \end{bmatrix}$$

$$\begin{cases} \dfrac{dE}{dt} = 0 \\ \dfrac{dI}{dt} = 0 \end{cases} \longrightarrow \text{Fixed point} \longrightarrow J = \begin{bmatrix} \dfrac{\partial f_E}{\partial E} & \dfrac{\partial f_E}{\partial I} \\ \dfrac{\partial f_I}{\partial E} & \dfrac{\partial f_I}{\partial I} \end{bmatrix} \quad \lambda_i$$

→ Stable

→ bistability_Unstable

$$\tau_E \frac{dv}{dt} = - - - - + \cancel{\eta(t)}$$



{ Pyramidal neuron
Excitatory interneurons
Inhibitory - - - -

Jansen-Rit

1) $y \rightarrow v_m$

2) $y_1, y_2 \rightarrow v_{SI}, v_{SE}$

3) $y_3, y_4, y_5 \rightarrow \frac{d(y_0, y_1, y_2)}{dt}$

$$v(t) = y_1 - y_2$$

$$\begin{cases} \dot{y}_0 = y_3 \\ \dot{y}_3 = \alpha\beta F(y_1 - y_2) - 2\alpha y_3 - \alpha^2 y_0 \\ \dot{y}_1 = y_4 \longrightarrow \dot{y}_4 = \alpha\beta\left(I + C_4 F(C_3 y_0)\right) - 2\alpha y_4 - \alpha^2 y_1 \\ \vdots \end{cases}$$

bifurcation ⇒ stable → limit cycle } → Epilepsy → Noise!

$$\dot{y}_2 \longrightarrow \dot{y}_{2,S} \equiv \dot{y}_2 + \sigma \xi(t)$$

Deter.

$$\frac{dX}{dt} = f(X, \mu)$$

S.V

C.P

t

Stoch.

$$\frac{dx}{dt} = f(x) + \sigma \eta(t)$$

noise vector

white noise

1D ODE

$$U(x) = \frac{x^4}{4} - 0.5x^2 \rightarrow \frac{dx}{dt} = -\nabla U(x) = x - x^3$$

SDE

1D

$$\frac{dx}{dt} = x - x^3 + \sigma\,\eta(t)$$

1) 2 stable → $x = \pm 1$

2) $x = 0$ $\quad x = -1$

$$\frac{dE}{dt} = -E + F(w_{EE}E - w_{EI}I + P_E) + \sigma_E \eta_E(t)$$

$$\eta(t) \sim \mathcal{N}(0,1)$$

if $\sigma_E \to 0$

oscillation

if $\sigma_E \to \infty \longrightarrow$

$$\frac{dx}{dt} = f(x) + \sigma\xi(t)$$

Effective potential

$$* \quad \frac{dx}{dt} = -\nabla U(x) + \sigma\eta(t) \longrightarrow \text{Langevin equ.}$$

if $\sigma \ll 1$

$$\langle T \rangle \sim e^{\left(\frac{\Delta U}{\sigma^2}\right)}$$

$$T = \inf\{t \geq 0 : x(t) \geq x_c\}$$

$$A \longrightarrow B$$