Classical Particle mech : $q_\alpha(t)$ $\qquad \alpha$: finite

" Field " : $\phi(\vec{r},t) \longrightarrow$

EM: $\phi, \vec{A} \rightarrow \vec{E} = -\nabla\phi - \partial_t \vec{A}$, $\vec{B} = \nabla \times \vec{A}$

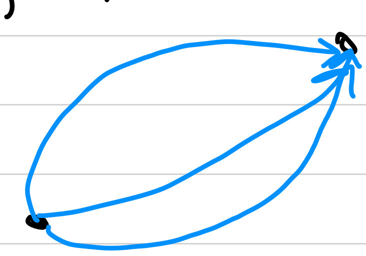
$\nabla \cdot \vec{B} = 0$, $\vec{E} = -\partial_t \vec{B}$

Lagrangian : $S = \int L(t)\, dt$ $\qquad L(t) = \int d^3\vec{r}\ \mathcal{L}(\phi, \partial_\mu\phi, t)$

$$S = \int d^4x\ \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\delta S = \int d^4x \left[\frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \underbrace{\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta(\partial_\mu\phi)}\right] = 0$$

$$\underbrace{\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi\right)} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\delta\phi$$



$\delta\phi(t_1) = \delta\phi(t_2) = 0$

$\delta\phi(\vec{r}\to\infty) \to 0$

$\phi_1 \longrightarrow \phi_2$

$$\boxed{\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0}$$

Electrostatic: $\phi(\vec{r})$: $\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$

$$\mathcal{L} = \frac{1}{2}\nabla\phi\cdot\nabla\phi - \frac{\rho}{\varepsilon_0}\phi$$

$$\delta S = \int \left[\nabla\phi\cdot\nabla\delta\phi - \frac{\rho}{\varepsilon_0}\delta\phi\right] d^3\vec{r}$$

$$= \int_V \underbrace{\left(-\nabla^2\phi - \frac{\rho}{\varepsilon_0}\right)} \delta\phi \, d^3\vec{r} + \cancel{\oint_{\partial V} \delta\phi \, \nabla\phi\cdot d\vec{a}}$$

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$

$i\hbar\,\partial/\partial t\,\psi = H\psi$; $H = \sqrt{p^2c^2 + m^2c^4}$; $p = -i\hbar\nabla$

$$H^2 = p^2c^2 + m^2c^4$$

$$\left.\begin{array}{l} H \to i\hbar\,\partial/\partial t \\ p = -i\hbar\nabla \end{array}\right\} \quad \underline{\left(i\hbar\,\partial/\partial t\right)^2\psi = \left(-i\hbar c\nabla\right)^2\psi + m^2c^2\psi}$$

Klein-Gordon: $\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{m^2}{2}\phi^2$

$\eta^{\mu\nu} = (+\,-\,-\,-)$; $T = \int d^3x\,\frac{1}{2}\dot{\phi}^2$

$$= \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad V = \int d^3x\,\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m\phi^2$$

$c^2dt^2 - dr^2$ $\quad dX = \begin{pmatrix} c\,dt \\ dx \\ dy \\ dz \end{pmatrix}$ $\quad dX \cdot dX = dX^T \eta \, dX$

$$\rightarrow \mathcal{L} = \frac{1}{2}\eta^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi) - \frac{m^2}{2}\phi^2$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2\phi \quad ; \quad \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi \equiv (\dot{\phi}, -\nabla\phi)$$

$$-\frac{\partial \mathcal{L}}{\partial \phi} + \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = 0$$

$$\boxed{\partial_\mu \partial^\mu \phi + m^2\phi = 0} \qquad \boxed{\frac{\partial^2}{\partial t^2}\phi - \nabla^2\phi + m^2\phi = 0}$$

$$\mathcal{L} = \frac{i}{2}(\psi^* \dot{\psi} - \dot{\psi}^* \psi) - \hbar \nabla\psi^* \cdot \nabla\psi - m\psi^*\psi$$

$$\frac{\partial \mathcal{L}}{\partial \psi^*} = \frac{i}{2}\dot{\psi} - m\psi \quad ; \quad \frac{\partial \mathcal{L}}{\partial \dot{\psi}^*} = -\frac{i}{2}\psi \quad ; \quad \frac{\partial \mathcal{L}}{\partial(\nabla\psi^*)} = -\nabla\psi$$

$$\boxed{i\,\partial_t \psi = -\nabla^2\psi + m\psi} \qquad \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = 0$$

$$i\hbar\,\partial_t \psi = \hat{H}\psi$$

Electrodynamics :

$$A = \begin{pmatrix} \phi \\ \vec{A} \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu A^\nu)(\partial_\mu A^\nu) + \frac{1}{2}(\partial_\mu A^\mu)^2 \quad ; \; \partial_\mu A^\nu = \eta^{\nu\sigma}\partial_\mu A_\sigma$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -\partial^\mu A^\nu + \partial_\sigma A^\sigma \eta^{\mu\nu} \qquad \frac{\partial \mathcal{L}}{\partial A_\mu} = 0$$

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu A_\nu)}\right)=0 \Rightarrow -\partial_\mu\partial^\mu A^\nu + \underline{\partial_\mu}\partial_\sigma A^\sigma \underbrace{\eta^{\mu\nu}}_{\partial^\nu} = 0$$

$$-\partial_\mu\partial^\mu A^\nu + \partial^\nu\partial_\mu A^\mu = 0$$

$$-\partial_\mu(\underbrace{\partial^\mu A^\nu - \partial^\nu A^\mu}_{F^{\mu\nu}}) = 0 \Rightarrow \underline{\underline{\partial_\mu F^{\mu\nu} = 0}}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Locality: $\mathcal{L} = \int d^3\vec{r}\, d^3\vec{r}'\, \phi(\vec{r})\,\phi(\vec{r}')$ $\times$

Lorentz invariance: $x^\mu \longrightarrow \underbrace{\Lambda^\mu{}_\nu x^\nu}$

Λ : $\Lambda^\mu{}_\sigma\,\eta^{\sigma\tau}\,\Lambda^\nu{}_\tau = \eta^{\mu\nu}$

$$x'\cdot x' = x\cdot x$$

$$x' = \Lambda x \rightarrow x^T\Lambda^T\eta\Lambda x = x^T\eta x \Rightarrow \Lambda^T\eta\Lambda = \eta$$

$\phi(x)$: sol. $\Longrightarrow$ $\phi(\Lambda^{-1}x)$: sol.

T : $x \longrightarrow x$ $\qquad \phi(\vec{r}) \longrightarrow \phi' = \phi(T^{-1}x)$

KG $$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{m^2}{2}\phi^2$$

$$\phi^2(x) \longrightarrow \phi(\underbrace{\Lambda^{-1}x}_{y})^2$$

$$\phi(x) \longrightarrow \phi(\Lambda^{-1}x) \qquad \partial_\mu\phi(x) \longrightarrow {\Lambda^{-1}}^\nu{}_\mu\,\partial_\nu\phi(\Lambda^{-1}x)$$

$$(\partial_\mu\phi)(\partial_\nu\phi)\,\eta^{\mu\nu} \longrightarrow \underline{{\Lambda^{-1}}^\rho{}_\mu}\;\partial_\rho\phi(y)\;\underline{{\Lambda^{-1}}^\sigma{}_\nu}\;\partial_\sigma\phi(y)\;\underline{\eta^{\mu\nu}}$$

$$\eta^{\rho\sigma}$$

$$= \partial_\rho\phi(y)\,\partial_\sigma\phi\;\eta^{\rho\sigma}$$

$$S = \int d^4x\;\mathcal{L}(y) = \int d^4y\;\mathcal{L}(y)$$

$$\partial_\mu J^\mu = 0 \qquad S = \int d^4x\;\mathcal{L}$$

$$\delta\phi(x) = X(\phi) \longrightarrow \delta\mathcal{L} = \partial_\mu F^\mu$$

$$\partial\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial_\mu\delta\phi$$

$$= \underbrace{\left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\right]}_{=0}\delta\phi + \underbrace{\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi\right)}_{} = \partial_\mu F^\mu$$

$$\partial_\mu \Big(\underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi - F^\mu}_{j^\mu} \Big) = 0 \quad ; \quad x^\nu \longrightarrow x^\nu - \varepsilon^\nu$$

$$\phi \longrightarrow \phi + \varepsilon^\nu \partial_\nu \phi$$