Para statistics :

$$|\psi\rangle = \Big[\big(|mns\rangle + |nsm\rangle + |smn\rangle\big) \pm \big(|nms\rangle + |snm\rangle + |msn\rangle\big)\Big]$$

↳ 2-dim

3 Particles in state $|m\rangle$, $|n\rangle$, $|s\rangle$

6 basis : 2 ⟶ 4 are left ! S_n

Hidden Numbers : $3 \times \underbrace{u\text{-quark}}_{1/2} \longrightarrow \underbrace{\Delta^{++}}_{3/2}$ $|\uparrow\uparrow\uparrow\rangle$

$$|R,\uparrow\rangle \otimes |G,\uparrow\rangle \otimes |B,\uparrow\rangle$$

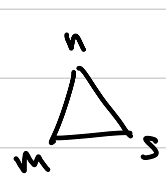
or Parastatistics !

3 identical particles :

$|abc\rangle \longrightarrow |cab\rangle$

Bosonic ⟶ invariant under swap

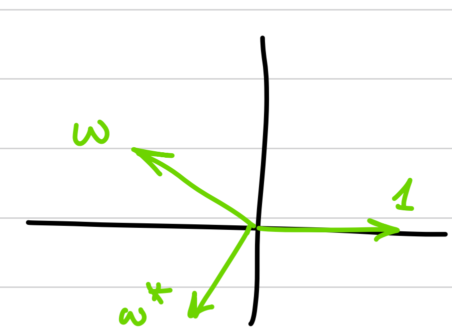
Fermionic ⟶ invariant under cyclic , swap : -1



: swap : Reflection

cyclic : Rotation

$$\omega := e^{\frac{2\pi i}{3}} = \frac{i\sqrt{3}-1}{2} \quad , \quad \omega^* = \frac{-i\sqrt{3}-1}{2}$$



$1+\omega+\omega^*=0 \quad , \quad \omega^*=\omega^2=\omega^{-1}$

$$\Phi_+ := \frac{|mns\rangle + \omega|nsm\rangle + \omega^*|smn\rangle}{\sqrt{3}}$$

$\longrightarrow$ orthogonal to Bosonic & Fermionic states

$$\Phi_- := \frac{|nms\rangle + \omega|snm\rangle + \omega^*|msn\rangle}{\sqrt{3}}$$

$|abc\rangle \to |cab\rangle$

Cyclic : $\Phi_+ \longrightarrow \dfrac{|smn\rangle + \omega|mns\rangle + \omega^*|nsm\rangle}{\sqrt{3}} = \omega\,\Phi_+$

$\hat{P}\,|n\rangle\otimes|m\rangle = |m\rangle\otimes|n\rangle$

$\underbrace{\hat{P}^2 = \mathbb{1}}$

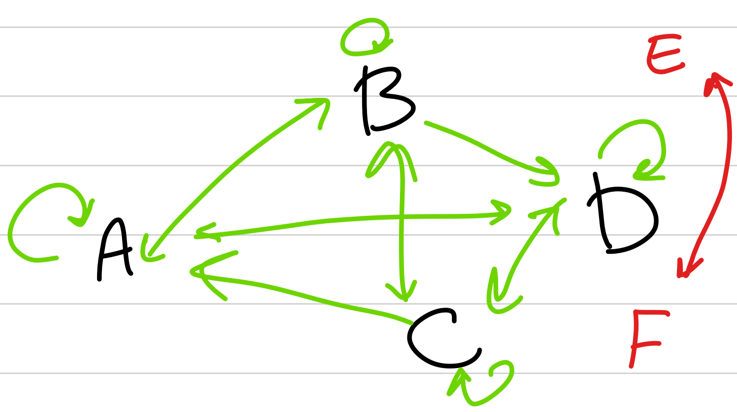
$$\left.\begin{array}{l}\underbrace{\Phi_+ \longrightarrow \omega\,\Phi_+} \\ \underbrace{\Phi_- \longrightarrow \omega^*\,\Phi_-}\end{array}\right\} \quad \Phi = \begin{pmatrix}\Phi_+ \\ \Phi_-\end{pmatrix}$$

$$\Phi \longrightarrow \begin{pmatrix}\omega & 0 \\ 0 & \omega^*\end{pmatrix}\Phi$$

$|abc\rangle \to |bac\rangle$

Swap : $\Phi_+ \longleftrightarrow \Phi_-$

$$\Phi = \begin{pmatrix}\Phi_+ \\ \Phi_-\end{pmatrix} \ : \ \Phi \longrightarrow \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\Phi$$



$$|n\,m\,s\rangle \xrightarrow{\ \xrightarrow{2\times\text{Cyclic}}\ |msn\rangle\ \xrightarrow{\text{Swap}}\ } |smn\rangle$$

$$\hookrightarrow \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}\omega & 0 \\ 0 & \omega^*\end{pmatrix}\begin{pmatrix}\omega & 0 \\ 0 & \omega^*\end{pmatrix}$$

unitary + only element that commutes with all other elements is $\hat{\mathbb{1}}$

$\hookrightarrow$ irreducible

reducible :

$$\begin{pmatrix} \boxed{\begin{matrix} 1 & 2 \\ 2 & 1 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \boxed{\begin{matrix} 3 & 4 \\ 4 & 5 \end{matrix}} \end{pmatrix}$$

$|1\rangle \longrightarrow |1\rangle + 2|2\rangle$

$|2\rangle \longrightarrow 2|1\rangle + |2\rangle$

$|3\rangle$

$|4\rangle$

if $\exists$ basis : $\forall D \in$ Representation : $D \rightarrow$ Block diagonal

then , $\exists \underset{\substack{\cap \\ \text{Rep}}}{A} \not\propto \hat{\mathbb{1}}$: $\forall D$: $[A, D] = 0$

$\longleftarrow$ $\exists A$. $[A, D] = 0$ $\longrightarrow$ $A = \begin{pmatrix} \boxed{\begin{matrix} \lambda_1 & \\ & \lambda_1 \end{matrix}} & \\ & \boxed{\begin{matrix} \lambda_2 & \\ & \lambda_2 \end{matrix}} \\ & & \ddots \end{pmatrix}$

$A|\lambda\rangle = \lambda|\lambda\rangle$

$A D|\lambda\rangle = D A|\lambda\rangle = \underline{\underline{\lambda}} D|\lambda\rangle \longrightarrow$ Block diagonal

$\longrightarrow$: $D = \begin{pmatrix} \boxed{} & \\ & \boxed{} \\ & & \ddots \end{pmatrix}$, $\hat{P} = \begin{pmatrix} \boxed{\mathbb{1}} & & 0 \\ 0 & \boxed{0} & \\ & & \ddots \end{pmatrix}$

$[\hat{P}, D] = 0$

Bosonic : 1

Fermionic : $\begin{cases} -1 & \text{even} \\ +1 & \text{odd} \end{cases}$

$\underline{\begin{pmatrix} \omega & 0 \\ 0 & \omega^* \end{pmatrix}}$

$\underline{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$

$$1 + 1 + 2^2 = \underline{\underline{6}} = 3!$$

$$\Psi_+ := \frac{|mns\rangle + \omega|nsm\rangle + \omega^*|smn\rangle}{\sqrt{3}}$$

$$\Psi_- := \frac{|nms\rangle + \omega|snm\rangle + \omega^*|msn\rangle}{\sqrt{3}}$$

$$\left.\begin{aligned} \Phi_+ &:= \frac{|nms\rangle + \omega^*|snm\rangle + \omega|msn\rangle}{\sqrt{3}} \\ \Phi_- &:= \frac{|mns\rangle + \omega^*|nsm\rangle + \omega|smn\rangle}{\sqrt{3}} \end{aligned}\right\} \begin{aligned} &\perp \text{ Fermion \& Boson} \\ &\perp \Psi_\pm \end{aligned}$$

$$\Lambda_+ := \frac{\Psi_+ + \Psi_-}{\sqrt{2}}, \quad \Lambda_- := i\,\frac{\Psi_+ - \Psi_-}{\sqrt{2}}$$

$$\Lambda = \begin{pmatrix} \Lambda_+ \\ \Lambda_- \end{pmatrix}$$

$$\text{cyclic}: \Lambda \to \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Lambda$$

$$\text{swap}: \Lambda \to -\frac{1}{2} \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} \Lambda.$$