۱۴ مهر

تقارن‌ها → داخلی

تقارن‌ها → فضا زمانی

$$\phi_a(x) \to \phi'_a(x) = \Big(e^{\overbrace{i\lambda^k \theta^k}^{\text{ماتریس}}}\Big)_{ab}\, \phi_b(x)$$

$$\mathcal{L}(\phi, \partial_\mu \phi, \ldots) = \mathcal{L}(\phi', \partial_\mu \phi', \ldots)$$

گروه آبلی : $U(1)$

$$\phi(x) \to e^{\alpha} \phi(x) \longrightarrow j^\mu \to \partial_\mu j^\mu = 0 \longrightarrow Q(T) = \int d^3x\, j^0$$

$$\boxed{j^k_\mu(x) = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi^a)} \lambda^k_{ab} \phi_b(x)}$$

global $\to$ local

تعمیم اصل نسبیت خاص از فضا-زمان به فضای داخلی میدان هست.

$$\phi_a(x) \to \phi'_a(x) = \Big(e^{i\lambda^k \theta^k(x)}\Big)_{ab} \phi_b(x)$$

$$U(1) \rightsquigarrow \phi(x) \longrightarrow \phi'(x) = e^{i\theta(x)} \phi(x)$$

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(|\phi|^2)$$

کلی‌ترین لاگرانژی مجازی که می‌توانیم بنویسیم چیست؟!

$$\partial_\mu \phi' = \partial_\mu \left(e^{i\theta(x)} \phi(x) \right) = e^{i\theta(x)} \left[\partial_\mu \phi + i \phi \partial_\mu \theta \right]$$

کار و خراب کن!

$$\partial_\mu \phi^* \partial^\mu \phi \; \times$$

برای اینکه کل موضوعات ناوردا بمانه:

مشتق هموردا

$$\partial_\mu \phi \longrightarrow D_\mu \phi \longrightarrow D_\mu \phi^* D^\mu \phi$$

تحت تبدیل پیمانه‌ای موضعی

$$e^{i\theta} D_\mu \phi$$

$$\left.\begin{array}{ll} x_\mu \longrightarrow & x_\mu + dx_\mu \\ \phi(x) & \phi(x+dx) \end{array}\right\} \quad \phi(x+dx) - \phi(x) = \delta\phi(x)$$

$$\delta\phi(x) = i A_\mu(x) dx^\mu \phi(x)$$

میدان برداری [illegible]

gauge field

$$D_\mu \phi(x) \equiv \partial_\mu \phi - ie A_\mu \phi = (\partial_\mu - ie A_\mu)\phi$$

A_μ تحت تبدیلات پیمانه‌ای چگونه عوض شود تا $D_\mu\phi \to e^{i\theta} D_\mu\phi$:

$$D_\mu \longrightarrow D'_\mu$$

$$\phi \longrightarrow \phi'$$

درخواست: $\underline{D\phi' = e^{i\theta} D\phi}$

$$(\partial_\mu - ieA'_\mu)(e^{i\theta}\phi) = e^{i\theta}(\partial_\mu - ieA_\mu)\phi$$

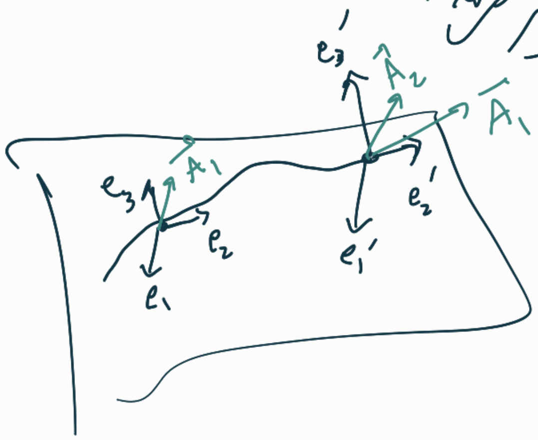
$$\Downarrow$$

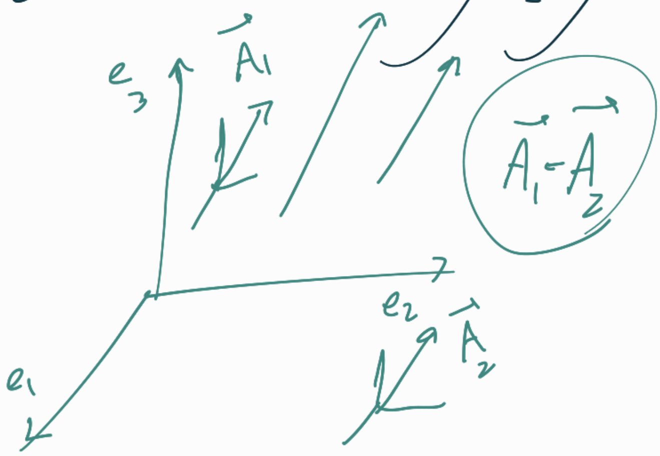
$$i\partial_\mu\theta - ieA'_\mu = -ieA_\mu$$

$$\boxed{A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\theta}$$

$\partial_\mu A$

تغییرپذیری $\phi(x)$ وجود داره که فقط بخاطر وجود این A_μ تغییر میکنه.





$$\partial_\mu (A_i e^i) \longrightarrow \partial_\mu A_i + 0$$

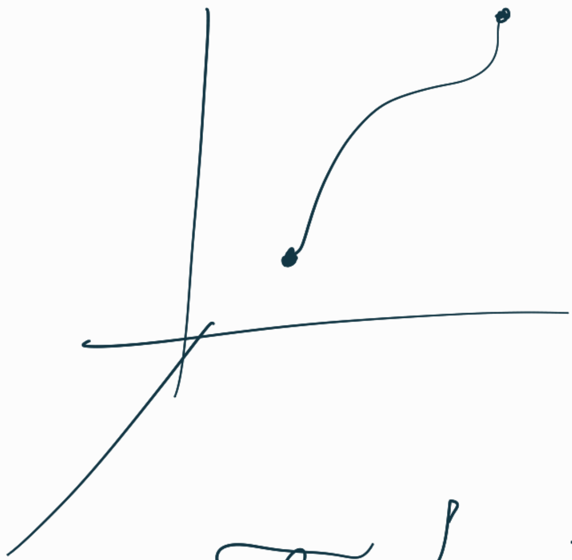
$$\longrightarrow \partial_\mu A_i + \partial_\mu e^i$$

$$\boxed{D_\mu \phi_c = 0} = (\partial_\mu - ie A_\mu)\phi_c = 0$$

$$\partial_\mu \phi_c = ie A_\mu \phi_c$$

$$\Longrightarrow \quad \phi_c(x) = e^{-ie \int_{\gamma(x,y)} dz_\mu A^\mu} \phi_c(y)$$

path-ordered exponential



$$z_\mu : [0,1] \longrightarrow \mathbb{R}^{1,3}$$

$$\mathcal{T}\left\{ e^{\int_0^s f(s')ds'} \right\} =$$

$$\sum_{n=0}^{\infty} \frac{1}{n!}\int_0^s ds_1' \cdots \int_0^s ds_n' \, \mathcal{T}\{f(s_1')\cdots f(s_n')\}$$

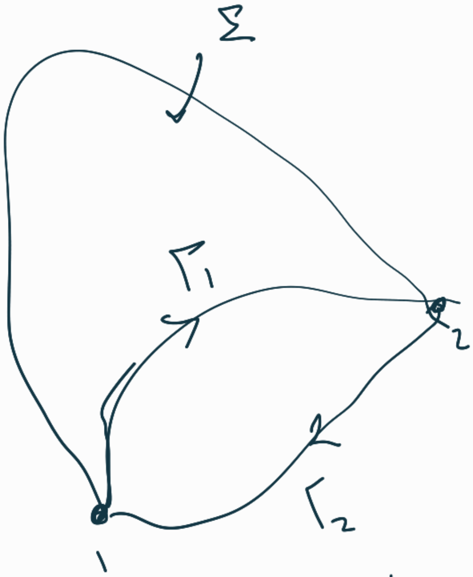
$$= \sum_{n=0}^{\infty} \int_0^s ds_1' \int_0^{s_1'} ds_2' \cdots \int_0^{s_{n-1}'} ds_n' \, f(s_n') \cdots f(s_1')$$

$$e\int_{\gamma} dz_\mu A^\mu \xrightarrow{\text{gauge}} e\int dz_\mu A^\mu + e\int dz_\mu \frac{1}{e}\partial^\mu \theta$$

$$= e\int dz_\mu A^\mu + \theta(y) - \theta(x)$$

$$\phi_c(y)\, e^{-ie\int_\gamma \cdots} \longrightarrow \phi_c(y)\, e^{-ie\int_\gamma \cdots}\, e^{-i\theta(y)}\, e^{i\theta(x)}$$

$$\equiv e^{i\theta}\phi_c(x)$$



$$\Delta\varphi = -e\int_{\Gamma_1} dz_\mu A^\mu + e\int_{\Gamma_2} dz_\mu A^\mu$$

$$\| $$

$$-e\oint_{\Gamma^+} dz^\mu A_\mu$$

$$\Gamma^+ = \Gamma_1 \cup \Gamma_2$$

$$\partial\Sigma = \Gamma^+$$

$$\Delta\varphi = -\frac{e}{2}\int_{\Sigma} dS_{\mu\nu} F^{\mu\nu} = -e\,\Phi(\Sigma)$$

$$\leftarrow F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

تانسور انحنا

$$F^{\mu\nu} = \frac{i}{e}\left[D^{\mu}, D^{\nu}\right]$$