Observables :

Fermionic states : $\dfrac{|m\rangle \otimes |n\rangle - |n\rangle \otimes |m\rangle}{\sqrt{2}}$

Bosonic states : $|n\rangle \otimes |n\rangle$, $\dfrac{|m\rangle \otimes |n\rangle + |n\rangle \otimes |m\rangle}{\sqrt{2}}$

$\hat{A}$: $\langle A\rangle = \langle \psi | A | \psi \rangle$; $|\psi\rangle = \dfrac{|m\rangle \otimes |n\rangle \pm |n\rangle \otimes |m\rangle}{\sqrt{2}}$

$$\langle A\rangle = \frac{1}{2}\left(A_{\underline{m}\underline{n}, \underline{mn}} + A_{nm,nm} \pm A_{mn,nm} \pm A_{nm,mn} \right)$$

$$A_{mn,rs} := \langle m| \otimes \langle n|\ \hat{A}\ |r\rangle \otimes |s\rangle \qquad A_{mn,rs} = A_{nm,sr}$$

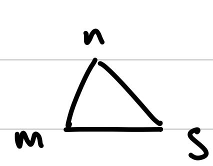
$$\langle A\rangle = A_{mn,mn} \underline{\pm} A_{mn,nm}$$

↳ Boson or Fermion

Three Particles :

$|mns\rangle := |m\rangle \otimes |n\rangle \otimes |s\rangle$

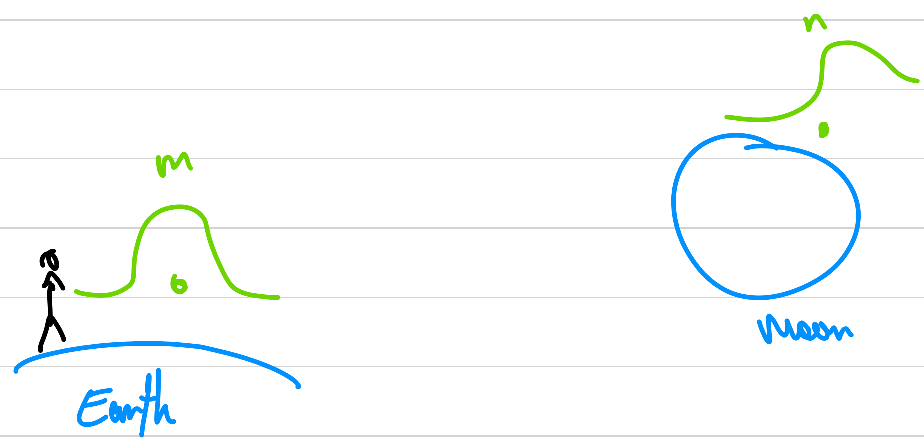
$$|\psi\rangle = \left[\left(|mns\rangle + |nsm\rangle + |smn\rangle \right) \pm \left(|nms\rangle + |snm\rangle + |msn\rangle \right) \right]$$

$=$ 

$$A_{mns,xyz} = A_{nsm,yzx} = A_{nms,yxz} = \cdots$$

$$\langle A\rangle = \left(A_{mns,mns} + A_{mns,nsm} + A_{mns,smn}\right) \pm \left(A_{mns,nms} + A_{mns,snm} + A_{mns,msn}\right)$$

Cluster separability :



$$\hat{A} = \sum_m A_n |m\rangle\langle m|$$

$$\| \hat{A} |n\rangle \| \ll 1 \longrightarrow \langle m| A |n\rangle \ll 1$$

$$\langle n| \hat{\mathbb{1}} |n\rangle = \langle n|n\rangle = 1 \quad !$$

m: local state   n: remote state

$$|\psi\rangle = \frac{|n\rangle\otimes|m\rangle \pm |m\rangle\otimes|n\rangle}{\sqrt{2}} \; ; \quad \hat{A} \text{. single-Particle operator}$$

$$\hookrightarrow A\otimes\mathbb{1} + \mathbb{1}\otimes A$$

$$\langle A\rangle = \langle\psi| (A\otimes\mathbb{1} + \mathbb{1}\otimes A) |\psi\rangle$$

$$= \langle m| A |m\rangle \rightarrow \text{cluster separability}$$

Three Particle : ...

$$\langle A\rangle = A_{mn,mn} \pm A_{mn,nm}$$