"gɑs"

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$ local gauge + Renorm + scalar

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right] \quad \left\{ F_{\mu\nu} = F^k_{\mu\nu}\lambda^k \right\}$$

$$F'_{\mu\nu} = U(x)\, F_{\mu\nu}\, U^{-1}(x)$$

$$\mathcal{L}_{Dirac}(\psi, \bar\psi) = \bar\psi(i\partial\!\!\!/ - m)\psi$$

$$\mathcal{L}_{gauge}(A) = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

minimal Coupling : total lagrangian be invariant under local gauge transformation

$$\mathcal{L}_{QED} = \bar\psi(iD\!\!\!/ - m)\psi - \frac{1}{4}F^2$$

$$= \underbrace{\bar\psi(i\partial\!\!\!/ - m)\psi + e\bar\psi\gamma_\mu\psi A^\mu}_{\mathcal{L}_{matter}} - \frac{1}{4}F^2$$

$$\psi_a \to \psi'_a = e^{i\theta}\psi_a \xrightarrow[\text{کوچک}]{\text{بی‌نهایت}} \underline{\delta\psi = i\theta\psi \ , \ \delta\bar\psi = -i\theta\bar\psi \ , \ \delta A_\mu = \tfrac{1}{e}\partial_\mu\theta}$$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\delta\partial_\mu\psi}\delta(\partial_\mu\psi) + (\psi\leftrightarrow\bar\psi) + \frac{\delta\mathcal{L}}{\delta(\partial_\mu A_\nu)}\delta\partial_\mu A_\nu + \frac{\delta\mathcal{L}}{\partial A_\mu}\delta A_\mu$$

حل کنیم

$$\Downarrow$$

پادمتقارن

$$\delta\mathcal{L} = \partial_\mu(j^\mu\theta) - \underbrace{\frac{1}{e}F^{\mu\nu}\partial_\mu\partial_\nu\theta}_{\text{متقارن}} \to 0 + \frac{\delta\mathcal{L}}{\delta A_\mu}\cdot\frac{1}{e}\cdot\partial_\mu\theta$$

$$\boxed{j^\mu = i\left[\frac{\delta\mathcal{L}}{\delta\partial_\mu\psi}\psi - \bar\psi\frac{\delta\mathcal{L}}{\delta\partial_\mu\bar\psi}\right]}$$

θ is arb

$$0 = j^\mu\partial_\mu\theta + \theta\partial_\mu j^\mu + \frac{\delta\mathcal{L}}{\delta A_\mu}\cdot\frac{1}{e}\partial_\mu\theta = \theta_{(x)}\left[\partial_\mu j^\mu\right] + \partial_\mu\theta\left[j^\mu + \frac{1}{e}\frac{\delta\mathcal{L}}{\delta A_\mu}\right]$$

$$\partial_\mu j^\mu = 0$$

$$J^\mu \equiv \frac{\delta\mathcal{L}}{\delta A_\mu(x)}$$

charge current

number current

$$\to J_\mu(x) = -e\,j_\mu(x) = -e\bar\psi\gamma_\mu\psi$$

Griener : Field Quantization

برای جزئیات محاسبات نظریه میدان مقدمه.

$$Q_0 = \int d^3\vec{x}\, J_0(x) = \int d^3x\, \psi^\dagger \psi$$

$$Q = -e\, Q_0 = \boxed{-e} \int d^3x\, \psi^\dagger \psi$$

این نوع نوشتن به ما می‌گوید

e همان بار الکتریکی است

که انتظار داشتیم.

مشابه برای QCD:

$$\mathcal{L} = \mathcal{L}_{matter} + \mathcal{L}_{gauge}$$

$$= \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}\,\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

Spacetime Symmetries → Energy-momentum tensor

$$x'_\mu = x_\mu + \delta x_\mu(x_\mu) \qquad \phi(x) \to \phi'(x) = \phi(x) + \delta\phi(x)$$

$$\phi(x) \longrightarrow \overset{1}{\phi'}(x') = \phi(x) + \boxed{\delta\phi(x)} + \boxed{\partial_\mu \phi\, \delta x^\mu}$$

تغییر میدان صرفاً به خاطر تغییرات داخلی

صرفاً ناشی از تغییرات مختصات

انتقال : $\delta x_\mu = a_\mu$, دوران : $\delta x_0 = 0$, $\delta x_i = \epsilon_{ijk}\theta_j x_k$, ...

$$d^4x' = J\, d^4x$$

$\hookrightarrow$ jacobian : $\left|\det \frac{\partial x'^\mu}{\partial x_\nu}\right|$

$$J = \left|\det \frac{\partial x'^\mu}{\partial x_\nu}\right| = \left|\det\left(g^\nu_\mu + \partial^\nu \delta x_\mu\right)\right| = 1 + tr(\partial^\nu \delta x_\mu) + \cdots$$

$$J = 1 + \partial^\mu \delta x_\mu + O(\delta x^2)$$

$$S = \int d^4x\, \mathcal{L} \rightarrow \delta S = \delta \int d^4x\, \mathcal{L}$$

$$= \int \delta(d^4x)\mathcal{L} + \int d^4x\, \delta\mathcal{L}$$

$$= \int d^4x \left[\partial_\mu \delta x^\mu \mathcal{L} + \underbrace{\partial_\mu \mathcal{L}\, \delta x^\mu}_{\text{off-shell}} + \frac{\delta \mathcal{L}}{\delta\phi}\delta\phi + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\partial_\mu\phi\right]$$

فضایی که در ادامه و معمولا ناظر به قضیه نوتر در کتاب‌ها و اینا ذکر می‌شود برای کمیت‌هایی است که لزوما تابع مشتق اول میدان باشند. حال on-shell :

$$\frac{\delta\mathcal{L}}{\delta\phi} = \partial_\mu \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}$$

$$\delta S = \int d^4x \left[\partial_\mu \delta x^\mu \mathcal{L} + \delta x^\mu \partial_\mu \mathcal{L} + \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta \phi \right) \right]$$

$$\delta_T \phi = \delta \phi + \partial_\mu \phi \, \delta x^\mu$$

Total

$$\Downarrow$$

$$\delta S = \int d^4x \left\{ \partial_\mu \underbrace{\left[\left(g^\mu_\nu \mathcal{L} - \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \partial_\nu \phi \right) \delta x^\nu \right]}_{\text{space-time}} + \partial_\mu \underbrace{\left[\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta_T \phi \right]}_{\text{internal}} \right\}$$

space-time + internal

جزئی نیست کل‌ها

Scalar field + translations:

$$\phi(x) = \phi'(x') \longrightarrow \delta_T \phi = 0 = \delta \phi + \partial_\mu \phi \, \delta x^\mu$$

$$\delta x_\mu = a_\mu$$

$$\delta S = \int d^4x \, \partial_\mu \left(g^{\mu\nu} \mathcal{L} - \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \partial^\nu \phi \right) a_\nu = \int d^4x \, \underbrace{\partial_\mu T^{\mu\nu}}_{=0} a_\nu$$

$$T^{\mu\nu} \equiv -g^{\mu\nu} \mathcal{L} + \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \partial^\nu \phi$$

تانسور انرژی - تکانه

$$\partial_\mu T^{\mu\nu} = 0$$

$$P^{\nu} \equiv \int d^3x \, T^{0\nu}(\vec{x}, x_0)$$

$$P^0_{(x_0)} = \int d^3x \, T^{00} = \int d^3x \Big(-\mathcal{L} + \underbrace{\frac{\delta \mathcal{L}}{\delta \partial_0 \phi}}_{\Pi} \partial^0 \phi \Big)$$

$$= \int d^3x \, \mathcal{H}$$

$$\vec{P} = \int d^3x \, \Pi(x) \, \partial \phi(x)$$

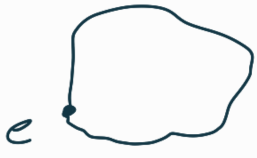
نگاهی عمیق‌تر و توپولوژیکی: EM

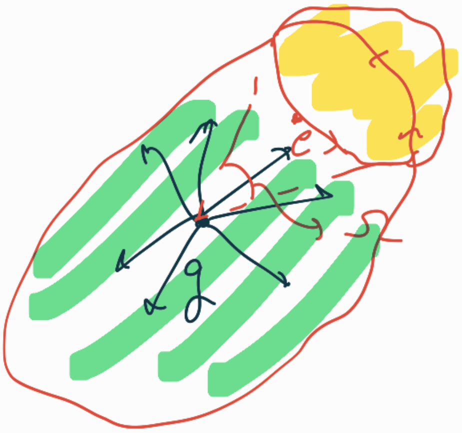
$$\vec{\nabla} \cdot \vec{B} = 0 \longrightarrow \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot \vec{B} \neq 0 \rightsquigarrow \vec{B} = \frac{\overbrace{g}^{\text{بار مغناطیسی}} \, \hat{n}}{4\pi r^2}$$

QM + Magnetic monopole $\longrightarrow$ Quantization of charge

$$\psi \longrightarrow e^{\frac{ie\alpha}{\hbar}} \psi \qquad \alpha = \oint \vec{A} \cdot d\vec{x}$$





$$\alpha = \frac{g\Omega}{4\pi}$$

$$\alpha' = \frac{g(4\pi - \Omega)}{4\pi}$$

$$e^{i\alpha e/\hbar} = e^{i\alpha' e/\hbar}$$

$$\frac{\alpha e}{\hbar} = \frac{\alpha' e}{\hbar} + 2\pi n$$

$$\longrightarrow \boxed{eg = 2\pi n \hbar}$$