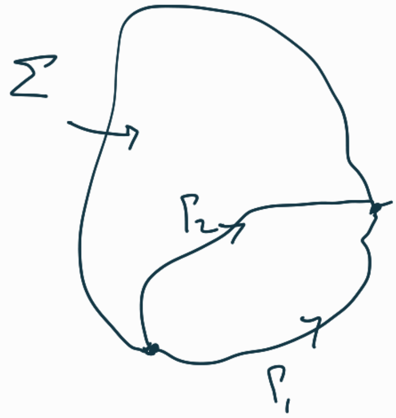
جلسه ۸



$$\Gamma^+ = \Gamma_1^+ \cup \Gamma_1^-$$

$$\mathcal{L} = \int F^{\mu\nu} F_{\mu\nu}$$

$$\int_{\Gamma_2^-} A_\mu dz_\mu = -\int_{\Gamma_2^+} A_\mu dz_\mu$$

$$dz^\mu \wedge dz^\nu$$

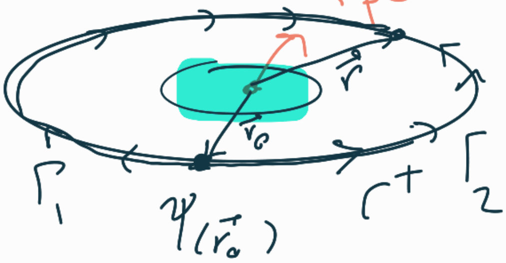
$$D\phi = 0 \quad \longrightarrow \quad \Delta\gamma = -\frac{e}{2}\int dS_{\mu\nu} F^{\mu\nu} = -e\phi(\Sigma)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \; ; \; F^{\mu\nu} = \frac{i}{e}[D^\mu, D^\nu]$$

اگر $F^{\mu\nu}$ در یک ناحیه فضا غیرصفر باشد، فاز میدان ϕ کلیتاً تعریف نمی‌شود. فاز ϕ به مسیر ربط دارد.

$$\vec{B} = B_0 \delta(\vec{r})\hat{k}$$

$$\varphi \neq 0$$



$$\phi = \int \vec{B} \cdot \vec{dS} \neq 0 = \phi_0$$

QM : $\mathcal{H}\psi = i\hbar \frac{\partial \psi}{\partial t}$

$$\frac{1}{2m}\left(\frac{\hbar}{i}\nabla + \frac{e}{c}\vec{A}\right)^2 \psi = i\hbar\frac{\partial \psi}{\partial t}$$

جواب معادله با A به فرم زیر:

نتیجه تبدیل پیمانه‌ای

$$\psi(\vec{r}) = e^{i\theta(\vec{r})}\psi_0(\vec{r})$$

$\phi = 0 \, , \, \vec{B} = 0$ ، در

$$-\frac{\hbar^2}{2m}\nabla^2\psi_0 = i\hbar\frac{\partial \psi_0}{\partial t}$$

$$\lim_{\vec{r} \to 0} \psi_0(\vec{r}, t) = 0$$

$$\left(\frac{\hbar}{i}\nabla + \frac{e}{c}A\right)\left(e^{i\theta}\psi_0\right) = e^{i\theta}\left(\hbar\nabla\theta + \frac{e}{c}\vec{A} + \frac{\hbar}{i}\nabla\right)\psi_0$$

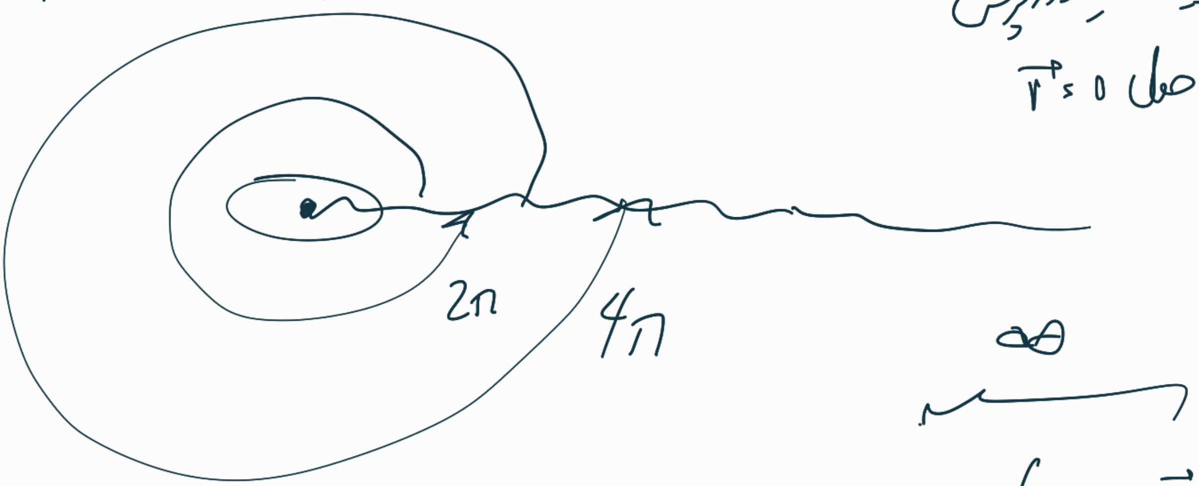
$$\Downarrow$$

$$\hbar\vec{\nabla}\theta + \frac{e}{c}\vec{A} = 0$$

$$\Rightarrow \quad \vec{\nabla}\theta = -\frac{e}{\hbar c}\vec{A}(\vec{r})$$

$$-\frac{e}{\hbar c}\oint_{\Gamma^+} d\vec{l}\cdot\vec{A} = \int_{\Gamma^+} d\vec{l}\cdot\vec{\nabla}\theta = \Delta\theta$$

تغییر θ در یک دور چرخش
حول $\vec{r}=0$



$$\vec{r}_0 \longrightarrow \vec{r} \quad : \quad \theta(\vec{r}) = \theta(\vec{r}_0) - \frac{e}{\hbar c}\int_{\Gamma_1(\vec{r}_0,\vec{r})} d\vec{x}\cdot\vec{A}$$

$$\langle\Gamma_1|\Gamma_2\rangle = \int d^2r\, \psi^*_{\Gamma_1}\psi_{\Gamma_2}$$

$$= \int d^2r\, |\psi_0|^2 e^{\frac{ie}{\hbar c}\left[\left(\int_{\Gamma_1} - \int_{\Gamma_2}\right) d\vec{l}\cdot\vec{A}\right]}$$

$$= \int d^2r\, |\psi_0|^2 e^{\frac{ie}{\hbar c}\int_{\Gamma^+} d\vec{l}\cdot\vec{A}}$$

$$= e^{\frac{ie}{\hbar c}\phi} \underbrace{\int d^2r \, |\Psi_0|^2}_{1} = e^{\frac{ie}{\hbar c}\phi}$$

$$\boxed{\langle \Gamma_1 | \Gamma_2 \rangle = e^{\frac{ie}{\hbar c}\phi}} \rightarrow \text{pure phase}$$

$$\frac{e}{\hbar c}\phi = 2\pi n \rightarrow \phi = n\frac{\hbar c}{e} = n\phi_0$$

flux quantum

Dirac + Witten : monopole magnetic fields
1931

$\Downarrow$

why charge is quan______ .

$$e^{i\alpha} e^{i\beta} - e^{i\beta} e^{i\alpha} = 0 \qquad U(1)$$

$$\Downarrow$$

$$A^\mu : \partial_\mu \to D_\mu$$

$\downarrow$

$U_1 U_2 - U_2 U_1 \neq 0$ Lie Group non-abelian

$$\phi'_a(x) \longrightarrow U_{ab}\phi_b(x)$$

$$\mathcal{L} = \partial_\mu \phi_a^* \partial^\mu \phi^a - V(\ldots)$$

$$\partial_\mu \phi' \longrightarrow \partial_\mu \left(U_{(x)} \phi_{(x)} \right) = \partial_\mu U \cdot \phi + U \partial_\mu \phi$$

$$= U \left[\partial_\mu \phi + U^{-1} \partial_\mu U \cdot \phi \right]_{(x)}$$

$$\begin{cases} \partial_\mu \phi' \neq U \partial_\mu \phi \\ \phi' = U \phi \end{cases}$$

درخواست

$$(D_\mu \phi)' = U (D_\mu \phi)$$

unique ?

$$D_\mu = \underbrace{I}_{\text{ماتریس واحد}} \partial_\mu - \underbrace{ig A_\mu(x)}_{\text{میدان پیمانه‌ای}}$$

$$(A_\mu(x))_{ab} = \underbrace{A^k_\mu(x)}_{\text{عدد}} \underbrace{\lambda^k_{ab}}_{\text{ماتریس}}$$

$\lambda^k_{ab} \rightarrow$ basis of the group generators

$$D_\mu \phi \longrightarrow D'_\mu \phi' = D' \left(U_{(x)} \phi_{(x)} \right)$$

$$= \left(\partial_\mu - ig A'_\mu(x) \right) (U \phi)$$

$$= U\left[\partial_\mu \phi + U^{-1}\partial_\mu U \phi - ig U^{-1} A'_\mu U \phi\right]_{(x)} = D'\phi'$$

$$\parallel$$

$$= U D_\mu \phi \quad = \quad U(\partial_\mu - igA_\mu)\phi$$

$$\Rightarrow \boxed{A'_\mu = U A_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}} \qquad N\times N$$

for $U(1)$ we have $U_{(x)} = e^{i\alpha(x)}$

$$A'_\mu = A_\mu + \frac{1}{g}\partial_\mu \theta$$

$$U_{(x)}\Big)_{ab} = e^{i\lambda^k \theta^k(x)}\Big]_{ab} = \delta_{ab} + i\lambda^k_{ab}\theta^k + \cdots$$

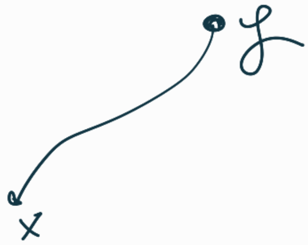
$$\phi \to U\phi$$

$$\delta\phi_a(x) \simeq i\lambda^k_{ab}\phi_b\theta^k + \cdots$$

$$A \to \cdots$$

$$\delta A^k_\mu = i f^{ksj} A^j_\mu \theta^s + \frac{1}{g}\partial_\mu\theta^k + \cdots$$

$$D_\mu^{ab}\phi_b = 0 \quad (\text{انتقال موازی})$$



$$\Rightarrow \partial_\mu \phi_a(x) = ig A_\mu^k \lambda^k_{ab} \phi_b(x)$$

$$\phi_a(y) = \phi_a(x) + ig\int dt \frac{dz_\mu}{dt} A^\mu(z)\phi$$

$$+ (ig)^2 \int_0^1 dt_1 \int_0^{t_1} dt_2 \frac{dz_{\mu_1}(t_1)}{dt_1}\frac{dz_{\mu_2}(t_2)}{dt_2} A^{\mu_1}_{(t_1)} A^{\mu_2}_{(t_2)} \phi(x)$$

$$+ \cdots + (ig)^n \int_0^1 \cdots \int$$

$$I_n = (ig)^n \int_0^1 dt_1 \int_0^{t_1} dt_2 \int \cdots \int_0^{t_{n-1}} dt_n\, F(t_1) \cdots F(t_n)$$

$$= \frac{(ig)^n}{n!} \hat{P}\left[\left(\int_0^1 dt\, F(t)\right)^n\right]$$

$$\phi(y) = \hat{P}\left[\underbrace{e^{ig\int_0^1 dt \frac{dz_\mu}{dt} A^\mu}}_{1+O+O^2+\cdots} \right]\phi(x)$$

$$\hat{P}\left[e^{ig\int dz^\mu A'_\mu}\right] = U_{(y)}\,\hat{P}\,e^{ig\int dz^\mu A_\mu}\,U^{-1}_{(x)}$$

$$\hat{W}_{\Gamma(x,x)} = \hat{P}\left[e^{ig\int_{\Gamma(x,x)} dz^\mu A_\mu}\right]$$

$$\hat{W} \longrightarrow U_{(x)}\,\hat{W}\,U^{-1}_{(x)}$$

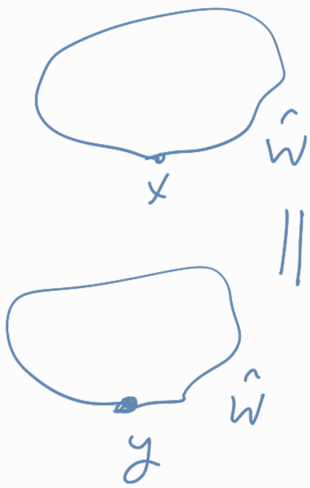
$$\hat{W}_\Gamma = \mathrm{tr}\,\hat{W}_{\Gamma(x,x)} = \mathrm{tr}\left[\hat{P}\left[e^{ig\int_\Gamma dz^\mu A_\mu}\right]\right]$$

$$\downarrow$$

$$\mathrm{tr}\left(U\hat{W}U^{-1}\right) = \mathrm{tr}\,W$$

<u>Non-local</u>

مستقل از x بلکه فقط به نوع Γ بستگی دارد.



$$U(1) \longrightarrow \alpha \int F^{\mu\nu} dS_{\mu\nu} = \Delta\varphi$$

$a(\Gamma)$.
$\ell(\Gamma)$

$$\hat{W} = I + \frac{ig}{2} \iint_{a^{\mu\nu}} dx^\mu \wedge dx^\nu F_{\mu\nu} + O(a^2)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

$U(1) = 0$

$$= i[D_\mu, D_\nu]$$

$$F'_{\mu\nu} = U F_{\mu\nu} U^{-1}$$ not gauge-inv

$$F^{\mu\nu} F_{\mu\nu} \longrightarrow \mathrm{tr}(F^{\mu\nu} F_{\mu\nu})$$ gauge in

$$\boxed{\mathcal{L} = -\frac{1}{4} \mathrm{tr} F_{\mu\nu} F^{\mu\nu}}$$