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T1 ________________	**64089**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2017
MCM/ICM
Abstract Sheet

Novel Approaches to Merge after Toll

Abstract

Shape, size and merging pattern are significant criteria for a toll plaza. Meanwhile, the different types of toll methods are a new design challenge with expressway focusing the most traffic pressure on a toll plaza. And, given that the construction cost of a toll plaza can be very high, so it is an urgent matter to tackle the merging of a toll plaza under the condition of least cost and best road capacity.

In order to study the influence of different merging methods on road capacity, we used the principle of cellular automata, the idea of difference to mesh the toll plaza and introduce the concept of cellular flow. We used the flow propagation from each cell to quantify the vehicle flow to establish a discrete traffic flow propagation model.

By controlling the change of the flow in the model and the length of the toll plaza, we found the variation of vehicle flow and density. We quantified the difference between autonomous vehicles and manual vehicles, and analyzed the situation in which the proportion of autonomous vehicles increases gradually. Using MatLab to simulate different merging patterns, we addressed the conclusion that a longer merging area can lead to a better traffic effect.

Our model has good robustness and feasibility to solve the problem of merging in toll plazas. It can be used in situations where there are a wide range of different vehicle flows. We provided a theoretical basis for traffic management depart- ments to better manage toll plazas, and will write to New Jersey Turnpike Authority to recommend our model.

Keywords: Cellular Automata; Differential Thoughts; Merging Pattern; Bottleneck; Multiple Merging Traffic Flow;

Novel Approaches to Merge after Toll

January 24, 2017

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Contents

1 Introduction

1.1 Background

Multi-lane divided limited-access toll highways use "barrier tolls" to collect tolls from vehicles. After payment at tollbooths in a toll plaza, vehicles have to fan in from a larger number of egress lanes to a smaller number of regular travel lanes. This results in the toll plaza causing traffic congestion. Hence, optimizing the design of a toll plaza to achieve the highest traffic efficiency in the smallest occupied area is an important matter. The current situation of toll plaza is shown in Fig.1.

(a) Close-up Image (b) Toll Plaza

Figure 1: Toll Station

1.2 Previous Work

Up to the present, there is a pretty common phenomenon in much of the world where the proportion of electronic toll collection is uncoordinated with manual toll collection. This leaves too many electronic tollbooths idle while the manual tollbooths are congested. After passing the tollbooth, almost all toll plazas have a fan-shaped area to achieve the merging of the cars, which leads to widespread traffic congestion in peak times. In addition, road traffic pressure increases with the increasing number of autonomous vehicles, which will also have particular impacts on road traffic flow.

1.3 Our Work

The easiest solution is to increase the length of the plaza which can reduce the pressure of merging, but it can also increase the area which means more construction cost. Thus, we need to find a design scheme with the best shape and the smallest shape size under the condition of optimal vehicle capacity. The merging pattern plays a key role in the shape size of the toll plaza so this is the reason why we will describe it in detail.

We studied the influence of different shapes, sizes and merging patterns on traffic congestion in a toll plaza and proposed a cellular automaton model based on discrete vehicle flow. Meanwhile, we consider the influence of the increasing number of autonomous vehicles and the proportions of conventional tollbooths, exact-change tollbooths and electronic toll collection booths to traffic capacity and the construction cost.

2 General Assumptions

In practice, the real situation is difficult to compute due to the complex environment. Thus simplifications and assumptions must be introduced to make the problem tractable.

- The vehicle flow pass through the tollbooth can be approximately seen as following Poisson distribution[1].
- We suppose that the exit and entrance is separated and set symmetric so we only study half of the plaza.
- We do not take into account traffic accidents, man-made interference and road obstruction in the simulation.
- In reality autonomous driving is different from manual driving, but in order to simplify the model, we assume that autonomous driving can be represented as manual driving in a low-flow condition.

3 Symbols and Notion's Definitions

- **Autonomous Vehicle**
  An autonomous vehicle is a vehicle that is capable of sensing its environment by using a variety of techniques such as radar, lidar, GPS, odometry, and computer vision and navigation without human input. Those techniques can interpret sensory information to identify appropriate navigation paths, as well as obstacles and relevant signage to arrive at the desired destination[4].
- **Electronic toll collection (ETC)**
  ETC aims to eliminate the delay on toll roads by collecting tolls electronically. ETC charges the owner of passing cars electronically without requiring them to stop[5].

- **Exact-change:**
  Exact-change means that you can pay in currency the exact amout of the fare without any change in return. A cash receipt is a paper voucher that you could receive to show that you paid a particular amount in cash.

Symbols	Definitions
B	Number of Tollbooth Lanes
L	Number of Conventional Lanes
Q	Traffic Flow
K	Traffic Density
K_c	Critical Density
K_J	Congestion Density
$MaxQ$	Maximum Traffic Flow
U	Discrete Map Set
u_i	Map Unit
S	Number of Vehicles
U_e	Edge Units Set
q_{out}	Throughput
Q_{out}	Sum of Throughput

4 Analysis of the Problem

Generally, the tollbooth area is generally a fan-shaped area (Fig.2) with B tollbooth lanes and L conventional lanes to merge, where $B > L$. The merging area is shown in Fig.2, and its capacity is relevant to its area[7].

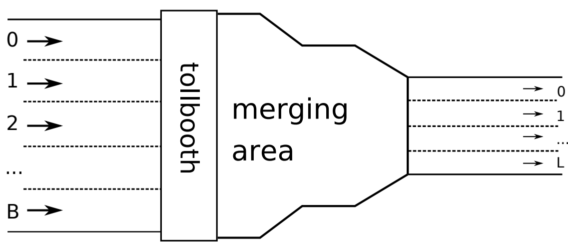


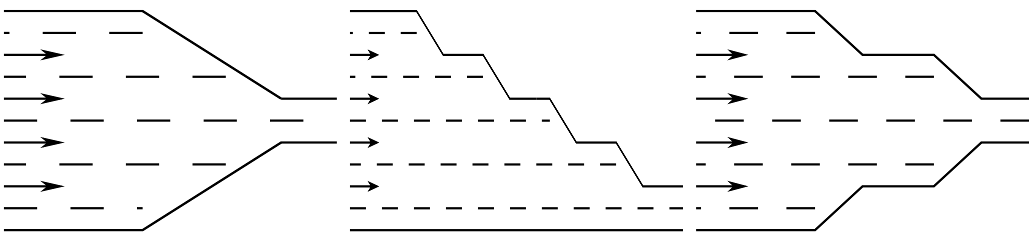
Figure 2: Merging Area

We assume that all vehicle flow which passes through the tollbooth follows a Poisson distribution which means the much closer to the tollbooth of the intermediate lane, the more vehicle flow it has. From the tollbooth after the merging in the fan-shaped area of the model, the merging pattern in the fan area after passing through the tollbooth can be divided into two types(Fig.3).

- Due to the different length of the toll plaza: Single or multiple merging patterns.

- Due to the different merging position: One-side or double-side merging patterns.

We had considered three ways to merge: 1) Single Merging; 2) One-side Multiple Merging; 3) Double-side Multiple Merging.



(a) Single Merging (b) One-side Multiple Merging (c) Double-side Multiple Merging

Figure 3: Merging Methods

The main factor to measure the capacity of the toll plaza is to consider the maximum traffic capacity when the vehicles pass through it, which means the maximum vehicle flow should be under no traffic congestion. Therefore, we discretize the merging area and establish a cellular automaton model.

5 The Establishment

The research of the capacity of traffic bottleneck generally based on traffic flow theory.[3][8]. We choose a new simulation method. In order to quantitatively simulate the influence of shape and area on the vehicle flow when merging, the toll plaza is divided into a finite and continuous unit cell. Each cell can be regarded as a cell and the value of each cell is the vehicle flow (the cells vehicle flow is defined as the number of vehicles passing on the cell per unit of time). Each cell dispersed in the regular grid follows the same motion rules and updates synchronously according to the determined local rules. A large number of cells form a dynamic systematic evolution by simple interaction.

Let U represents discrete map set. Then the each cell is u_i, with conditions

$$0 < i < m$$

and the cell's density is

$$K_i = D(u_i)$$

Evolution rules from the first column of the left side generate the flow of Q, all the propagation directions of the flow are as shown(Fig.4) All cells propagate its flow according to propagation direction and the status of the surrounding

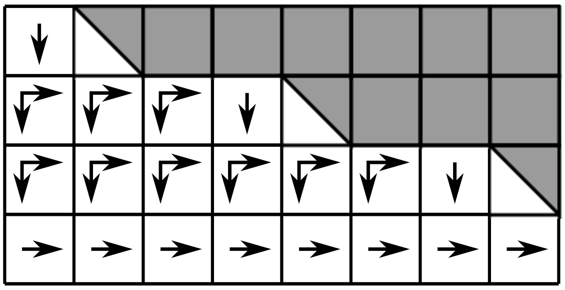


Figure 4: Evolution Rules

lattice. The number of flow propagation is controlled by a function with figure as follows (Fig.5)

$$F(K) = \frac{2}{1+e^K} \tag{1}$$

For the propagation flow Q, we have:

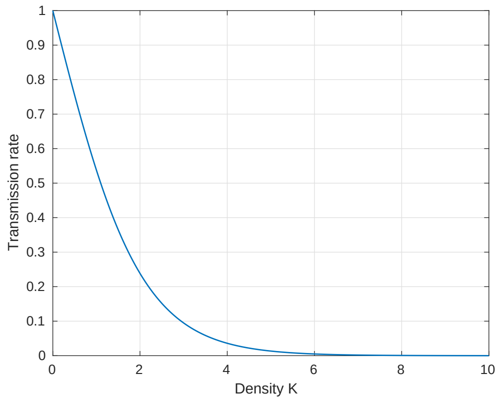


Figure 5: Propagation Coefficient Function

$$Q = K \cdot F(K) \tag{2}$$

So the relation of Density K and Flow Q is shown in Fig.6. It can be proved as a right relation from past research[6]. Where K_c is critical density, K_j is congestion density, $MaxQ$ is most probable propagation flow. It can be seen in Fig.7(a), that the vehicle flow of each cell up to 7 is considered saturated. If a cell has only one propagation direction, this cell just needs to simply determine how much vehicle flow to the next cell by the propagation function. As shown in Fig.7(a), the darker the color the greater the flow. Then we will get the equation

$$\begin{aligned} Q_i &= K_i \cdot F(K_{i+1}) \\ K_{i+1} &= K_i + Q_i \\ K_i &= K_i - Q_i \end{aligned}$$

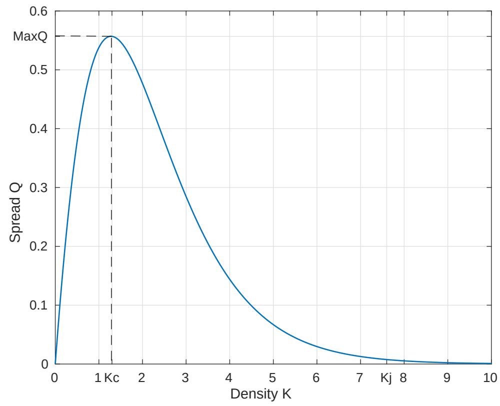


Figure 6: Spread Flow Q for Density K

If a cell has a different choice which means two propagation directions, the cell will first make a decision of a major propagation direction according to the different flows in propagation direction. For instance: If the flow below the cell is less while more in the front, the main part of flows propagation will focus on the bottom. Pseudocode is shown in Algorithm 1 and propagation is shown in Fig.7(b), furthermore, $Q' > Q$.

Figure 7: Spread Rules

When it comes to the end of the cycle of evolution, set the density of each cell u_i as K_i, then the number of vehicles S is

$$S = K_0 + K_1 + \cdots + K_m = \sum_{i=0}^{i \in U} K_i \tag{3}$$

The out-flow q_{out} of the toll plaza can be calculated by the following equation. Let $U_e = \{u_{e1}, u_{e2}, \ldots, u_{en}\}$ represents the cell set in the exit.

$$q_{out} = \sum_{u_i' = u_{e1}}^{u_i' \in U_e} D(u_i') \cdot F(D(u_i')) \tag{4}$$

Algorithm 1 Flow Control

Require: Self density k, Forward density kl, Downword density kd
Ensure: The result of flow control
  function FLOWCTL(k, kd, kl)
    if $kd < kl$ **then**
      $k, kd \leftarrow$ DOWNWARD(k, kd)
      $k, kl \leftarrow$ FORWARD(k, kl)
    else
      $k, kl \leftarrow$ FORWARD(k, kl)
      $k, kd \leftarrow$ DOWNWARD(k, kd)
    end if
    return k, kd, kl
  end function

Then, in the period of $t \in (0, T)$, let Q_{out} represents the total out-flow of toll plaza

$$\begin{aligned} Q_{out} &= \sum_{t=o}^{t=T} q_{out} \\ &= \sum_{t=0}^{t=T} \sum_{u'_i=u_{e1}}^{u'_i \in U_e} D(u'_i) \cdot F(D(u'_i)) \end{aligned} \tag{5}$$

When the road traffic increases, the probability of an accident will increase.

6 The Model Results

6.1 The Choice of Merging Patterns

The distribution of vehicle flow to the tollbooth follows the Poisson distribution(Eq.6) after the vehicle passes through the tollbooth. When$\lambda = 6$, Poisson distribution is shown in Fig.8.

$$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{6}$$

In order to approach the optimal shape of the merging area, we quantitatively analyzed the situation of $B = 10$ and $L = 2$. The probability of arriving vehicles is shown below, x axis represents tollbooth Bi, y axis represents the arrive probability of tollbooth B_i .

Using above-mentioned model to simulate the influence of length of merging area for traffic capacity so that separate the merging area into single merging (small) and multiple merging (long). The simulation result is shown below:

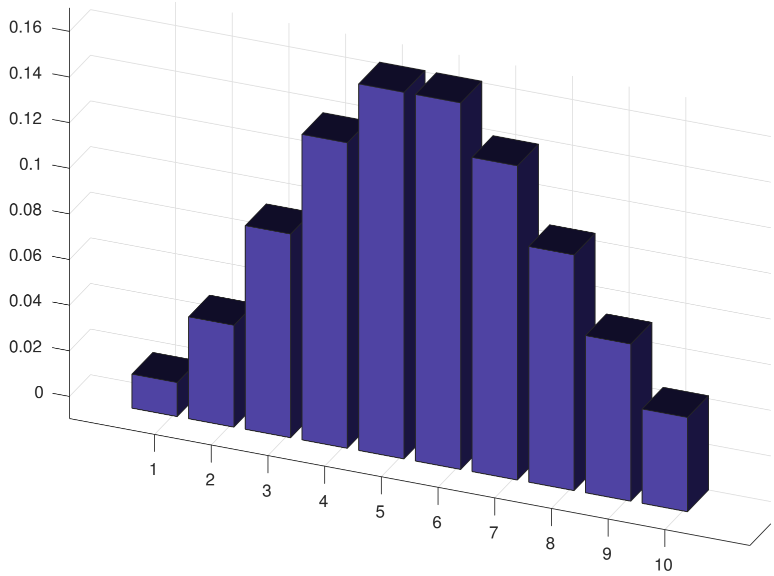


Figure 8: Poisson Distribution

6.1.1 Single Merging

Single merging refers to a kind of merging pattern that vehicles only need one time to merge from B tollbooth lanes to L conventional routes. Under this circumstance, multiple lanes merge at the same time easily prone to traffic congestion even accidents. Using MatLab to simulate the above method, results are shown in Fig.9, the final out-flow is 71.4.

6.1.2 Multiple Merging

Multiple merging refers to a kind of merging pattern that vehicles need more than one time to merge from B tollbooth lanes to L conventional routes. Under this circumstance, the number of merging depends on lanes. In this process, drivers can choose lanes with less vehicles and a more convenient route to merge. Using MatLab to simulate the above method, results are shown in Fig.10, the final out-flow is 93.1.

According to the above-mentioned comparison of one-side and double-side merging patterns, we can see that the overall throughput of the double-side road is larger than the one-side road under the condition of a certain area of the toll station. Comparing the single merging pattern to the multiple merging pattern, it is obvious that the overall throughput of the multiple merging pattern is more than the single merging pattern. It can be seen from the multiple merging patterns simulation, a longer merging area can bear a larger vehicle flow, while too long an area will cause too much construction cost.

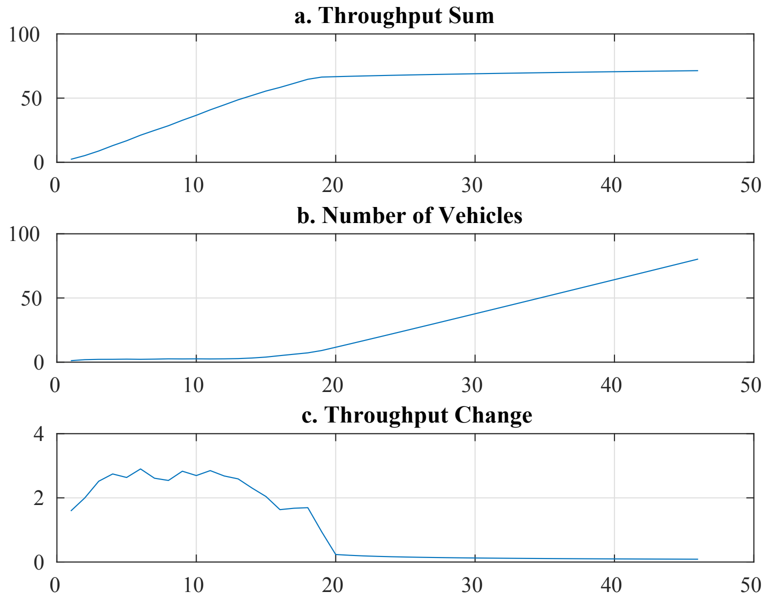


Figure 9: Result of Single Merging

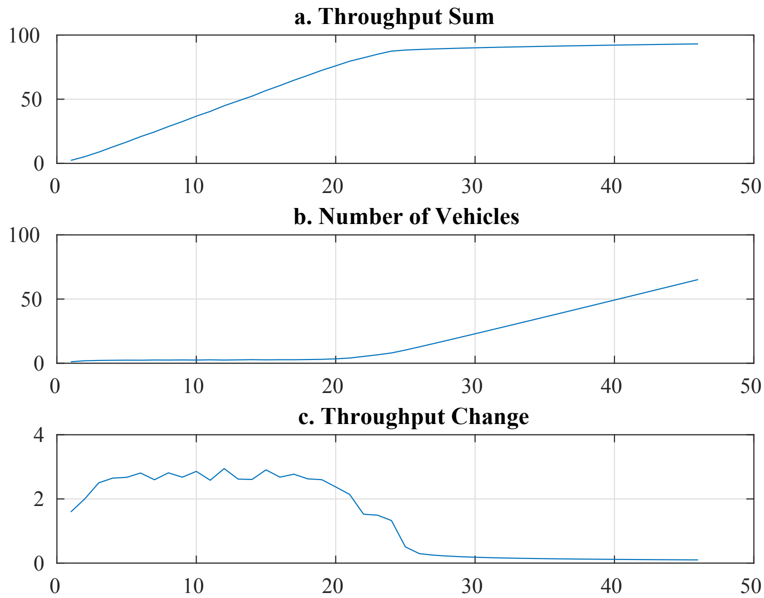


Figure 10: Result of Multiple Merging

6.2 Accident Prevention

The probability of the occurrence of accidents will increase when the traffic density increases[9] The traffic density obtained from the simulation is shown in Fig.11. Vehicle flow is used to determine on which roads the probability of an accident is higher and carry out advance prevention.

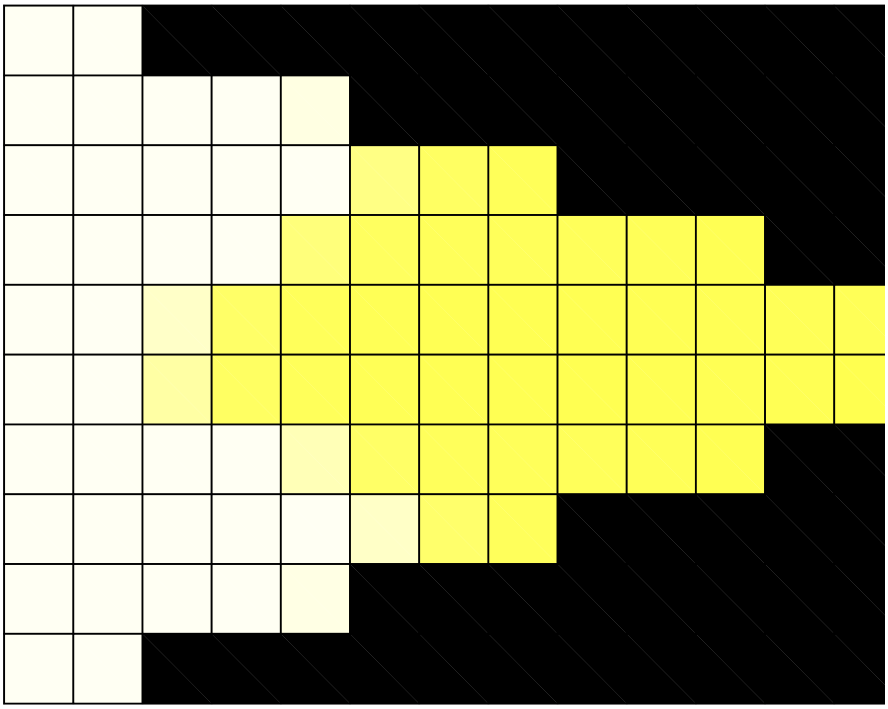


Figure 11: Density Map

In this figure, the black block represents the after-toll area, the yellow block represents the vehicle flow with time-moving. The darker of the color, the more flow of vehicle.

6.3 The Influence of the Number of Vehicle Flow

According to the above-mentioned model, we study the transit efficiency of tollbooths in small and large vehicle flow respectively. When it comes to the small vehicle flow which means vehicle density is in a low status, vehicles are free to choose a convenient, safe and suitable tollbooth to pay the toll and leave in a unimpeded lane. In the situation mentioned above, vehicles go through the toll plaza at a fast speed. As we can see in Fig.12, when the traffic flow is small, the road capacity is maintained at the highest level.

When the traffic volume is large, the road traffic density increases, and the road is more crowded, as shown in Fig.13. It can be seen when the traffic flow is high, the road capacity is quickly saturated.

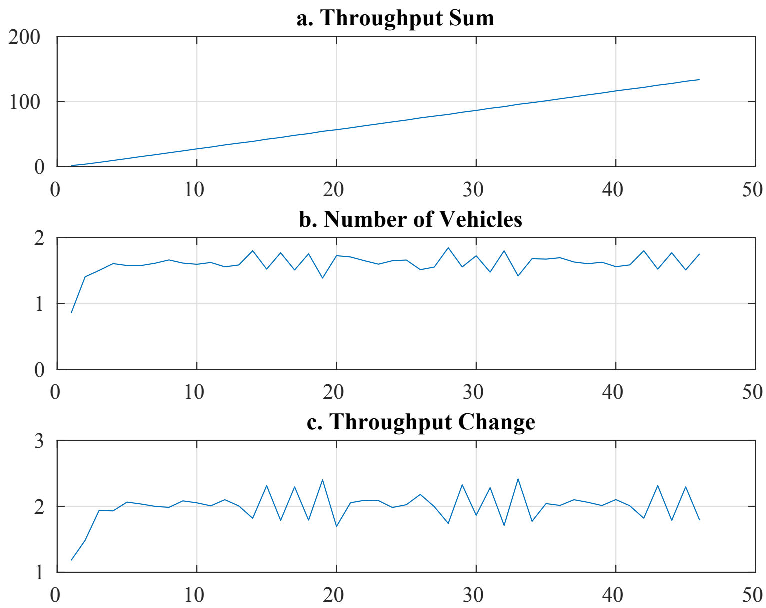


Figure 12: Result in Low Q Condition

For the same section of a road, different sizes of vehicle flow will also make an influence on the shape, size, and merging patterns of the toll plaza. Vehicle flow means the number of vehicles on the road. A low traffic flow means a small number of vehicles on the road, and as it is not crowded, vehicles are free to choose the tollbooth and merging road, but this could reduce road utilization. If the toll plaza area is too large with too many tollbooths, it will cause a waste of resources and the construction costs are too high. A large traffic flow means a big number of vehicles on the road and as it is crowded, vehicles are not free to choose the tollbooth and merging road.

With the maturity of autonomous vehicles technology, the number of autonomous vehicle on the road is increasing gradually. The working principle of autonomous vehicles make it keep a safe distance from the surrounding vehicles and obstructions in order to ensure safety. Moreover, when the traffic volume is large, their speed decreases.

In Fig.12 and 13, Throughput Sum represents total vehicle changes with the variation of time; Number of Vehicles represents the time varying vehicle density in toll plaza; Throughput Change represents the time varying vehicle flow which passes through the toll plaza.

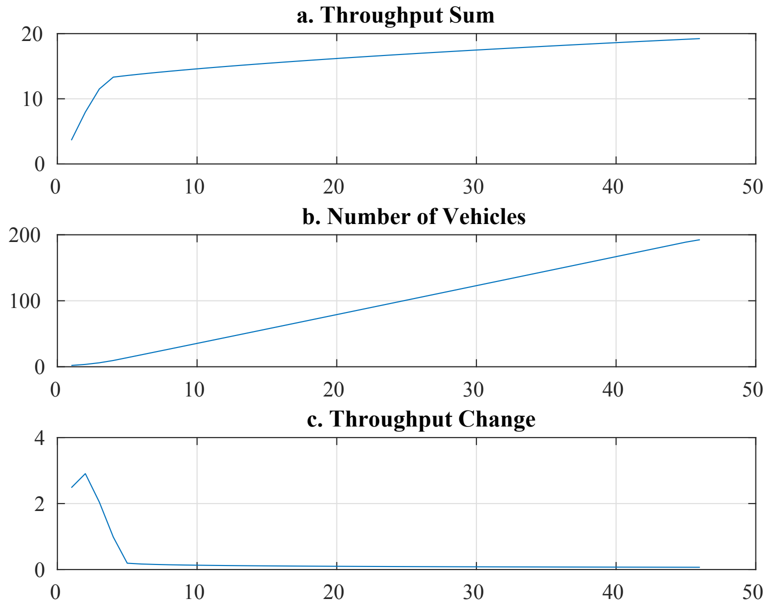


Figure 13: Result in High Q Condition

6.4 The Influence of Autonomous Vehicles

Autonomous vehicles are an emerging technology. With the increasing number of autonomous vehicles , due to the character of autonomous driving technology, the road capacity will be reduced. The autonomous vehicles recognize the distance from other vehicles through sensors and keep the distance and speed. The main differences between autonomous vehicles and manual driving are below. Through the sensor, autonomous vehicles can sense the distance be- tween the vehicle and the surrounding vehicle distance or obstructions to keep vehicles and passenger safe. If the road has large density and flow like in a toll plaza, autonomous vehicles will keep a safe distance so that it will reduce the road capacity. Increasing numbers of autonomous vehicles will reduce the road saturation density.

Table 1: The Difference of Autonomous Vehicles and Manual Vehicles

	Autonomous vehicles	Manual vehicles
Principle	Sense the surroundings by sensor	Sense the surroundings by human
In a large vehicle flow	Keep a safe distance(big)	Small safe distance

6.5 The Influence of Exact-Change & ETC

When the vehicle passes from the traditional toll station, drivers need to stop accepting the bill and pay the tollbooth staff cash, waiting for change. Service time is longer, and easily influenced by staff efficiency.

When passing the exact-change tollbooth, drivers can only pay coins and do not wait for change. Compared to the traditional service time, the time for this kind of service is decreased. When passing the tollbooth by ETC, drivers do not need cash and to stop the vehicle for bill, which reduces the waiting time and improves the road traffic capacity so that the road traffic density increases[2]. The flow chart of the different payment methods is shown in Fig.14. Increasing the electronic payment lanes will increase the roads vehicle flow. The Fig.15 is after ETC lanes opened(Open additional ETC lanes would increases vehicle flow of the road. The following figure shows the difference after ETC lanes opened).

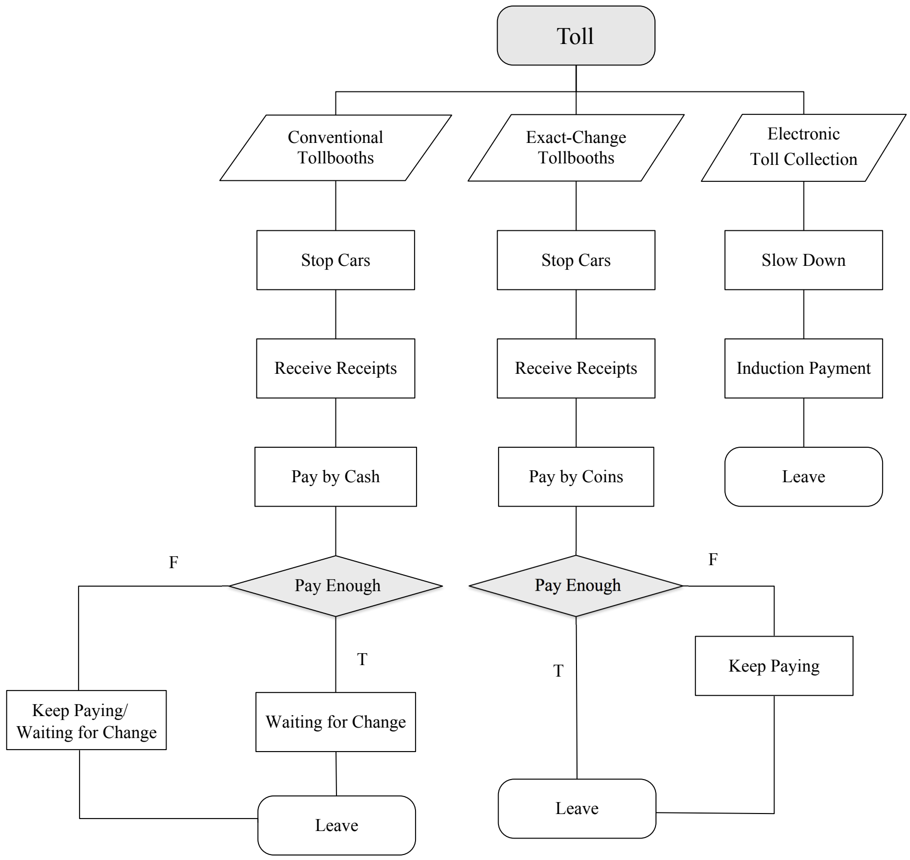


Figure 14: Flow Chart of Three Different Toll Methods

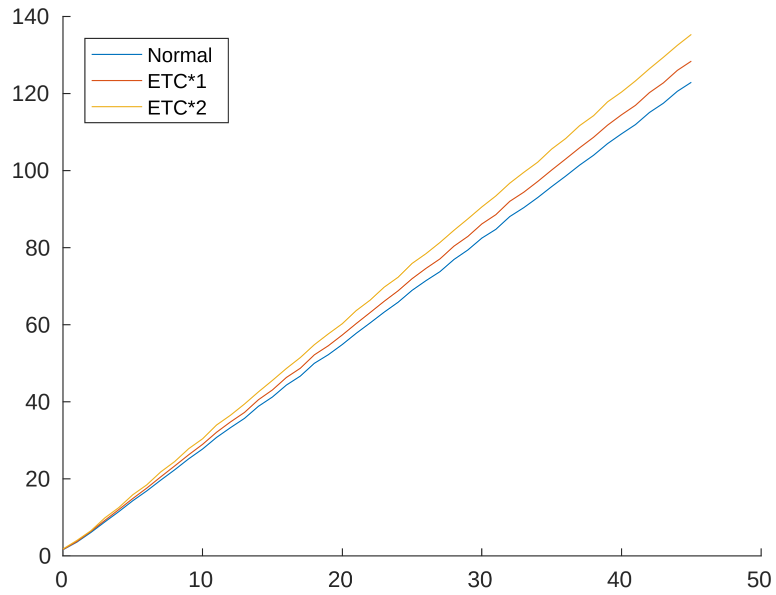


Figure 15: Contrast With or Without ETC

7 Conclusions

We propose a more efficient solution to the toll merging problem with a better shape. It has certain theoretical reference and guidance value to solve the existing problem of traffic congestion and high construction cost in the process of merging.

Given the toll plazas demand of traffic throughput, discrete traffic flow propagation model can be easily to sum the vehicle flow (sum each cells vehicle flow to get the overall vehicle flow directly). Furthermore, discuss the change of vehicle flow under the condition of large and small flow and analyze several kinds of merging methods, we address a conclusion that the multiple double-sides merging is optimal. With the increase of the number of autonomous vehicles, the vehicle flow of the road is reduced, which imposes higher requirements on the capacity of the toll plaza. Different toll methods have great influence on road capacity and additional electronic toll collection booths can increase traffic density.

Our model has good robustness and feasibility to solve the problem of merging in toll plaza and has been tested. Howeverthe intelligence of the autonomous vehicles was neglected when solving the problem of increasing proportion of autonomous vehicles. Our model only considered the distance between the vehicle and the obstruction shows its limitation and still needs to carry out a further study in the future.

8 Strengths and weaknesses

8.1 Strengths

- Our model utilizes the idea of difference. In the process of vehicles passing, vehicle flow changes at any time so that the overall consideration of the vehicle flow change is very difficult and has no practical significance. Hence, we mesh the toll plaza, breaking its entirety into cells because we can then fully analyze the flow change of each cell and then get the overall flow changes.

- Based on the improved cellular automata model, using its continuous transfer (0 represents no, 1 represents have) to calculate the toll plazas vehicle flow. Making the value of each cell continuous and giving the valuation of vehicle flow to each cell,the transfer between cells extends to the transfer of vehicle flow.

- Given the difference between autonomous driving and manual driving, we simplify the condition of autonomous driving as manual driving with less vehicle flow to quantify the difference between them.

8.2 Weaknesses

The toll plazas planning model based on cellular automata does not have the advantage to solve the area, so it is difficult to solve the construction cost, and it can only measure the rationality of the toll area from the angle of vehicle flow.

9 Letter to New Jerseys Turnpike Authority

Dear chief of the New Jersey Turnpike Authority,

I am writing on behalf of myself and Team #64089. We would like to take this opportunity to introduce our new method of toll plaza in expressway which can relieve the traffic congestion happened nowadays.

The state of New Jersey is the most densely populated state in the United States. Expressway, as the most important way to travel, its significance of complete layout, construction and post-management is very important. We are very pleased to see the construction and maintenance of expressway road and the popularization of ETC technology, which has further facilitated our people's life.

While in the further study of toll plaza, we found there are still some problems in the expressway now.

Up to now, there is a quiet normal phenomenon that the proportion of electronic toll collection is uncoordinated with manual toll collection. New Jersey's expressway road which makes too many electronic tollbooths idle and manual tollbooth congested. After passing the tollbooth, all vehicles pass through a fan-shaped area to implement the merging which leads to a widespread traffic congestion phenomenon in the peak time. In addition, road traffic pressure increases with the increasing number of autonomous vehicles, meanwhile, it also have a certain impact on the road traffic flow.

We established a mathematical model, optimized and improved the existing problems. Our method is to redesign a way of merging after toll in the expressway. The problem of existing toll plaza is easy to lead to traffic congestion and accident. We simulated the different shape, size and merging pattern, put forward an new measures to tackle the traffic congestion and we tested the validity of it.

Admittedly, what we have discussed in our study is far from complete. It still needs a further study. But thank you for reading. We are looking forward to seeing positive changes if you think our method is feasible. We hope to be able to get your recognition and contribute to the optimization of traffic in New Jersey.

Sincerely yours

COMAP Control #64089

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