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T1 ________________	**64089**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2017
MCM/ICM
Summary Sheet

Modeling merge after toll

Abstract

Shape, size and merging pattern are significant criteria for a toll plaza. The different types of toll way are design challenge width expressways focuses the most traffic pressure on toll plaza. Besides, given that construction cost of a toll plaza can be very high, it is an urgent matter to tackle the merging of toll plaza under the condition of the least cost and the best road capacity. In order to study the influence of different merging methods on road capacity, we used the principle of cellular automata, the idea of difference to mesh the toll plaza and introduce the concept of cellular flow. Using the flow propagation from each cell to quantify the vehicle flow to establish a discrete traffic flow propagation model

By controlling the change of the flow in the model and the length of the toll plaza, we find the variation of vehicle flow and density. Quantify the difference between autonomous vehicles and manual vehicles and analyze the situation when the autonomous vehicles proportion of autopilot increasing gradually. Using MatLab to simulate different merging patterns, we address the conclusion that the longer area of the merging area can lead to a better traffic effect

Our model has good robusticity and feasibility to solve the problem of merging in toll plaza. It can be widely used in toll plaza which in different vehicle flow.

We provide a theoretical basis for the traffic management department to better manage the toll plaza and write to New Jersey Turnpike Authority to recommend our model.

Keywords: Cellular Automata; differential thinking; merging pattern; bottleneck; multiple merging traffic flow;

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January 24, 2017

Abstract

Shape, size and merging pattern are significant criteria for a toll plaza. The different types of toll way are design challenge width expressways focuses the most traffic pressure on toll plaza. Besides, given that construction cost of a toll plaza can be very high, it is an urgent matter to tackle the merging of toll plaza under the condition of the least cost and the best road capacity. In order to study the influence of different merging methods on road capacity, we used the principle of cellular automata, the idea of difference to mesh the toll plaza and introduce the concept of cellular flow. Using the flow propagation from each cell to quantify the vehicle flow to establish a discrete traffic flow propagation model

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Contents

1 Introduction

1.1 Background

Multi-lane divided limited-access toll highways use "barrier tolls" to collect tolls from vehicles. After the payment from tollbooth, vehicles have to fan in from the larger number of egress lanes to the smaller number of regular travel lanes. A toll plaza is the area to build for facilitates the traffic congestion in this process. Hence, optimize the construction scheme of a toll plaza to achieve the highest traffic efficiency in a smaller occupied area is an urgent matter.

1.2 Previous Work

Up to the present, there is a pretty common phenomenon that the proportion of electronic toll collection is uncoordinated with manual toll collection in the vast majority area of the world which makes too many electronic tollbooths idle and manual tollbooth congested. After passing the tollbooth, almost all toll plazas is to take a fan-shaped area to implement the merging of the car which leads to a widespread traffic congestion phenomenon in the peak time. In addition, road traffic pressure increases with the increasing number of autonomous vehicles, meanwhile, it also have a certain impact on the road traffic flow.

1.3 Our Work

The easiest solution is to increase the length of the plaza which can reduce the pressure of merging, meanwhile, it can also increase the area which means more construction cost. Thus, we need to find a design scheme with best shape and the smallest shape size under the condition of optimal vehicle capacity. Besides, the merging pattern plays a key role to the shape size of the toll plaza so this is the reason why we will describe it in detail.

We study the influence of different shapes, sizes and merging patterns on traffic congestion in a toll plaza and propose a cellular automaton model based on discrete vehicle flow. Meanwhile, we consider the influence of the increasing number of autonomous vehicles and the proportions of conventional tollbooths, exact-change tollbooths and electronic toll collection booths to traffic capacity and the construction cost.

An autonomous vehicle is a vehicle that is capable of sensing its environment by using a variety of techniques such as radar, lidar, GPS, odometry, and computer vision and navigating without human input. Those techniques can interpret sensory information to identify appropriate navigation paths, as well as obstacles and relevant signage to arrive the desired destination[3].

2 General Assumptions

In practice, the real situation is difficult to compute due to the complex environment. Thus simplifications and assumptions must be introduced to make the problem tractable.

- The vehicle flow pass through the tollbooth can be approximate seen as following Poisson distribution.
- We suppose that the exit and entrance is separated and set symmetric so we only study the half of the plaza.
- We do not take into account traffic accidents, man-made interference and road obstruction in the simulation.
- In fact in reality the autonomous driving is different from manual driving, but in order to simplify the model, we assume that autonomous driving can be represented as manual driving in a low-flow condition.

3 Symbols and Definitions

Symbols	Meaning
B	Number of tollbooth lanes
L	Number of conventional routes
Q	Traffic flow
K	Traffic density
K_c	Critical density
K_J	Jam density
$MaxQ$	Maximum traffic flow
U	Discrete map set
u_i	Map unit
S	Number of Vehicles
U_e	Edge units set
q_{out}	Throughput
Q_{out}	Sum of Throughput

4 Analysis of the Problem

Generally, toll booth area is generally a fan-shaped area (Fig.1) with B tollbooth lanes and L conventional routes to merge, where B > L. The merging area is shown in Fig.1, and its capacity is relevant to its area[5].

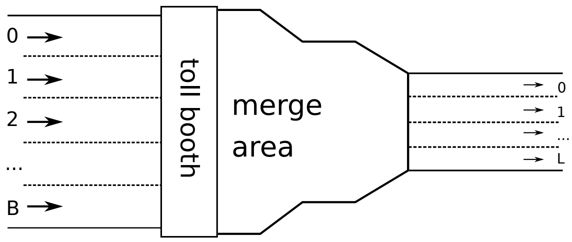
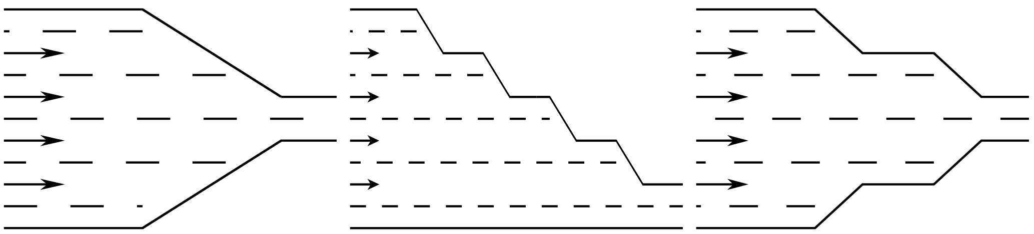


Figure 1: merge area

We assume that all vehicle flow which pass through the tollbooth follow a Poisson distribution which means the much closer to the tollbooth of the intermediate lane, the more vehicle flow it has. From the toll station after the merger in the fan-shaped area of the model, the merging pattern in the fan area after passing through the tollbooth can be divided into two types :(Fig.2)

- Due to the different length of the toll plaza: Single or multiple merging patterns.
- Due to the different merging position: One-side or double-side merging patterns.

We had considered 3 way to merge, 1) direct merge, 2) single side multiple merge, 3) double side multiple merge.



(a) single merging (b) one-side mutiple mering (c) double-side multiple merging

Figure 2: merging ways

The main factor to measure the capacity of the toll plaza is to consider the maximum traffic capacity when the vehicle through it, which means the maximum vehicle flow should be under no traffic congestion. Therefore, discretizing the merging area and establish a cellular automaton model.

5 The Establishment

The research of the capacity of traffic bottleneck generally based on traffic flow theory.[2][6]. We choose a new simulation method. In order to quantitatively simulate the influence of shape and area on the vehicle flow when merging, the toll plaza is divided into a finite and continuous unit cell. Each cell can be regarded as a cell and the value of each cell is the vehicle flow (the cells vehicle flow is defined as the number of vehicles passing on the cell per unit of time). Each cell dispersed in the regular grid follows the same motion rules and updates synchronously according to the determined local rules. A large number of cells form a dynamic systematic evolution by simple interaction.

Let U represent discrete map set. Then the each cell is u_i, with conditions

$$0 < i < m$$

and the cell's density is

$$K_i = D(u_i)$$

.

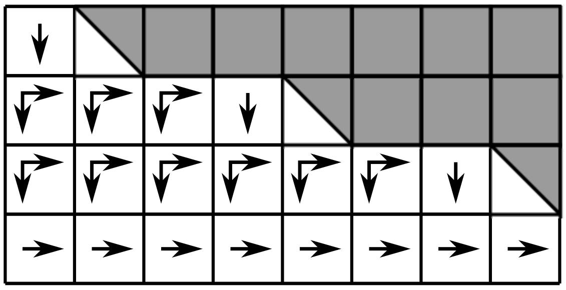


Figure 3: road discrete

Evolution rules From the first column of the left side generate the flow of Q, all the propagation directions of the flow are as shown:(Fig.3) All cells propagate its flow according to propagation direction and the status of surrounding lattice. The number of flow propagated controlled by a function with figure as follows:(Fig.4)

$$F(K) = \frac{2}{1 + e^K} \tag{1}$$

For the propagation flow Q, we have:

$$Q = K \cdot F(K) \tag{2}$$

So the relation of Density K and Flow Q is shown in Fig.5. It can be proved as a right relation from past research[4]. . where K_c is critical density, K_j is congestion density, $MaxQ$ is most probable propagation flow. It can be seen in Fig.6(a), the vehicle flow of each cell up to 7 is considered saturated. If a cell has only one propagation direction, this cell just need simply determine how much vehicle flow to the next cell by the propagation function. As shown in Fig.6(a), the darker the color the greater the flow. Then we will get the equation

$$\begin{aligned} Q_i &= K_i \cdot F(K_{i+1}) \\ K_{i+1} &= K_i + Q_i \\ K_i &= K_i - Q_i \end{aligned}$$

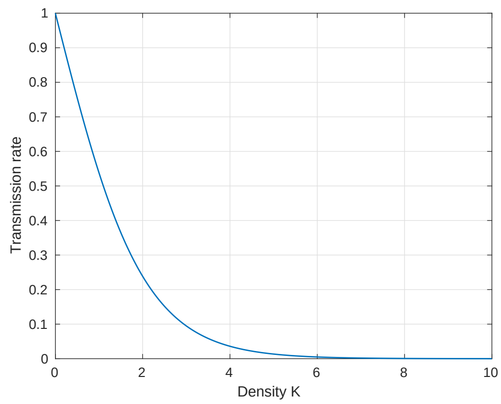


Figure 4: s function

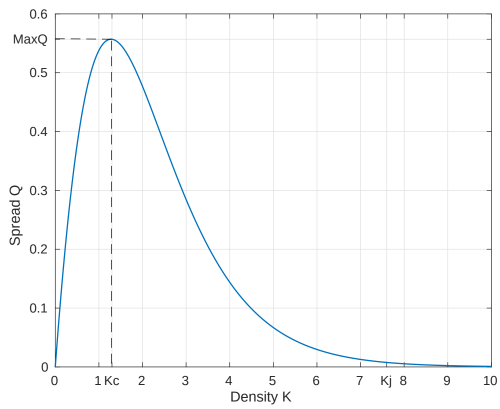


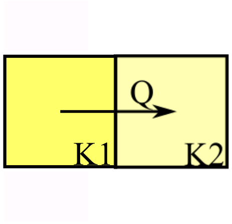
Figure 5: Spread flow Q for density K

If a cell has a different choice which means two propagation directions, the cell will first make a decision of a major propagation direction according to the different flows in propagation direction. For instance: If the flow below the cell is less while more in the front, the main part of flows propagation will focus on the bottom. Pseudocode is shown in Algorithm 1 and propagation are shown in Fig.6(b), furthermore, $Q' > Q$.

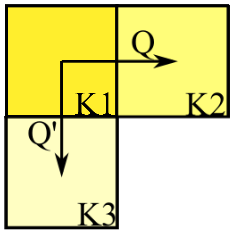
When it comes to the end of the cycle of evolution, set the density of each cell u_i as K_i, then the number of vehicles S is

$$S = K_0 + K_1 + \cdots + K_m = \sum_{i=0}^{i \in U} K_i \tag{3}$$

The out-flow q_{out} of the toll plaza can be calculated by the following equation.



(a)



(b)

Figure 6: Spread Mode

Algorithm 1 Flow Control

Require: Self density k, Forward density kl, Downword density kd
Ensure: The result of flow control

```
function FlowCtl(k, kd, kl)
    if kd < kl then
        k, kd ← Downward(k, kd)
        k, kl ← Forward(k, kl)
    else
        k, kl ← Forward(k, kl)
        k, kd ← Downward(k, kd)
    end if
    return k, kd, kl
end function
```

Let $U_e = \{u_{e1}, u_{e2}, \ldots, u_{en}\}$ represents the cell set in the exit.

$$q_{out} = \sum_{u_i'=u_{e1}}^{u_i' \in U_e} D(u_i') \cdot F(D(u_i')) \tag{4}$$

Then, in the period of $t \in (0, T)$, let Q_{out} represents the total out-flow of toll plaza

$$\begin{aligned} Q_{out} &= \sum_{t=o}^{t=T} q_{out} \\ &= \sum_{t=0}^{t=T} \sum_{u_i'=u_{e1}}^{u_i' \in U_e} D(u_i') \cdot F(D(u_i')) \end{aligned} \tag{5}$$

When the road traffic increases, the probability of an accident will increase.

6 The Model Results

6.1 The Choice of Merging Patterns

The distribution of vehicle flow to tollbooth follows the Poisson distribution (Eq.6) after the vehicle passes through the tollbooth. When$\lambda = 6$, Poisson distribution is shown in Fig.7.

$$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{6}$$

In order to approach the optimal shape of the merging area, quantitatively analyzed the situation of $B = 10$ and $L = 2$. The probability of arriving vehicles is shown below, x axis represents tollbooth Bi, y axis represents the arrive probability of tollbooth B_i .

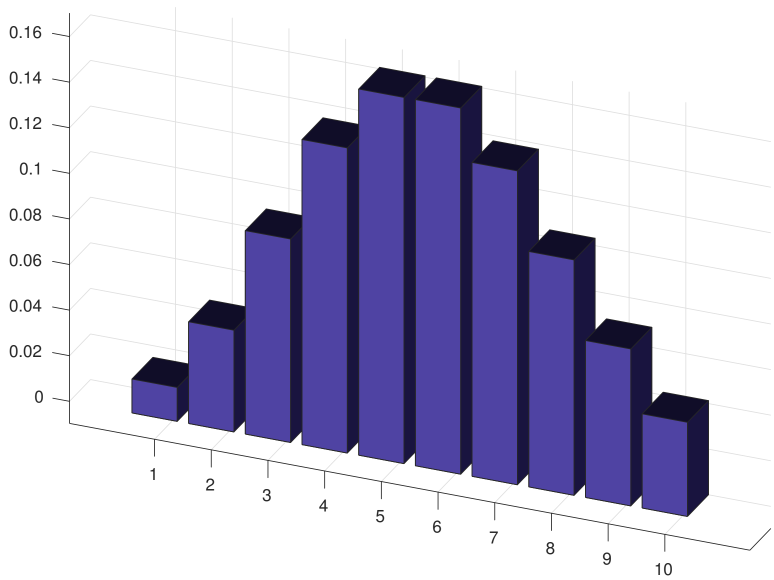


Figure 7: poisson distribution

Using above-mentioned model to simulate the influence of length of merging area for traffic capacity so that separate the merging area into single merging (small) and multiple merging (long). The simulation result is shown below:

6.1.1 Single Merging

Single merging refers to a kind of merging pattern that vehicles only need one time to merge from B tollbooth lanes to L conventional routes. Under this circumstance, multiple lanes merge at the same time easily prone to traffic congestion even accidents. Using MatLab to simulate the above method, results are shown in Fig.8, the final out-flow is 71.4.

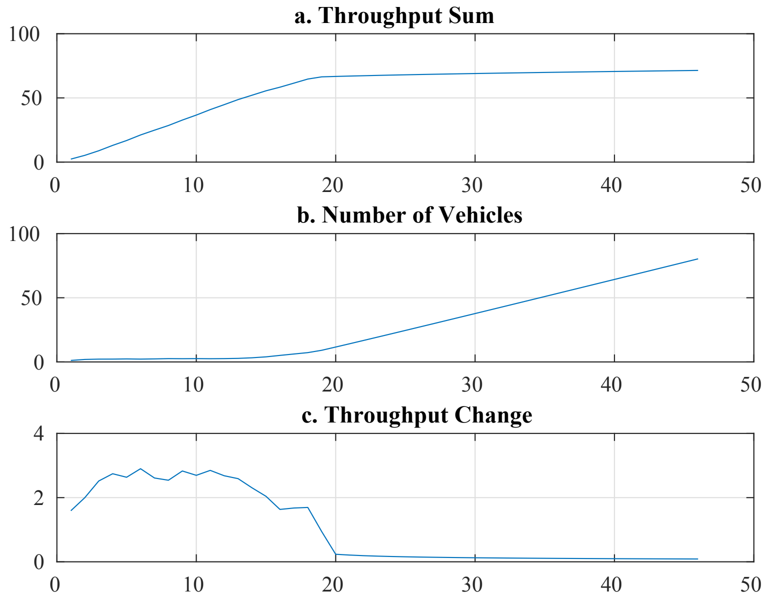


Figure 8: direct out

6.1.2 Multiple Merging

Multiple merging refers to a kind of merging pattern that vehicles need more than one time to merge from B tollbooth lanes to L conventional routes. Under this circumstance, the number of merging depends on lanes. In this process, drivers can choose less-vehicles lanes and more convenient route to merge. Using MatLab to simulate the above method, results are shown in Fig.9, the final out-flow is 93.1.

According to the above-mentioned comparison of one-side and double-side merging pattern, we can see that the overall throughput of the double-side road is larger than the one-side road under the condition of a certain area of the toll station, Comparing single merging pattern to multiple merging pattern, it is obvious that the overall throughput of multiple merging pattern is more than single merging pattern. It can be seen from the multieginitemizeple merging patterns simulation, longer merging area can bear the larger vehicle flow, while too long area will cause too much construction costs.

6.2 Accident Prevention

The probability of occurrence of accidents will increase with the traffic density increases[7] The traffic density is obtained from the simulation is shown in Fig.10. According to vehicle flow to determine which roads probability of an accident is higher and do advance prevention

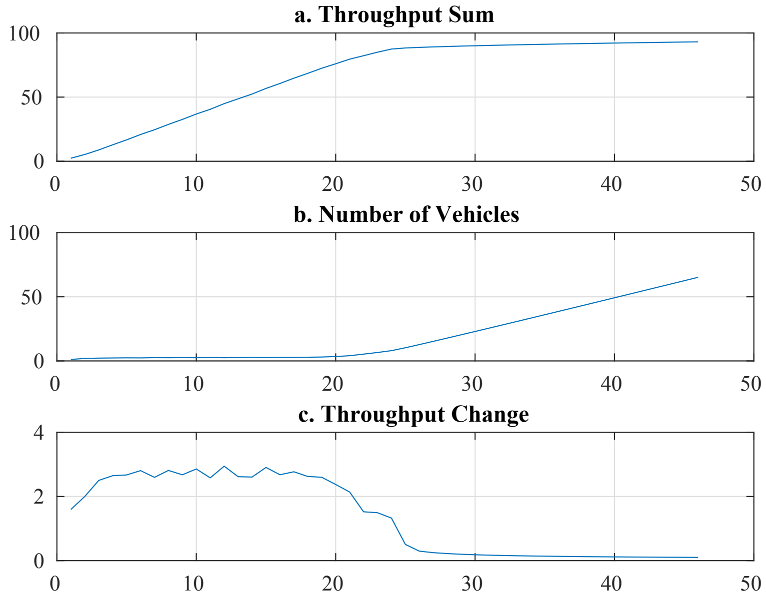


Figure 9: multiple out

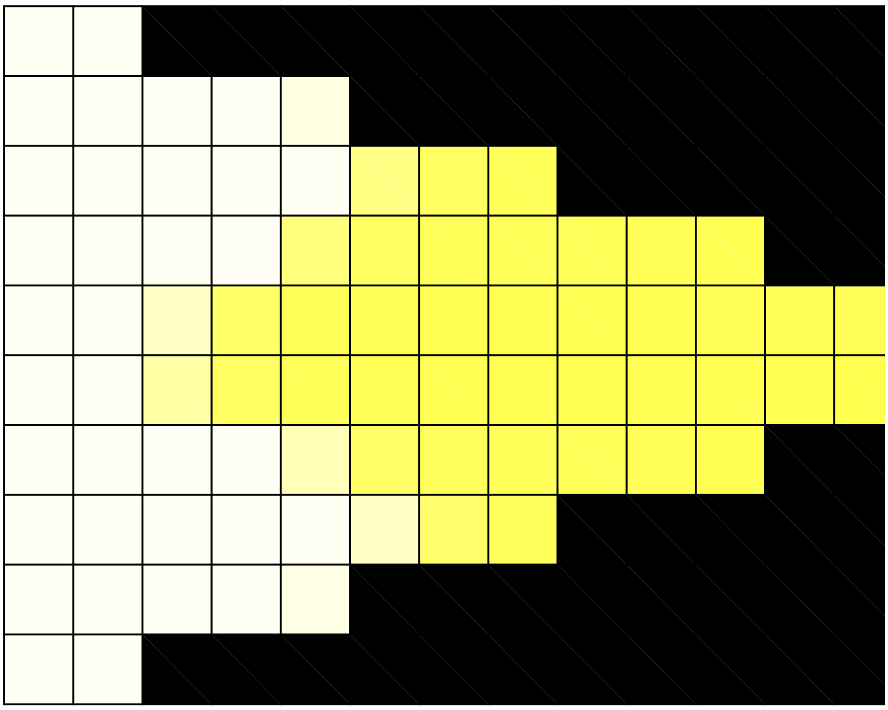


Figure 10: Density map

In this figure, the black block represents the after-toll area, the yellow block represents the vehicle flow with time-moving. The darker of the color, the more flow of vehicle.

6.3 The Influence of the Number of Vehicle Flow

According to the above-mentioned model, study the transit efficiency of tollbooths in small and large vehicle flow respectively. When it comes to the small vehicle flow which means vehicle density is in a low status, vehicles are free to choose convenient, safe and suitable tollbooth to pay toll and left in a unimpeded lane. In the situation mentioned above, vehicles go through the toll plaza in a fast speed. As we can see in Fig.11, when the traffic flow is small, the road capacity is maintained at the highest level.

When the traffic volume is large, the road traffic density, the road is more crowded, the results are shown in Fig.12. It can be seen when the traffic flow is high, the road capacity quickly saturated.

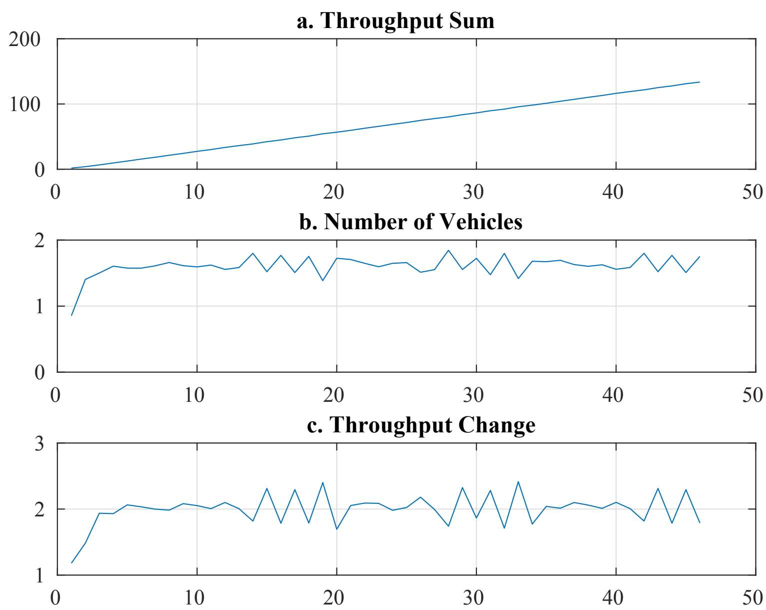


Figure 11: low q

For the same section of a road, different sizes of vehicle flow will also make an influence on the shape, size and merging patterns of toll plaza. Vehicle flow means the number of vehicles on road. Little number of traffic flow means to small number of vehicles on road and not crowded, vehicles are free to choose tollbooth and merging road, but this could reduce road utilization. If the toll plaza area is too large to have too many tollbooths, it will cause the waste of resources and construction costs are too high; large number of traffic flow means to big number of vehicles on road and crowded, vehicles are not free to choose tollbooth and merging road.

With the maturity of autonomous vehicles technology, the number of autonomous vehicle on the road increases gradually. The working principle of autonomous vehicles make it keeps a safe distance from the surrounding vehicles and obstructions in order to ensure their own safety. Moreover, when the traffic volume is large, their speed decreases.

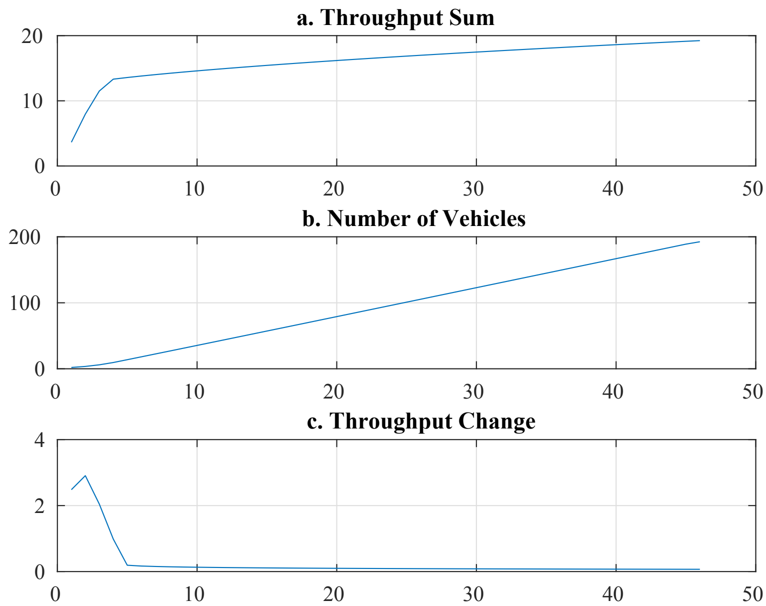


Figure 12: high q

In Fig.11 and 12, throughput sum represents total vehicle changes with the variation of time; q in plaza represents the time varying vehicle density in toll plaza; current out q represents the time varying vehicle flow which pass through the toll plaza.

6.4 The Influence of Autonomous Vehicles

Autonomous vehicles are an emerging technology. With the number of autonomous vehicles gradually increased, due to the character of autonomous driving technology, the road capacity will be reduced. The Autonomous vehicles recognize the distance from the other vehicle through the sensor and keep the distance and speed. The main differences between Autonomous vehicles and manual driving are at below: Through the sensor, autonomous vehicles can sense the distance between the vehicle and the surrounding vehicle distance or obstructions to keep vehicles and passenger safe. In the road of large density and flow like toll plaza, autonomous vehicles will keep a safe distance so that it will reduce the road capacity. Joining autonomous vehicles will reduce the road saturation density.

6.5 The Influence of Exact-Change & ETC

When the vehicle passes from the conventional toll station, drivers need to stop accepting the bill and pay the tollbooth staff cash, waiting for change. Service time is longer, and easy to be influenced by staffs' efficiency. When passing the exact-change tollbooth, drivers can only pay coins and do not wait for change. Comparing to the conventional service time, this kind of service time has decreased. When passing the tollbooth by ETC, drivers do not need cash and to stop vehicle for bill, which reduce the waiting time and improve the road traffic capacity so that the road traffic density increases[1]. The flow chart of the different payment methods are shown in Fig.13. Increasing the electronic payment lanes will increase the roads' vehicle flow. The Fig.14 is after ETC lanes opened(Open additional ETC lanes would increases vehicle flow of the road. The following figure shows the difference after ETC lanes opened).

7 Conclusions

We propose a more efficient solution to the toll merging problem with a better shape. It has certain theoretical reference and guidance value to solve the existing problem of traffic congestion and high construction cost in the process of merging.

Given the toll plazas demand of traffic throughput, discrete traffic flow propagation model can be easily to sum the vehicle flow (sum each cells vehicle flow to get the overall vehicle flow directly). Furthermore, discuss the change of vehicle flow under the condition of large and small flow and analyze several kinds of merging methods, we address a conclusion that the multiple double-sides merging is optimal. With the increase of the number of autonomous vehicles, the vehicle flow of the road is reduced, which imposes higher requirements on the capacity of the toll plaza. Different toll methods have great influence on road capacity and additional electronic toll collection booths can increase traffic density.

Table 1: The difference of autonomous vehicles and manual vehicles

	Autonomous vehicles	Manual vehicles
Principle	Sense the surroundings by sensor	Sense the surroundings by human
In a large vehicle flow	Keep a safe distance(big)	Small safe distance

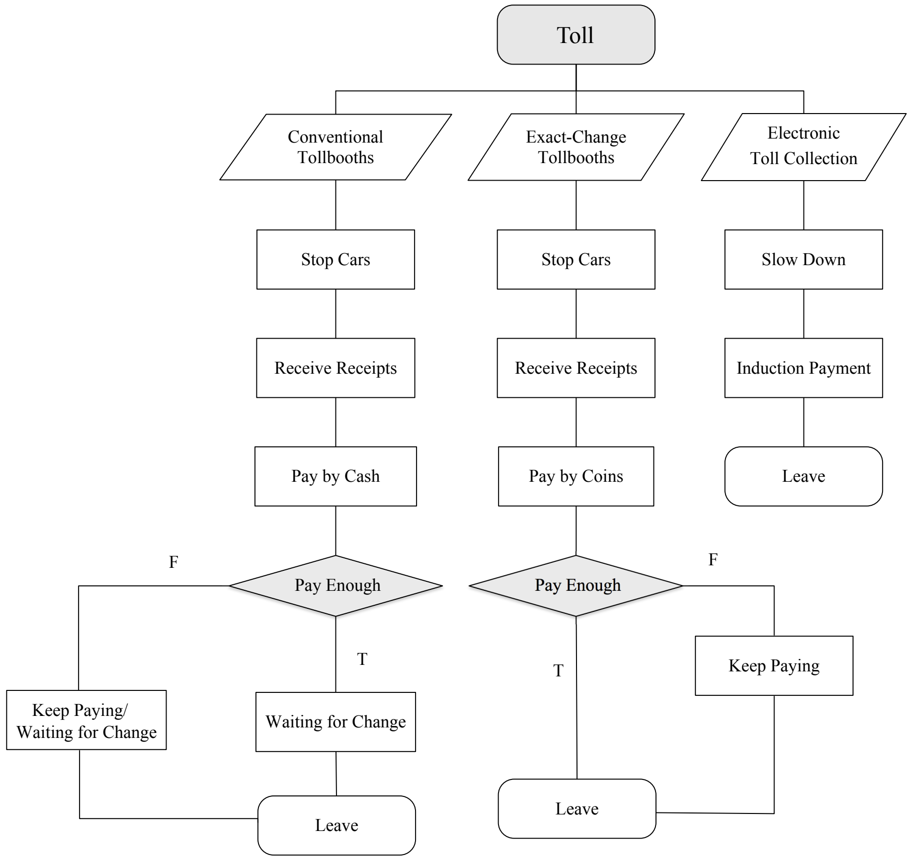


Figure 13: flow chart of three different toll methods

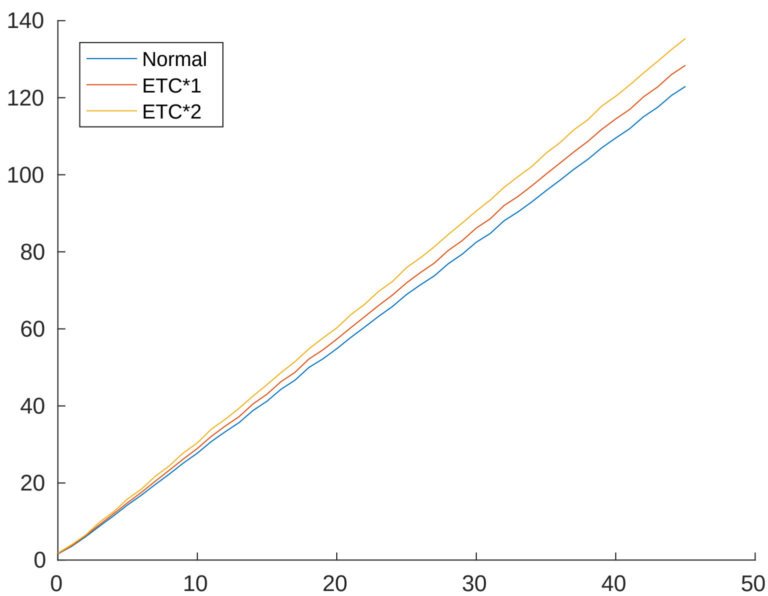


Figure 14: etc

Our model has good robustness and feasibility to solve the problem of merging in toll plaza and has been tested. Howeverthe intelligence of the autonomous vehicles was neglected when solving the problem of increasing proportion of autonomous vehicles. Our model only considered the distance between the vehicle and the obstruction shows its limitation and still needs to carry out a further study in the future.

8 Strengths and weaknesses

8.1 Strengths

- Our model utilizes the idea of difference. In the process of vehicles passing, vehicle flow changes at any time so that the overall consideration of the vehicles flow change is very difficult and has no practical significance. Hence, we will mesh the toll plaza, brake the entirety in to limited cells because it can fully analyze the flow change of each cell and then get the overall flow changes.
- Based on the improved cellular automata model, using its continuous transfer (0 represents no, 1 represents have) to contact the toll plazas vehicle flow. Making the value of each cell continuous and giving the valuation of vehicle flow to each cell. The transfer between cells extends to the transfer of vehicle flow.
- Given the difference between autonomous driving and manual driving, we simplify the condition of autonomous driving as the manual driving while in a less vehicle flow to quantify the difference between them.

8.2 Weaknesses

The toll plazas planning model based on cellular automata does not have the advantage to solve the area, so it is difficult to solve the construction cost, and it can only measure the rationality of the toll area from the angle of vehicle flow.

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