

SCED Formulation

Sojin Park, Ross Baldick, and Hunyoung Shin

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Nomenclature

Acronyms

PFR	Primary Frequency Response
QSS	Quasi-Steady State

Sets/Indices

$i \in \mathcal{I}$	Set of synchronous generators
$j \in \mathcal{J}$	Set of inverter-based resources

Decision Variables

P_k^E	Power dispatch of resource $k \in \mathcal{I} \cup \mathcal{J}$ [MW]
$P_k^{\text{PFR,r}}$	Power reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for ramp response of PFR [MW]
$P_k^{\text{PFR,d}}$	Power reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for droop response of PFR [MW]
P_k^{In}	Power reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for inertia service [MW]
P^{In}	Total power reservation for inertia [MW]
$P^{\text{PFR,r}}$	Total power reservation for ramp response of PFR [MW]
$P^{\text{PFR,d}}$	Total power reservation for droop response of PFR [MW]
E_k^{PFR}	Energy reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for PFR [MWh]
E_j	State of charge of resource $j \in \mathcal{J}$ in dispatch interval [MWh]
ΔP^L	Magnitude of largest contingency for which security is to be guaranteed [MW]
D_j	Proportionality factor between change in power and rate of change of frequency of resource $j \in \mathcal{J}$ [MW/(Hz/s)]
Loss_j^E	Power loss term for energy service due to a single-directional efficiency of resource $j \in \mathcal{J}$ [MW]
$\text{Loss}D_j^E$	Power loss term of resource $j \in \mathcal{J}$ in discharging state [MW]
$\text{Loss}C_j^E$	Power loss term of resource $j \in \mathcal{J}$ in charging state [MW]
E_k^{In}	Energy reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for inertia service [MWh]

Data/Parameters

P^D	Power demand of the load [MW]
η_j	Round-trip efficiency of resource $j \in \mathcal{J}$ [%]
T^{PFR}	Maximum required activation time of PFR [seconds]
$\overline{\Delta f}$	Maximum allowable magnitude of the rate of change of frequency [Hz/s]
$\overline{\Delta f}_{\text{Nad}}$	Maximum allowable frequency deviation from nominal frequency at the frequency nadir [Hz]
$\overline{\Delta f}_{\text{QSS}}$	Maximum allowable frequency deviation from nominal frequency at Quasi-Steady-State [Hz]
\underline{P}_k	Minimum power capacity of resource $k \in \mathcal{I} \cup \mathcal{J}$ [MW]
\overline{P}_k	Maximum power capacity of resource $k \in \mathcal{I} \cup \mathcal{J}$ [MW]
$\overline{P}_k^{\text{PFR,r}}$	Maximum PFR capability of resource $k \in \mathcal{I} \cup \mathcal{J}$ for ramp response [MW]
$\overline{P}_k^{\text{PFR,d}}$	Maximum PFR capability of resource $k \in \mathcal{I} \cup \mathcal{J}$ for droop response [MW]
$\overline{P}_i^{\text{Gov}}$	Maximum governor-controlled capacity of resource $i \in \mathcal{I}$ [MW]
D_i	Proportionality factor between change in power and RoCoF of resource $i \in \mathcal{I}$ [MW/(Hz/s)]
\overline{D}_j	Maximum proportionality factor between change in power and RoCoF of resource $j \in \mathcal{J}$ [MW/(Hz/s)]
\underline{E}_j	Minimum state of charge of resource $j \in \mathcal{J}$ [MWh]
\overline{E}_j	Maximum state of charge of resource $j \in \mathcal{J}$ [MWh]
E_{j0}	Initial state of charge of resource $j \in \mathcal{J}$ [MWh]
f_0	Nominal system frequency [Hz]
ΔT	Dispatch interval [minutes]

Formulation

- **Objective Function**

$$\min \sum_{i \in \mathcal{I}} C_i(P_i^E, P_i^{\text{PFR,r}}, P_i^{\text{PFR,d}}) + \sum_{j \in \mathcal{J}} C_j(P_j^E, P_j^{\text{PFR,r}}, P_j^{\text{PFR,d}}, P_j^{\text{In}}).$$

- **System-wide Constraints**

$$\begin{aligned} \sum_{i \in \mathcal{I}} P_i^E + \sum_{j \in \mathcal{J}} P_j^E &= P^D \\ P^{\text{In}} &= \sum_{k \in \mathcal{K}} P_k^{\text{In}} \\ P^{\text{PFR,r}} &= \sum_{k \in \mathcal{K}} P_k^{\text{PFR,r}} \\ P^{\text{PFR,d}} &= \sum_{k \in \mathcal{K}} P_k^{\text{PFR,d}} \\ P_k^E &\leq \Delta P^L, \quad \forall k \in \mathcal{I} \cup \mathcal{J} \\ P^{\text{In}} &\geq \Delta P^L \\ \left(\frac{P^{\text{In}}}{2\overline{\Delta f'}} \cdot \frac{P^{\text{PFR,r}}}{T^{\text{PFR}}} \right) &\geq \frac{(\Delta P^L)^2}{4\overline{\Delta f}_{\text{Nad}}} \\ P^{\text{PFR,d}} &\geq \Delta P^L. \end{aligned}$$

- **Individual Constraints**

– for a synchronous generator $i \in \mathcal{I}$

$$\begin{aligned} \underline{P}_i &\leq P_i^E \leq \overline{P}_i \\ P_i^E + P_i^{\text{PFR,r}} &\leq \overline{P}_i^{\text{Gov}} \\ P_i^E + P_i^{\text{PFR,d}} &\leq \overline{P}_i \\ P_i^{\text{PFR,r}} &\leq \overline{P}_i^{\text{PFR,r}} \\ P_i^{\text{PFR,d}} &\leq \overline{P}_i^{\text{PFR,d}} \\ P_i^{\text{PFR,d}} &\leq P_i^{\text{PFR,r}} \\ P_i^{\text{In}} &= D_i \overline{\Delta f'}. \end{aligned}$$

– for an inverter-based resource $j \in \mathcal{J}$

$$\begin{aligned}
& \underline{P}_j \leq P_j^E \leq \bar{P}_j \\
& P_j^E + P_j^{\text{PFR,r}} + P_j^{\text{In}} \leq \bar{P}_j \\
& \underline{P}_j \leq P_j^E - \frac{P_j^{\text{In}}}{\eta_j} \\
& P_j^{\text{PFR,r}} \leq \bar{P}_j^{\text{PFR,r}} \\
& P_j^{\text{PFR,d}} \leq \bar{P}_j^{\text{PFR,d}} \\
& P_j^{\text{PFR,d}} \leq P_j^{\text{PFR,r}} \\
& P_j^{\text{In}} = D_j \overline{\Delta f'} \\
& 0 \leq D_j \leq \bar{D}_j \\
& \underline{E}_j + \frac{E_j^{\text{In}}}{\sqrt{\eta_j}} + \frac{E_j^{\text{PFR}}}{\sqrt{\eta_j}} \leq E_{j0} \\
& \underline{E}_j + \frac{E_j^{\text{In}}}{\sqrt{\eta_j}} + \frac{E_j^{\text{PFR}}}{\sqrt{\eta_j}} \leq E_j \\
& E_j^{\text{PFR}} = \frac{1}{2} T^{\text{PFR}} P_j^{\text{PFR,r}} + P_j^{\text{PFR,d}} (\Delta T - T^{\text{PFR}}) \\
& E_j^{\text{In}} = D_j \overline{\Delta f}_{\text{Nad}} \\
& E_j = E_{j0} - (P_j^E + Loss_j^E) \Delta T \\
& \underline{E}_j \leq E_j \leq \bar{E}_j \\
& LossD_j^E \leq Loss_j^E \\
& LossC_j^E \leq Loss_j^E \\
& LossD_j^E = \left(\frac{1}{\sqrt{\eta_j}} - 1 \right) P_j^E \\
& LossC_j^E = (\sqrt{\eta_j} - 1) P_j^E.
\end{aligned}$$

• Others

$$\begin{aligned}
& P_i^E, P_i^{\text{PFR,d}}, P_j^{\text{PFR,d}}, P_i^{\text{PFR,r}}, P_j^{\text{PFR,r}}, P_i^{\text{PFR}}, P_j^{\text{PFR}}, P_i^{\text{In}}, P_j^{\text{In}}, E_j^{\text{PFR}}, E_j^{\text{In}}, E_j, D_j, \\
& P^{\text{In}}, P^{\text{PFR,r}}, P^{\text{PFR,d}}, \Delta P^L \geq 0, \quad \forall i, j.
\end{aligned}$$