SCED Formulation

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Nomenclature

Acronyms

PFR	Primary Frequency Response
QSS	Quasi-Steady State

Sets/Indices

$i \in \mathcal{I}$	Set of synchronous generators
$j \in \mathcal{J}$	Set of inverter-based resources

Decision Variables

P_k^{E}	Power dispatch of resource $k \in \mathcal{I} \cup \mathcal{J}$ [MW]
$P_k^{\mathrm{PFR,r}}$	Power reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for ramp response of PFR [MW]
$P_k^{\mathrm{PFR,d}}$	Power reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for droop response of PFR [MW]
P_k^{In}	Power reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for inertia service [MW]
P^{In}	Total power reservation for inertia [MW]
$P^{\mathrm{PFR,r}}$	Total power reservation for ramp response of PFR [MW]
$P^{\mathrm{PFR,d}}$	Total power reservation for droop response of PFR [MW]
E_k^{PFR}	Energy reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for PFR [MWh]
E_j	State of charge of resource $j \in \mathcal{J}$ in dispatch interval [MWh]
ΔP^{L}	Magnitude of largest contingency for which security is to be guaranteed [MW]
D_j	Proportionality factor between change in power and rate of change of frequency of resource $j \in \mathcal{J}$ [MW/(Hz/s)]
$Loss_j^{\mathrm{E}}$	Power loss term for energy service due to a single-directional efficiency of resource $j \in \mathcal{J}$ [MW]
$LossD_j^{\mathrm{E}}$	Power loss term of resource $j \in \mathcal{J}$ in discharging state [MW]
$LossC_j^{\mathrm{E}}$	Power loss term of resource $j \in \mathcal{J}$ in charging state [MW]
E_k^{In}	Energy reservation of resource $k \in \mathcal{I} \cup \mathcal{J}$ for inertia service [MWh]

Data/Parameters

P^{D}	Power demand of the load [MW]
η_j	Round-trip efficiency of resource $j \in \mathcal{J}$ [%]
T^{PFR}	Maximum required activation time of PFR [seconds]
$\overline{\Delta f'}$	Maximum allowable magnitude of the rate of change of frequency [Hz/s]
$\overline{\Delta f}_{\mathrm{Nad}}$	Maximum allowable frequency deviation from nominal frequency at the frequency nadir [Hz]
$\overline{\Delta f}_{\mathrm{QSS}}$	Maximum allowable frequency deviation from nominal frequency at Quasi-Steady-State [Hz]
$\underline{P}_k$	Minimum power capacity of resource $k \in \mathcal{I} \cup \mathcal{J}$ [MW]
$\overline{P}_k$	Maximum power capacity of resource $k \in \mathcal{I} \cup \mathcal{J}$ [MW]
$\overline{P}_k^{\mathrm{PFR,r}}$	Maximum PFR capability of resource $k \in \mathcal{I} \cup \mathcal{J}$ for ramp response [MW]
$\overline{P}_k^{\mathrm{PFR,d}}$	Maximum PFR capability of resource $k \in \mathcal{I} \cup \mathcal{J}$ for droop response [MW]
$\overline{P}_i^{\mathrm{Gov}}$	Maximum governor-controlled capacity of resource $i \in \mathcal{I}$ [MW]
D_i	Proportionality factor between change in power and RoCoF of resource $i \in \mathcal{I}$ [MW/(Hz/s)]
$\overline{D}_j$	Maximum proportionality factor between change in power and RoCoF of resource $j \in \mathcal{J}$ [MW/(Hz/s)]
$\underline{E}_j$	Minimum state of charge of resource $j \in \mathcal{J}$ [MWh]
$\overline{E}_j$	Maximum state of charge of resource $j \in \mathcal{J}$ [MWh]
E_{j0}	Initial state of charge of resource $j \in \mathcal{J}$ [MWh]
f_0	Nominal system frequency [Hz]
ΔT	Dispatch interval [minutes]

Formulation

- **Objective Function**

$$\min \quad \sum_{i\in\mathcal{I}} \mathrm{C}_i(P_i^{\mathrm{E}}, P_i^{\mathrm{PFR,r}}, P_i^{\mathrm{PFR,d}}) + \sum_{j\in\mathcal{J}} \mathrm{C}_j(P_j^{\mathrm{E}}, P_j^{\mathrm{PFR,r}}, P_j^{\mathrm{PFR,d}}, P_j^{\mathrm{In}}).$$

- **System-wide Constraints**

$$\sum_{i\in\mathcal{I}} P_i^{\mathrm{E}} + \sum_{j\in\mathcal{J}} P_j^{\mathrm{E}} = P^{\mathrm{D}}$$
$$P^{\mathrm{In}} = \sum_{k\in\mathcal{K}} P_k^{\mathrm{In}}$$
$$P^{\mathrm{PFR,r}} = \sum_{k\in\mathcal{K}} P_k^{\mathrm{PFR,r}}$$
$$P^{\mathrm{PFR,d}} = \sum_{k\in\mathcal{K}} P_k^{\mathrm{PFR,d}}$$
$$P_k^{\mathrm{E}} \le \Delta P^{\mathrm{L}}, \quad \forall k \in \mathcal{I}\cup\mathcal{J}$$
$$P^{\mathrm{In}} \ge \Delta P^{\mathrm{L}}$$
$$\left(\frac{P^{\mathrm{In}}}{2\overline{\Delta f'}} \cdot \frac{P^{\mathrm{PFR,r}}}{T^{\mathrm{PFR}}}\right) \ge \frac{(\Delta P^{\mathrm{L}})^2}{4\overline{\Delta f}_{\mathrm{Nad}}}$$
$$P^{\mathrm{PFR,d}} \ge \Delta P^{\mathrm{L}}.$$

- **Individual Constraints**

  - **for a synchronous generator** $i \in \mathcal{I}$

$$\underline{P}_i \le P_i^{\mathrm{E}} \le \overline{P}_i$$
$$P_i^{\mathrm{E}} + P_i^{\mathrm{PFR,r}} \le \overline{P}_i^{\mathrm{Gov}}$$
$$P_i^{\mathrm{E}} + P_i^{\mathrm{PFR,d}} \le \overline{P}_i$$
$$P_i^{\mathrm{PFR,r}} \le \overline{P}_i^{\mathrm{PFR,r}}$$
$$P_i^{\mathrm{PFR,d}} \le \overline{P}_i^{\mathrm{PFR,d}}$$
$$P_i^{\mathrm{PFR,d}} \le P_i^{\mathrm{PFR,r}}$$
$$P_i^{\mathrm{In}} = D_i\overline{\Delta f'}.$$

– **for an inverter-based resource** $j \in \mathcal{J}$

$$\underline{P}_j \leq P_j^{\mathrm{E}} \leq \overline{P}_j$$
$$P_j^{\mathrm{E}} + P_j^{\mathrm{PFR,r}} + P_j^{\mathrm{In}} \leq \overline{P}_j$$
$$\underline{P}_j \leq P_j^{\mathrm{E}} - \frac{P_j^{\mathrm{In}}}{\eta_j}$$
$$P_j^{\mathrm{PFR,r}} \leq \overline{P}_j^{\mathrm{PFR,r}}$$
$$P_j^{\mathrm{PFR,d}} \leq \overline{P}_j^{\mathrm{PFR,d}}$$
$$P_j^{\mathrm{PFR,d}} \leq P_j^{\mathrm{PFR,r}}$$
$$P_j^{\mathrm{In}} = D_j \overline{\Delta f'}$$
$$0 \leq D_j \leq \overline{D}_j$$
$$\underline{E}_j + \frac{E_j^{\mathrm{In}}}{\sqrt{\eta_j}} + \frac{E_j^{\mathrm{PFR}}}{\sqrt{\eta_j}} \leq E_{j0}$$
$$\underline{E}_j + \frac{E_j^{\mathrm{In}}}{\sqrt{\eta_j}} + \frac{E_j^{\mathrm{PFR}}}{\sqrt{\eta_j}} \leq E_j$$
$$E_j^{\mathrm{PFR}} = \frac{1}{2} T^{\mathrm{PFR}} P_j^{\mathrm{PFR,r}} + P_j^{\mathrm{PFR,d}} (\Delta T - T^{\mathrm{PFR}})$$
$$E_j^{\mathrm{In}} = D_j \overline{\Delta f}_{\mathrm{Nad}}$$
$$E_j = E_{j0} - (P_j^{\mathrm{E}} + Loss_j^{\mathrm{E}}) \Delta T$$
$$\underline{E}_j \leq E_j \leq \overline{E}_j$$
$$LossD_j^{\mathrm{E}} \leq Loss_j^{\mathrm{E}}$$
$$LossC_j^{\mathrm{E}} \leq Loss_j^{\mathrm{E}}$$
$$LossD_j^{\mathrm{E}} = (\frac{1}{\sqrt{\eta_j}} - 1) P_j^{\mathrm{E}}$$
$$LossC_j^{\mathrm{E}} = (\sqrt{\eta_j} - 1) P_j^{\mathrm{E}}.$$

- **Others**

$$P_i^{\mathrm{E}}, P_i^{\mathrm{PFR,d}}, P_j^{\mathrm{PFR,d}}, P_i^{\mathrm{PFR,r}}, P_j^{\mathrm{PFR,r}}, P_i^{\mathrm{In}}, P_j^{\mathrm{In}}, E_j^{\mathrm{PFR}}, E_j^{\mathrm{In}}, E_j, D_j$$
$$P^{\mathrm{In}}, P^{\mathrm{PFR,r}}, P^{\mathrm{PFR,d}}, \Delta P^{\mathrm{L}} \geq 0 \quad \forall i, j$$