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DAA (CSA 0672)

1 a)

$x(n) = x(n-1) + 5$ for $n>1$ $x(1)=0$

$n=2$ ∴ $x(2) = x(2-1)+5$

$= x(0)+5$

$\boxed{x(2) = 5}$

$n=3$ ∴ $x(3) = x(3-1)+5$

$= x(2)+5$

$\boxed{x(3) = 10}$

$n=4$ $x(4) = x(4-1)+5$

$= x(3)+5$

$\boxed{x(4) = 15}$

∴ Each term is 5 more than previous term

So, $x(n) = 5n-5$ for $n>1$

b)

$x(n) = 3x(n-1)$ for $n>1$ $x(1)=4$

$x(2) = 3x(2-1)$

$= 3(4)$

$\boxed{x(2) = 12}$

$x(3) = 3x(3-1)$

$= 3(12)$

$\boxed{x(3) = 36}$

$x(4) = 3x(4-1)$

$= 3x(3)$

$= 3(36)$

$\boxed{x(4) = 108}$

Each term is 3 times the previous term

So, $x(n) = 4 * 3^{n-1}$ for $n>1$

c) $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$

$\underline{n=2}$
$n = 2^1 = 2$

$x(2) = x(2/2) + 2$

$\boxed{x(2) = 3}$

for this recursion relation

$2^n - 1 = 2^{2k} - 1$

$\boxed{n = 2^k}$, $\boxed{x(2k) = 2^{2k} - 1}$

$\underline{n=4}$
$n = 2^2 = 4$

$x(4) = x(4/2) + 4$

$\boxed{x(4) = 7}$

$\underline{n=8}$
$n = 2^3 = 8$

$x(8) = x(8/2) + 8$

$= x(4) + 8$

$\boxed{x(8) = 15}$

d) $x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$

$x(1) = 1$

$n = 3^1 = 3$
$n = 3^2 = 9$
$n = 3^3 = 27$

$x(3) = x(3/3) + 1$

$= x(1) + 1$

$\boxed{x(3) = 2}$

$x(9) = x(9/3) + 1$

$= x(3) + 1$

$\boxed{x(9) = 3}$

$\boxed{x(n) = \log_3 n}$

$\boxed{x(3k) = \log_3 3^k}$

$x(27) = x(27/3) + 1$

$= x(9) + 1$

$\boxed{x(27) = 4}$

2 i) $T(n) = T(n/2) + 1$ when $n = 2^k$ for all $k \geq 0$

By using substitution method

$T(n) = T(n/2) + 1$ — ①

$T(n/2) = T(n/2^2) + 1$ — ②

sub ② in ①

$T(n) = \{T(n/2^2) + 1\} + 1$ — ③

$T(n) = T(n/2^2) + 2$ — ④

$T(n) = T(n/2^3) + 3$ — ⑤

$T(n) = T(n/2k) + k$

Assume $\frac{n}{2k} = 1$, $n = 2^k$

$n = \log n$

$T(n) = T(1) + \log n$

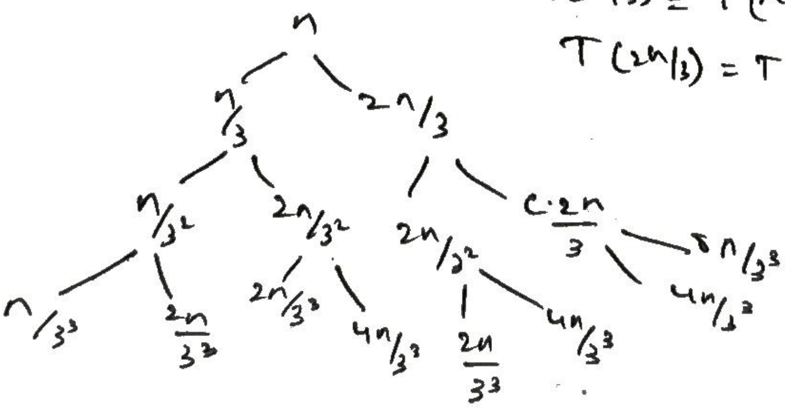
$T(n) = 1 + \log n$

$\Rightarrow O(\log n)$

ii) $T(n) = T(n/3) + T(2n/3) + cn$

$T(n/3) = T(n/3^2) + T(\frac{2n}{3^2}) + c \cdot n/3$

$T(2n/3) = T(2n/3^2) + T(4n/3^2) + c\frac{2n}{3}$



$n/3k = 1$, $k = \log_3 n$ ($\log_3 n \rightarrow$ with base 3)

$= c.n \log_3 n$

$= O(n \log n)$

3) a) What does this algorithm compute?

Ans The algorithm find the minimum value in the array, efficiently breaking down the problem into smaller sub-problem.

$n=1 \rightarrow$ There is only one element

b) Setup a recurrance relation for algorithm basic operation count and solve it.

Ans $T(n) = T(n-1) + 1$ where $n > 1$

$T(1) = 0$ (no comparision).

$T(n) = T(1) + T(n-1)$

$= 0 + (n-1)$

$= n-1$

Time complexity $= O(n)$

4) Analyse the order of growth $f(n) = 2n^2+5$ & $g(n) = 7n$ use the $\Omega(g(n))$ notation

	$f(n)$	$g(n)$
	$2n^2+5$	$7n$
$n=1$	7	7
$n=2$	13	14
$n=3$	23	21

$n \geq 3$ $f(n) \geq g(n)$ i.e

$f(n)$ is always greater or equal to $g(n)$ when $n_0 \geq 3$

$f(n) = \Omega(g(n))$