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RECSYS SERIES

Recommendation System Series Part 6: The 6 Variants of Autoencoders for Collaborative Filtering

Advancing Recommendations With Autoencoders

Update: This article is part of a series where I explore recommendation systems in academia and industry. Check out the full series: [Part 1](#), [Part 2](#), [Part 3](#), [Part 4](#), [Part 5](#), and [Part 6](#).

Many recommendation models have been proposed during the last few years. However, they all have their limitations in dealing with data sparsity and cold-start issues.

- The **data sparsity** occurs when the recommendation performance drops significantly if the interactions between users and items are very sparse.
- The **cold-start issues** occur when the model can't recommend new users and new items.

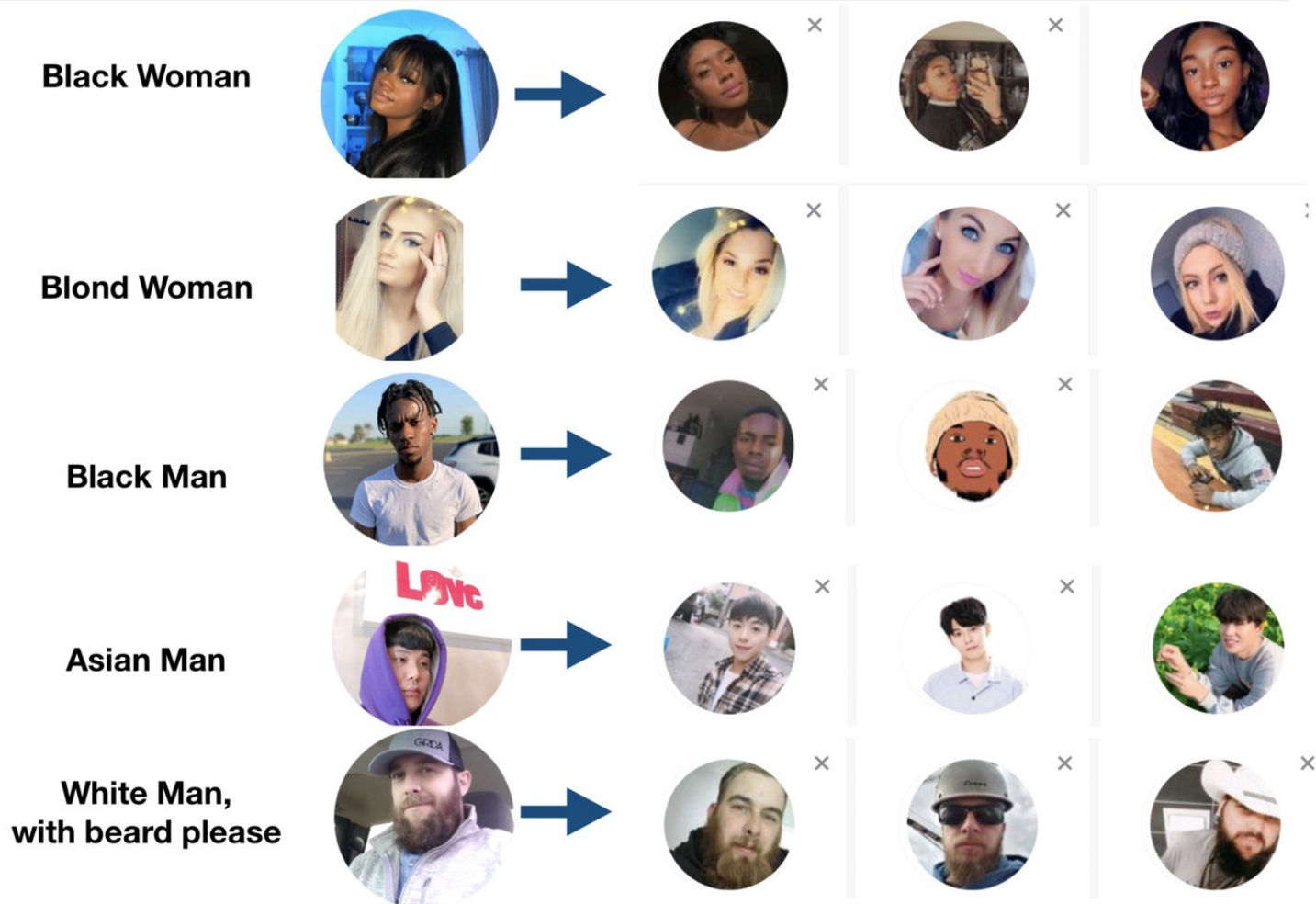
To solve these problems, recent approaches have exploited side information about users or items. However, the improvement of the recommendation performance is not significant due to the limitations of such models in capturing the user preferences and item features.





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Rebecca Heilweil — There's something strange about TikTok recommendations
(<https://www.vox.com/recode/2020/2/25/21152585/tiktok-recommendations-profile-look-alike>)

Auto-encoder is a type of neural network suited for unsupervised learning tasks, including generative modeling, dimensionality reduction, and efficient coding. It has shown its superiority in learning underlying feature representation in many domains, including computer vision, speech recognition, and language modeling. Given that knowledge, new recommendation architectures have incorporated autoencoder and thus brought more opportunities in re-inventing user experiences to satisfy customers.

- While traditional models deal only with a single data source (rating or text), auto-encoder based models can handle **heterogeneous** data sources (rating, audio, visual, video).
- Auto-encoder has a **better understanding** of the user demands and item features, thus leading to higher recommendation accuracy than traditional models.
- Furthermore, auto-encoder helps the recommendation model to be **more adaptable** in multi-media scenarios and more effective in handling input noises than traditional models.

In this post and those to follow, I will be walking through the creation and training of



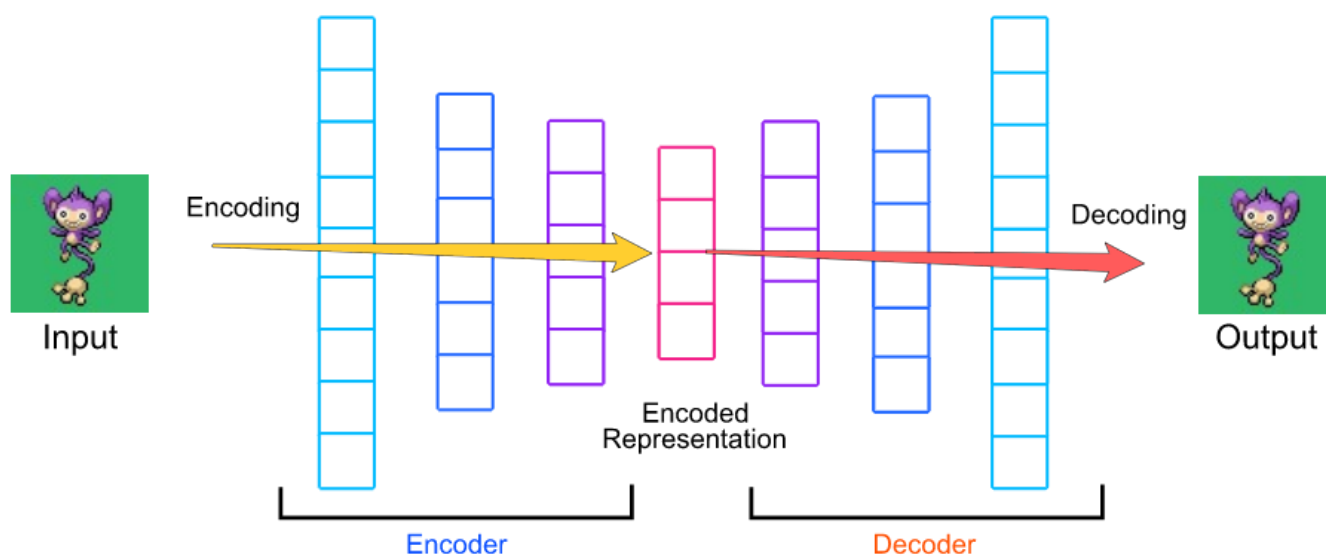


- [Part 2](#) provided a careful review of the ongoing research initiatives concerning the strengths and application scenarios of these models.
- [Part 3](#) provided a couple of research directions that might be relevant to the recommendation system scholar community.
- [Part 4](#) provided the nitty-gritty mathematical details of 7 variants of matrix factorization that you can construct: ranging from the use of clever side features to the application of Bayesian methods.
- [Part 5](#) provided the architecture design of 5 variants of multi-layer perceptron based collaborative filtering models, which are discriminative models that can interpret the features in a non-linear fashion.

In Part 6, I explore the use of **Auto-Encoders** for collaborative filtering. More specifically, I will dissect six principled papers that incorporate Auto-Encoders into their recommendation architecture. But first, let's walk through a primer on auto-encoder and its variants.

A Primer on Auto-encoder and Its Variants

As illustrated in the diagram below, a vanilla auto-encoder consists of an input layer, a hidden layer, and an output layer. The input data is passed into the input layer. The input layer and the hidden layer constructs an encoder. The hidden layer and the output layer constructs a decoder. The output data comes out of the output layer.





$$h = f(x) = s_f(Wx + b)$$

Equation 1

where s_f is an activation function, W is the weight matrix, and b is the bias vector.

The decoder decodes the hidden representation h back to a reconstruction x' by another function g :

$$x' = g(h) = s_g(W'h + b')$$

Equation 2

where s_g is an activation function, W' is the weight matrix, and b' is the bias vector.

The choices of s_f and s_g are non-linear, for example, Sigmoid, TanH, or ReLU. This allows auto-encoder to learn more useful features than other unsupervised linear approaches, say Principal Component Analysis.

I can train the auto-encoder to minimize the reconstruction error between x and x' via either the squared error (for regression tasks) or the cross-entropy error (for classification tasks).

This is the formula for the squared error:

$$SE(x, x') = ||x - x'||^2$$

Equation 3

This is the formula for the cross-entropy error:

$$CE(x, x') = - \sum_{i=1}^n (x_i \log x'_i + (1 - x_i) \log(1 - x'_i))$$

Equation 4

Finally, it is always a good practice to add a regularization term to the final reconstruction error of the auto-encoder:





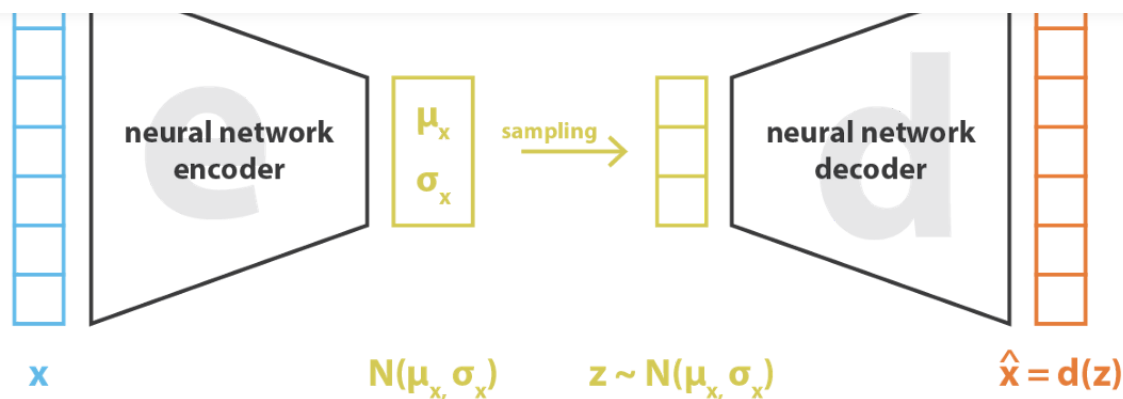
$$\left(\frac{1}{x} \right)$$

Equation 5

The reconstruction error function above can be optimized via either stochastic gradient descent or alternative least square.

There are many variants of auto-encoders currently used in recommendation systems. The four most common are:

- **Denoising Autoencoder (DAE)** corrupts the inputs before mapping them into the hidden representation and then reconstructs the original input from its corrupted version. The idea is to force the hidden layer to acquire more robust features and to prevent the network from merely learning the identity function.
- **Stacked Denoising Autoencoder (SDAE)** stacks several denoising auto-encoder on top of each other to get higher-level representations of the inputs. The training is usually optimized with greedy algorithms, going layer by layer. The apparent disadvantages here are the high computational cost of training and the lack of scalability to high-dimensional features.
- **Marginalized Denoising Autoencoder (MDAE)** avoids the high computational cost of SDAE by marginalizing stochastic feature corruption. Thus, it has a fast training speed, simple implementation, and scalability to high-dimensional data.
- **Variational Autoencoder (VAE)** is an unsupervised latent variable model that learns a deep representation from high-dimensional data. The idea is to encode the input as a probability distribution rather than a point estimate as in vanilla auto-encoder. Then VAE uses a decoder to reconstruct the original input by using samples from that probability distribution.



$$\text{loss} = ||x - \hat{x}||^2 - \text{KL}[N(\mu_x, \sigma_x), N(0, I)] = ||x - d(z)||^2 - \text{KL}[N(\mu_x, \sigma_x), N(0, I)]$$

Variational Autoencoder Architecture

Okay, it's time to review the different auto-encoder based recommendation framework!

1 — AutoRec

One of the earliest models that consider the collaborative filtering problem from an auto-encoder perspective is **AutoRec** from “[Autoencoders Meet Collaborative Filtering](#)” by Suvash Sedhain, Aditya Krishna Menon, Scott Sanner, and Lexing Xie.

In the paper's setting, there are m users, n items, and a partially filled user-item interaction/rating matrix R with dimension $m \times n$. Each user u can be represented by a partially filled vector r_u and each item i can be represented by a partially filled vector r_i . AutoRec directly takes user rating vectors r_u or item rating r_i as input data and obtains the reconstructed rating at the output layer. There are two variants of AutoRec depending on two types of inputs: item-based AutoRec (**I-AutoRec**) and user-based AutoRec (**U-AutoRec**). Both of them have the same structure.

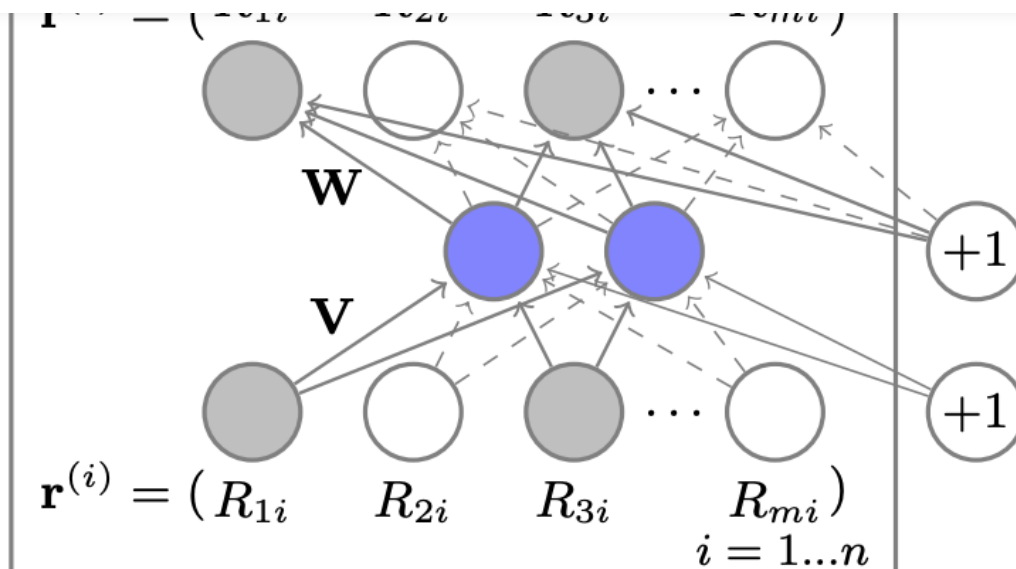


Figure 1: Item-based AutoRec model. We use plate notation to indicate that there are n copies of the neural network (one for each item), where \mathbf{W} and \mathbf{V} are tied across all copies.

Suvesh Sedhain et al. — AutoRec: Autoencoders Meet Collaborative Filtering
<https://dl.acm.org/doi/10.1145/2740908.2742726>

Figure 1 from the paper illustrates the structure of I-AutoRec. The shaded nodes correspond to observed ratings, and the solid connections correspond to weights that are updated for the input r_i .

Given the input r_i , the reconstruction is:

$$h(r^{(i)}, \theta) = f(W \cdot g(V \cdot r^{(i)} + \mu) + b)$$

Equation 6

where f and g are the activation functions, and the parameter θ includes W , V , μ , and b .

AutoRec uses only the **vanilla auto-encoder structure**. The objective function of the model is similar to the loss function of auto-encoder:

$$\operatorname{argmin}_{\theta} \sum_{i=1}^N ||r^{(i)} - h(r^{(i)}; \theta)||_O^2 + \lambda \cdot \operatorname{reg}$$



Here are some important things about AutoRec:

- I-AutoRec generally performs better than U-AutoRec. This is because the average number of ratings for each item is much more than the average number of ratings given by each user.
- Different combinations of activation functions affect the performance of AutoRec considerably.
- Increasing the number of hidden neurons or the number of layers improves model performance. This makes sense as expanding the dimensionality of the hidden layer allows AutoRec to have more capacity to simulate the input features.
- Adding more layers to formulate a deep network can lead to slight improvement.

The TensorFlow code of the AutoRec model class is given below for illustration purpose:

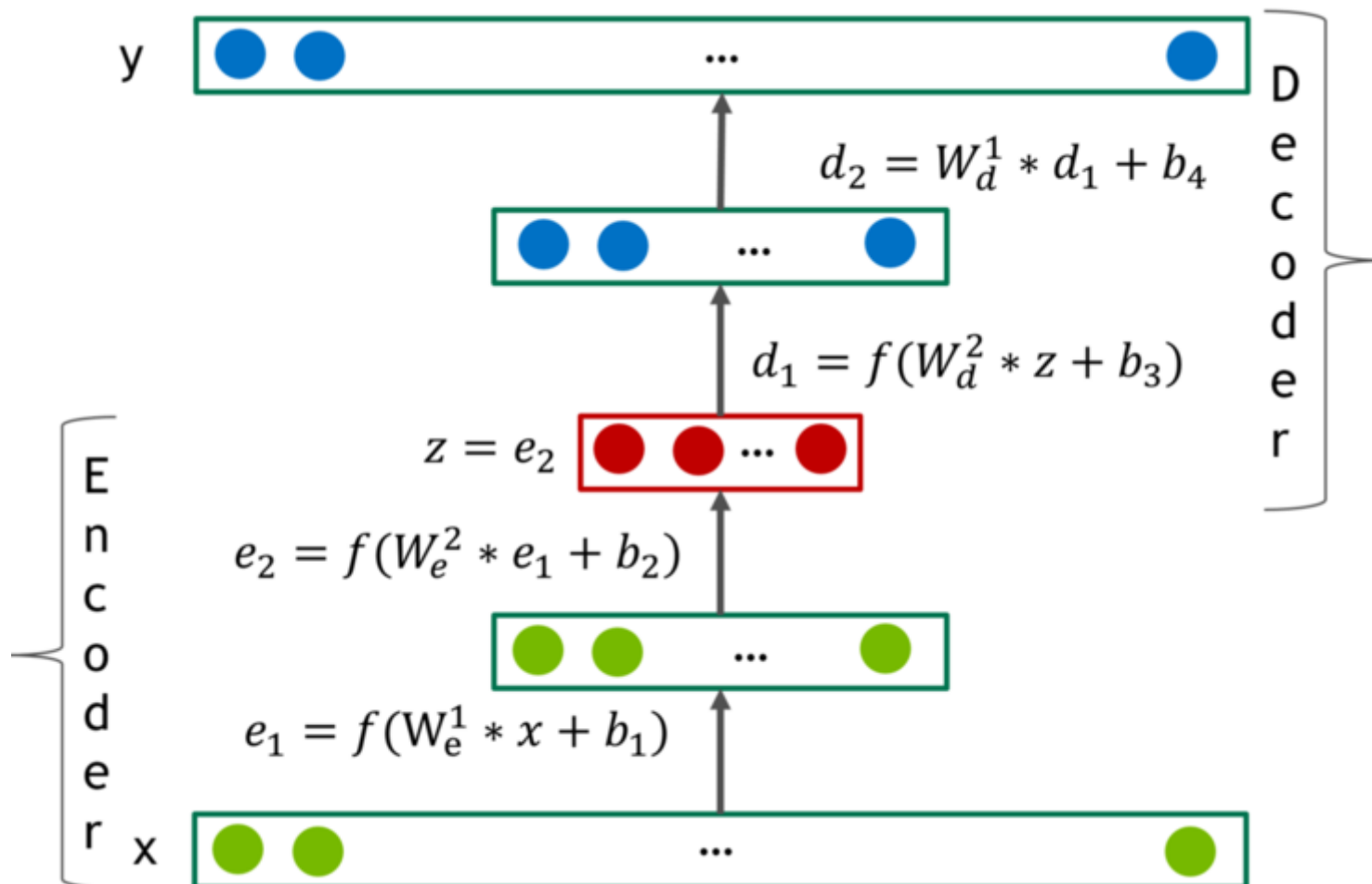


For my TensorFlow implementation, I trained AutoRec architecture with a hidden layer of 500 units activated by a sigmoid non-linear function. Other hyper-parameters include a learning rate of 0.001, a batch size of 512, the Adam optimizer, and a lambda regularizer of 1.

2 — DeepRec

DeepRec is a model created by Oleisii Kuchaiev and Boris Ginsburg from NVIDIA, as seen in “[Training Deep Autoencoders for Collaborative Filtering](#).” The model is inspired by the AutoRec model described above, with several important distinctions:

- The network is much deeper.
- The model uses “scaled exponential linear units” (SELUs).
- The dropout rate is high.
- The authors use iterative output re-feeding during training.





and e_2 , while the decoder has 2 layers d_1 and d_2 . They are fused together on the representation z . The layers are represented as $f(W * x + b)$, where f is some non-linear activation function. If the range of the activation function is smaller than that of the data, the last layer of the decoder should be kept linear. The authors found it to be very important for activation function f in hidden layers to contain a non-zero negative part, and use SELU units in most of their experiments.

2.2 — Loss Function

Since it doesn't make sense to predict zeros in user's representation vector x , the authors optimize the **Masked Mean Squared Error** loss:

$$MMSE = \frac{m_i * (r_i - y_i)^2}{\sum_{i=0}^n m_i}$$

Equation 8

where r_i is the actual rating, y_i is the reconstructed rating, and m_i is a mask function such that $m_i = 1$ if r_i is not 0 else $m_i = 0$.

2.3 — Dense Re-feeding

During forward pass and inference pass, the model takes a user represented by his vector of ratings from the training set x . Note that x is very sparse, while the output of the decoder $f(x)$ is dense and contains rating predictions for all items in the corpus. Thus, to explicitly enforce fixed-point constraint and perform dense training updates, the authors augment every optimization iteration with **an iterative dense re-feeding step** as follows:

1. During the initial forward pass, given sparse input x , the model computes the dense output $f(x)$ and the MMSE loss using equation 8.
2. During the initial backward pass, the model computes the gradients and updates the weights accordingly.
3. During the second forward pass, the model treats $f(x)$ as a new data point and thus computes $f(f(x))$. Both $f(x)$ and $f(f(x))$ become dense. The MMSE loss now has all m as non-zeros.
4. During the second backward pass, the model again computes the gradients and updates the



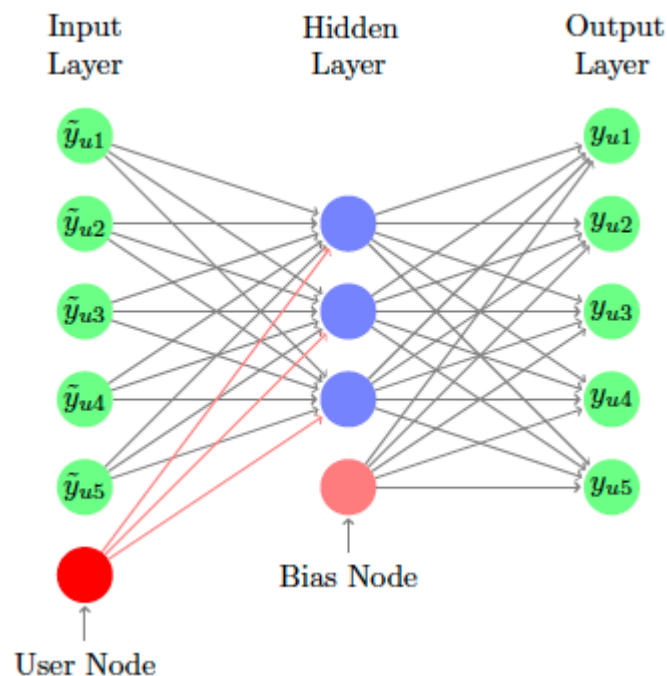
For [my TensorFlow implementation](#), I trained DeepRec with the following architecture: $[n, 512, 512, 1024, 512, 512, n]$. So n is the number of ratings that the user has given, the encoder has 3 layers of size $(512, 512, 1034)$, the bottleneck layer has size 1024, and the decoder has 3 layers of size $(512, 512, n)$. I trained the model using stochastic gradient descent with a momentum of 0.9, a learning rate of 0.001, a batch size of 512, and a dropout rate of 0.8. Parameters are initialized via the Xavier initialization scheme.

3 — Collaborative Denoising Auto-encoder





- The input of CDAE is not user-item ratings, but partially observed implicit feedback r (user's item preference). If a user likes a movie, the corresponding entry value is 1, otherwise 0.
- Unlike the previous two models that are used for rating prediction, CDAE is principally used for ranking prediction (also called Top-N preference recommendations).



Yao Wu et al. — Collaborative Denoising Autoencoders For Top-N Recommender Systems
(<https://dl.acm.org/doi/10.1145/2835776.2835837>)

3.1 — Model

The figure above shows a sample structure of CDAE, which consists of 3 layers: the input, the hidden, and the output.

- There are a total of $I + 1$ nodes in the input layer. The first I nodes represent user preferences, and each node of these I nodes corresponds to an item. The last node is a user-specific node denoted by the red node in the figure above, which means different users have different nodes and associated weights.
- Here y_u is the I -dimensional feedback vector of user u on all the items in I . y_u is a sparse binary vector that only has non-zero values: $y_{ui} = 1$ if i has been rated by user u and $y_{ui} = 0$ otherwise.
- There are $K (<< I) + 1$ nodes in the hidden layer. The blue K nodes are fully connected to the nodes of the input layer. The pink additional node in the hidden layer captures the bias effects.





The corrupted input $\mathbf{r_corr}$ of CDAE is drawn from a conditional Gaussian distribution $\mathbf{p}(\mathbf{r_corr} | \mathbf{r})$. The reconstruction of $\mathbf{r_corr}$ is formulated as follows:

$$h(r_{corr}) = f(W_2 \cdot g(W_1 \cdot r_{corr} + V_u + b_1) + b_2)$$

Equation 9

where W_1 is the weight matrix corresponding to the encoder (going from the input layer to the hidden layer), W_2 is the weight matrix corresponding to the decoder (going from the hidden layer to the output layer). V_u is the weight matrix for the red user node, while both b_1 and b_2 are the bias vectors.

3.2 — Loss Function

The parameters of CDAE are learned by minimizing the average reconstruction error as follows:

$$\min_{W_1, W_2, V_u, b_1, b_2} \frac{1}{M} \sum_{u=1}^M E_{p(r_{corr}|r)} [l(r_{corr}, h(r_{corr}))] + \lambda \cdot Regularization$$

Equation 10

The loss function $L(\mathbf{r_corr}, \mathbf{h}(\mathbf{r_corr}))$ in the equation above can be square loss or logistic loss. CDAE uses the square L2 norm to control the model complexity. It also (1) applies stochastic gradient descent to learn the model's parameters and (2) adopts AdaGrad to automatically adapt the training step size during the learning procedure.

The authors also propose a **negative sampling technique** to extract a small subset from items that user did not interact with for reducing the time complexity substantially without degrading the ranking quality. At inference time, CDAE takes a user's existing preference set (without corruption) as input and recommends the items with the largest prediction values on the output layer to that user.

The PyTorch code of the CDAE architecture class is given below for illustration purpose:





For [my PyTorch implementation](#), I used a CDAE architecture with a hidden layer of 50 units. I trained the model using stochastic gradient descent with a learning rate of 0.01, a batch size of 512, and a corruption ratio of 0.5.

4 — Multinomial Variational Auto-encoder

One of the most influential papers in this discussion is “[Variational Autoencoders for Collaborative Filtering](#)” by Dawen Liang, Rahul Krishnan, Matthew Hoffman, and Tony Jebara from Netflix. It proposes a variant of VAE for recommendation with implicit data. In particular, the authors introduced **a principled Bayesian inference approach** to estimate model parameters and show favorable results than commonly used likelihood functions.

The paper uses U to index all users and I to index all items. The user-by-item interaction matrix is called X (with dimension $U \times I$). The lower case x_u is a bag-of-words vector with the number of clicks for each item from user u . For implicit feedback, this matrix is binarized to have only 0s and 1s.



The generative process of the model is seen in equation 11 and broken down as follows:

- For each user u , the model samples a K -dimensional latent representation z_u from a standard Gaussian prior.
- Then it transforms z_u via a non-linear function f_θ to produce a probability distribution over I items $\pi(z_u)$.
- f_θ is a multi-layer perceptron with parameters θ and a softmax activation function.
- Given the total number of clicks from user u , the bag-of-words vector x_u is sampled from a multinomial distribution with probability $\pi(z_u)$.

The log-likelihood for user u (conditioned on the latent representation) is:

$$\log p_\theta(x_u | z_u) = \sum_i x_{ui} \log \pi_i(z_u)$$

Equation 12

The authors believe that **the multinomial distribution** is suitable for this collaborative filtering problem. Specifically, the likelihood of the interaction matrix in equation 11 rewards the model for putting probability mass on the non-zero entries in x_u . However, considering that $\pi(z_u)$ must sum to 1, the items must compete for a limited budget of probability mass. Therefore, the model should instead assign more probability mass to items that are more likely to be clicked, making it suitable to achieve a solid performance in the top- N ranking evaluation metric of recommendation systems.

4.2 — Variational Inference

In order to train the generative model in equation 11, the authors estimate θ by approximating the intractable posterior distribution $p(z_u | x_u)$ via **variational inference**. This method approximates the true intractable posterior with a simpler variational distribution $q(z_u)$ — which is a fully diagonal Gaussian distribution. The objective of variational inference is to optimize the free variational parameters $\{\mu_u, \sigma_u^2\}$ so that the Kullback-Leiber divergence $\text{KL}(q(z_u) || p(z_u | x_u))$ is minimized.

The issue with variational inference is that the number of parameters to optimize $\{\mu_u, \sigma_u^2\}$ grows





$$g_{\phi}(x_u) = [\mu_{\phi}(x_u), \sigma_{\phi}(x_u)]$$

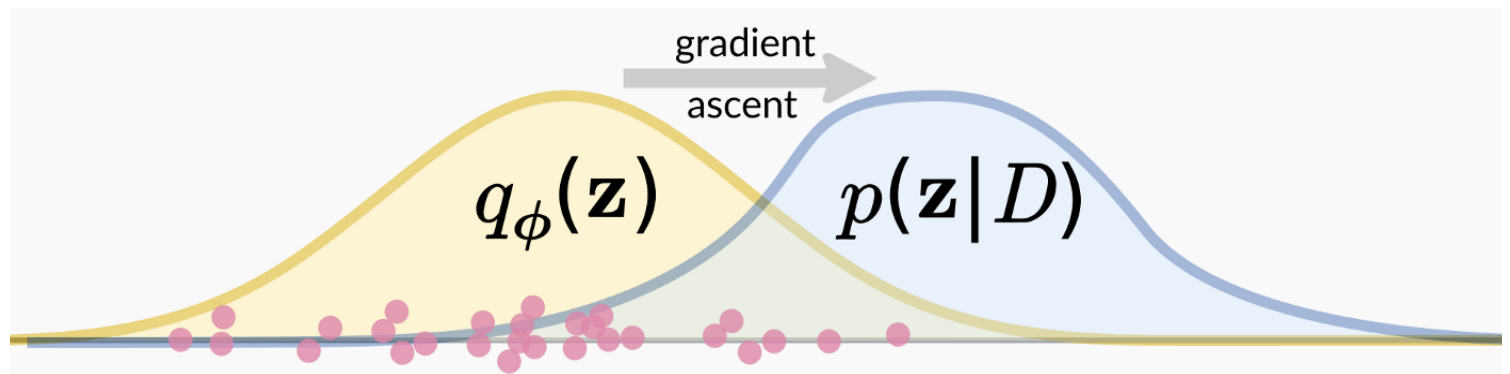
Equation 13

This function is parameterized by ϕ — in which both $\mu_{\phi}(x_u)$ and $\sigma_{\phi}(x_u)$ are vectors with K dimensions. The variational distribution is then set as follows:

$$q_{\phi}(z_u | x_u) = N(\mu_{\phi}(x_u), \text{diag}\{\sigma_{\phi}^2(x_u)\})$$

Equation 14

Using the input x_u , the inference model returns the corresponding variational parameters of variational distribution $q_{\phi}(z_u | x_u)$. When being optimized, this variational distribution approximates the intractable posterior $p_{\phi}(z_u | x_u)$.

Variational Gradient (<https://matsen.fredhutch.org/general/2019/08/24/vbpi.html>)

To learn latent-variable models with variational inference, the standard approach is to lower-bound the log marginal likelihood of the data. The objective function to maximize for user u now becomes:

$$\log p(x_u; \theta) \geq E_{q_{\phi}(z_u | x_u)}[\log p_{\theta}(x_u | z_u)] - KL(q_{\phi}(z_u | x_u) || p(z_u)) = L(x_u; \theta, \phi)$$

Equation 15

Another term to call this objective is **evidence lower bound (ELBO)**. Intuitively, we should be able to obtain an estimate of ELBO by sampling $z_u \sim q_{\phi}$ and optimizing it with a stochastic gradient ascent. However, we cannot differentiate ELBO to get the gradients with respect to ϕ . The **reparametrization trick** comes in handy here:



Essentially, we isolate the stochasticity in the sampling process, and thus the gradient with respect to ϕ can be back-propagated through the sampled z_u .

From a different perspective, the first term of equation 15 can be interpreted as **reconstruction error** and the second term of equation 15 can be interpreted as **regularization**. The authors, therefore, extend equation 15 with an additional parameter β to control the strength of regularization:

$$L_{\beta}(x_u; \theta, \phi) = E_{q_{\phi}(z_u|x_u)}[\log p_{\theta}(x_u|z_u)] - \beta \cdot KL(q_{\phi}(z_u|x_u)||p(z_u))$$

Equation 17

This parameter β engages a tradeoff between how well the model fits the data and how close the approximate posterior stays to the prior during learning. The authors tune β via **KL annealing**, a common heuristic used for training VAEs when there is concern that the model is being under-utilized.

4.3 — Prediction

Given a user's click history x , the model ranks all items based on the un-normalized predicted multinomial probability $f_{\phi}(z)$. The latent representation z for x is simply the mean of the variational distribution $z = \mu_{\phi}(x)$.

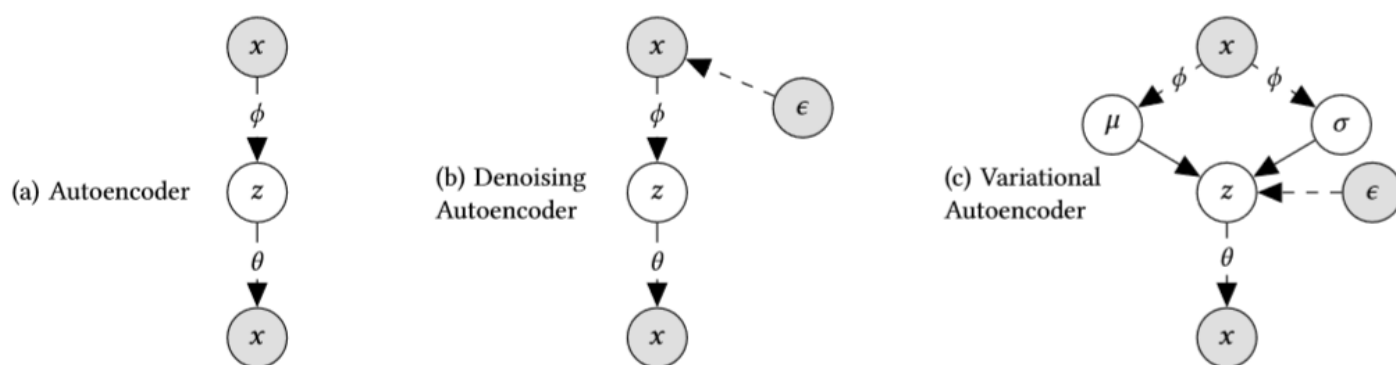


Figure 2: A taxonomy of autoencoders. The dotted arrows denote a sampling operation.

Dawen Liang et al. — Variational Autoencoders for Collaborative Filtering (<https://arxiv.org/abs/1802.05814>)

Figure 2 from the paper provides a unified view of different variants of autoencoders.

- 2a is the vanilla auto-encoder architecture, as seen in AutoRec and DeepRec.





distribution, as explained in detail above.

The PyTorch code of the MultVAE architecture class is given below for illustration purpose:

For my PyTorch implementation, I keep the architecture for the generative model f and the inference model g symmetrical and use an MLP with 1 hidden layer. The dimension of the latent representation K is set to 200 and of other hidden layers is set to 600. The overall architecture for a MultVAE now becomes $[I \rightarrow 600 \rightarrow 200 \rightarrow 600 \rightarrow I]$, where I is the total number of items. Other model details include tanH activation function, dropout rate with probability 0.5, Adam





Giuseppe Manco, Ettore Ritacco, and Vikram Pudi propose an extension of MultVAE by exploring the rich information present in the past preference history. They introduce a **recurrent version of MultVAE**, where instead of passing a subset of the whole history regardless of temporal dependencies, they pass the consumption sequence subset through a recurrent neural network. They show that **handling temporal information** is crucial for improving the accuracy of VAE.

5.1 — Setting

The problem setting is exactly similar to that of the MultVAE paper: U is a set of users, I is a set of items, and X is the user-item preference matrix with dimension $U \times I$. The principal difference is that SVAE considers precedence and temporal relationships within the matrix X .

- X induces a natural ordering relationship between items: $i <_u j$ has the meaning that $x_{\{u, i\}} > x_{\{u, j\}}$ in the rating matrix.
- They assume the existence of timing information T , where the term $t_{\{u, i\}}$ represents the time when i was chosen by u . Then $i <_u j$ denotes that $t_{\{u, i\}} > t_{\{u, j\}}$.
- They also introduce a temporal mark in the elements of x_u : $x_{\{u(t)\}}$ represents the t -th item in I_u in the sorting induced by $<_u$, whereas $x_{\{u(1:t)\}}$ represents the sequence from $x_{\{u(1)\}}$ to $x_{\{u(t)\}}$.

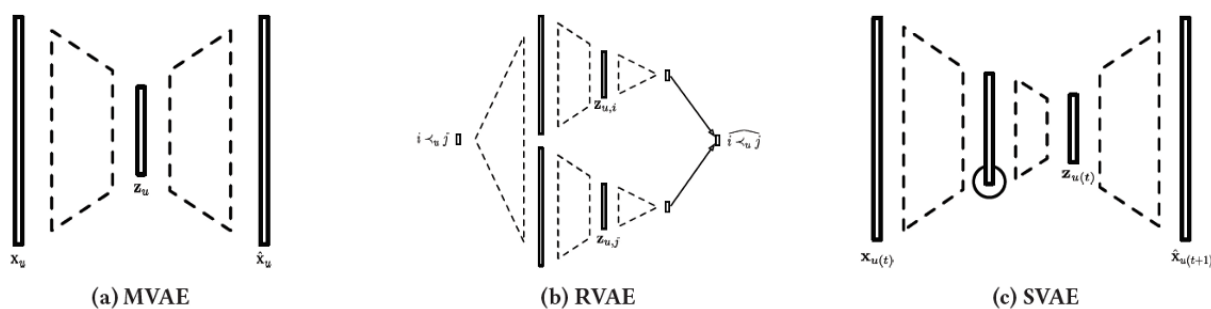


Figure 1: Variational architectures. Terms with hat represent decoding reconstruction. Dotted boxes represent neural layers.

Noveen Sachdeva et al. — Sequential Variational Autoencoders for Collaborative Filtering
(<https://arxiv.org/abs/1811.09975>)

5.2 — Model

The figure above from the paper shows the architectural difference between MultVAE, SVAE, and another model called RVAE (which I won't discuss here). Looking at the SVAE architecture, I can observe the recurrent relationship occurring in the layer upon which $z_{\{u(t)\}}$ depends. The basic idea behind SVAE is that **latent variable modeling should be able to express temporal dynamics and hence causalities and dependencies among preferences in a user's history**.





probability is:

$$P(x_{(1:T)}) = \prod_{t=0}^{T-1} P(x_{(t+1)} | x_{(1:t)})$$

Equation 18

This probability represents a recurrent relationship between $x_{(t+1)}$ and $x_{(1:t)}$. Thus, the model can handle each timestep separately.

Recall the generative process in equation 11, we can add a timestamp t as seen below:

$$z_{u(t)} \sim N(0, I_K); \pi(z_{u(t)}) \sim \text{softmax}\{f_\phi(z_{u(t)})\}; x_{u(t)} \sim \text{Mult}(1, \pi(z_{u(t)}))$$

Equation 19

Equation 19 results in the joint likelihood:

$$P(x_{u(1:T)}, z_{u(1:T)}) = \prod_t P(x_{u(t)} | z_{u(t)}) P(z_{u(t)})$$

Equation 20

The posterior likelihood in equation 20 can be approximated with a factorized proposal distribution:

$$Q_\lambda(z_{u(1:T)} | x_{u(1:T)}) = \prod_t q_\lambda(z_{u(t)} | x_{(1:t-1)})$$

Equation 21

where the right-term side is a Gaussian distribution whose parameters μ and σ depend upon the current history $x_{u(1:t-1)}$, by means of a recurrent layer h_t :

$$\mu_\lambda(t), \sigma_\lambda(t) = \phi_\lambda(h_t); h(t) = \text{RNN}_\lambda(h_{t-1}, x_{u(t-1)})$$

Equation 22





Equation 23

5.3 — Prediction

In this SVAE model, the proposal distribution introduces a dependency of the latent variable from a recurrent layer, which allows us **to recover the information from the previous history**. Given a user history $x_{\{u(1:t-1)\}}$, we can use equation 22 and set $z = \mu_{\{\lambda\}}(t)$, upon which we can devise the probability for the $x_{\{u(t)\}}$ by means of $\pi(z)$.

The PyTorch code of the SVAE architecture class is given below for illustration purpose:





histories of the users in the training set. During the evaluation, for each user in the validation/test set, they split the **time-sorted** user history into two parts, *fold-in* and *fold-out* split.

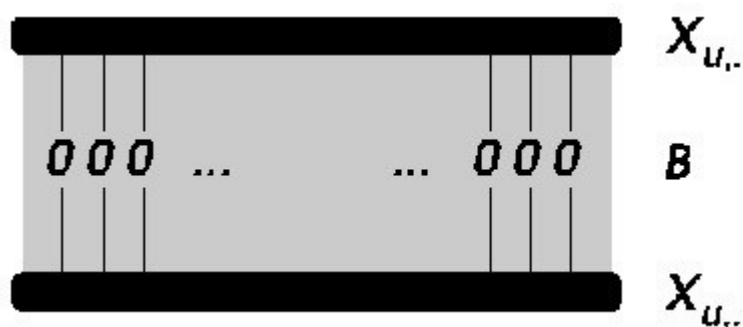
- The *fold-in* split learns the necessary representations and recommends items.
- These items are then evaluated with the *fold-out* split of the user history using metrics such as Precision, Recall, and Normalized Discounted Cumulative Gain.

For [my PyTorch implementation](#), I follow the same code provided by the authors.

- The SVAE architecture includes an embedding layer of size 256, a recurrent layer (Gated Recurrent Unit) with 200 cells, and two encoding layers (of size 150 and 64) and finally two decoding layers (of size 64 and 150).
- The number K of latent factors for the VAE is set to be 64.
- The model is optimized with Adam and weight decay is set to be 0.01.

6 — Embarrassingly Shallow Auto-encoders

Harald Steck's "[Embarrassingly Shallow Autoencoders for Sparse Data](#)" is a fascinating one that I want to bring into this discussion. The motivation here is that, according to his literature review, *deep models with a large number of hidden layers typically do **not** obtain a notable improvement in ranking accuracy in collaborative filtering, compared to 'deep' models with only one, two, or three hidden layers.* This is a stark contrast to other areas like NLP or computer vision.



Harald Steck — Embarrassingly Shallow Autoencoders for Sparse Data (<https://arxiv.org/abs/1905.03375>)

6.1 — Model

Embarrassingly Shallow Auto-encoders (ESAE) is a linear model *without* a hidden layer. The (binary) input vector X vector indicates which items a user has interacted with, and ESAE's objective is to predict the best items to recommend to that user in the output layer (as seen in the





The item-item weight matrix B represents the parameters of ESAE. Here, the self-similarity of an item in the input layer with itself in the output layer is omitted, so that ESAE can generalize effectively during the reconstruction step. Thus, the diagonal of this weight-matrix B is constrained to 0 ($\text{diag}(B) = 0$).

For an item j and a user u , we want to predict $S_{\{u, j\}}$, where $X_{\{u, .\}}$ refers to row u and $B_{\{., j\}}$ refers to column j :

$$S_{u,j} = X_{u,.} \cdot B_{.,j}$$

Equation 24

6.2 — Objective Function

With respect to $\text{diag}(B) = 0$, ESAE has the following convex objective for learning the weights B :

$$\min_B ||X - XB||_F^2 + \lambda \cdot ||B||_F^2$$

Equation 25

Here are important notes about this convex objective:

- $||.||$ denotes the Frobenius norm. This squared loss between the data X and the predicted scores XB allows for a **closed-form solution**.
- The hyper-parameter λ is the L2-norm regularization of the weights B .
- The constraint of a zero diagonal helps avoid the trivial solution $B = I$ where I is the identity matrix.

In the paper, Harald derived a closed-form solution from the training objective in equation 25. He argues that the traditional neighborhood-based collaborative filtering approaches are based on conceptually incorrect item-item similarity matrices, while the ESAE framework utilizes **principled** neighborhood models. I won't go over the math derivation here, but you should take a look at section 3.1 from the paper for the detail.

Notably, ESAE's similarity matrix is based on the *inverse* of the given data matrix. As a result, the learned weights can also be negative and thus the model can learn the **dissimilarities** between items (besides the similarities). This proves to be essential to obtain good **ranking accuracy**.





6.3 — Algorithm

The Python code of the learning algorithm is given above. The training requires only the item-item matrix $G = X^T * X$ as input, instead of the user-item matrix X . This is very efficient if the size of G is smaller than the size of X .

For my PyTorch implementation, I set the L2-Norm regularization hyper-parameter λ to be 1000, the learning rate to be 0.01, and the batch size to be 512.





repository: <https://github.com/khanhnamle1994/transfer-rec/tree/master/Autoencoders-Experiments>.

- The dataset is **MovieLens 1M**, similar to the two previous experiments that I have done using [Matrix Factorization](#) and [Multilayer Perceptron](#). The goal is to predict the ratings that a user will give to a movie, in which the ratings are between 1 to 5.
- For the AutoRec and DeepRec models, the evaluation metric is **Masked Root Mean Squared Error (RMSE)** in a rating prediction (regression) setting.
- For the CDAE, MultVAE, SVAE, and ESAE models, the evaluation metrics are **Precision**, **Recall**, and **Normalized Discounted Cumulative Gain (NDCG)** in a ranking prediction (classification) setting. As explained in the sections above, these models work with implicit feedback data, where ratings are binarized into 0 (less than equal to 3) and 1 (bigger than 3).
- The results were captured in **Comet ML**. For those that are not familiar, it is a fantastic tool that keeps track of model experiments and logs all necessary metrics in a single dashboard.

The result table is at the bottom of my repo's [README](#):

Model	Epochs	RMSE	Precision@100	Recall@100	NDCG@100	Runtime
AutoRec	500	0.910				35m16s
DeepRec	500	0.9310				54m24s
CDAE	141		0.0894	0.4137	0.2528	17m29s
MultVAE	55		0.0886	0.4115	0.2508	6m31s
SVAE	50		8.18	58.4987	38.0714	6h37m19s
ESAE	50		0.0757	0.4181	0.2561	10m12s

Model Evaluation

For **rating** prediction:

- AutoRec performs better than DeepRec: lower RMSE and shorter runtime.
- This is quite surprising, as DeepRec is a deeper architecture than AutoRec.





- Between the remaining three models: CDAE has the highest Precision@100, ESAE has the highest Recall@100 and NDCG@100, and MultVAE has the shortest runtime.

Conclusion

In this post, I have discussed the nuts and bolts of Auto-encoders and their use in collaborative filtering. I also walked through 6 different papers that use Auto-encoders for the recommendation framework: (1) AutoRec, (2) DeepRec, (3) Collaborative Denoising Auto-encoder, (4) Multinomial Variational Auto-encoder, (5) Sequential Variational Auto-encoder, and (6) Embarrassingly Shallow Auto-encoder.

There are several **emerging research directions** that are happening in this area:

- When facing different recommendation requirements, it is important to incorporate auxiliary information to help understand users and items to further improve the performance of recommendation. The capacity of auto-encoders to process heterogeneous data sources brings great opportunities in recommending diverse items with unstructured data such as text, images, audio, and video features.
- Many effective unsupervised learning techniques based on auto-encoders have recently emerged: weighted auto-encoders, ladder variational auto-encoders, and discrete variational auto-encoders. Using these new auto-encoders variants will help improve the recommendation performance even further.
- Besides collaborative filtering, one can integrate the auto-encoders paradigm with content-based filtering and knowledge-based recommendation methods. These are largely under-explored areas that have the potential for progress.

Stay tuned for future blog posts of this series that explore different modeling architectures that have been made for collaborative filtering.

References

- *[Autoencoders Meet Collaborative Filtering](#)*. Suvash Sedhain, Aditya Krishna Menon, Scott Sanner, and Lexing Xie. May 2015.
- *[Training Deep Autoencoders for Collaborative Filtering](#)*. Oleksii Kuchaiev and Boris Ginsburg. August 2017.
- *[Collaborative Denoising Autoencoders for Top-N Recommender Systems](#)*. Yao Wu, Christopher





- *[Sequential Variational Autoencoders for Collaborative Filtering](#)*. Naveen Sachdeva, Giuseppe Manco, Ettore Ritacco, and Vikram Pudi. November 2018.
- *[Embarrassingly Shallow Autoencoders for Sparse Data](#)*. Harald Steck. May 2019.

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