Modeling of PM synchronous motor

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Introduction

Synchronous motors are synchronous machines used to convert electrical power to mechanical power. To begin to understand synchronous motor operation, it is reasonable to start with the investigation in its mathematical model. The goal of this part is to study the mathematical model of permanent magnet synchronous motor (PMSM) and model it in SIMULINK application package.

Parameters

In the **Table 1**, the parameters of the PMSM, Initial conditions, external voltages, and external load torque are represented.

Explanation	Symbol [unit]	Value
Variant number	-	2
Input voltage amplitude	$A[V]$	136
Electrical angular frequency of input power supply	$W[rad/s]$	74
Stator resistance	$R[\Omega]$	6.25
Stator inductance	$L[H]$	0.030
The rotor inertia	$J_m[kg.m^2]$	0.00027
The constant flux generated by permanent	$\lambda_m[Wb]$	0.32
Number of poles pairs	n_p	5
Load torque	$\tau_{L1}[N.m]$	0.151
The viscous friction coefficient	$B[N.m.s]$	0
Initial currents at stator phases	$i_{\alpha\beta\_0}[A]$	$\begin{bmatrix}0 & 0\end{bmatrix}^T$
Initial value of mechanical angular frequency	$\omega_0[rad/s]$	0
Initial angle of the rotor	$\theta_0[rad]$	0

Table 1: Parameters of the PMSM, Initial conditions, external voltages, and external load torque.

Theoretical part

Considering Clarke forward transformation, the equivalent model of the unsaturated, non-salient PMSM in stationary two-phase coordinate system associated with stator winding is given by:

$$\dot{\lambda}_{\alpha\beta} = v_{\alpha\beta} - Ri_{\alpha\beta} \tag{1}$$

$$J_m\dot{\omega} = -B\omega + \tau_e - \tau_L \tag{2}$$

$$\dot{\theta} = \omega \tag{3}$$

Where $\lambda_{\alpha\beta} \in \mathbb{R}^2$ is the total flux, $i_{\alpha\beta} \in \mathbb{R}^2$ is the stator current, $v_{\alpha\beta} \in \mathbb{R}^2$ is the stator voltages, $R \in \mathbb{R}_+^*$ is the stator windings resistance, $J_m \in \mathbb{R}_+^*$ is the rotor inertia, $\theta \in [0, 2\pi[$ is the rotor angular position , $\omega \in \mathbb{R}$ is the rotor mechanical angular velocity, $B \in \mathbb{R}_+$ is the viscous friction coefficient, $\tau_L \in \mathbb{R}$ is the load torque, and $\tau_e \in \mathbb{R}$ is the generated electrical torque by the motor.

The electric angular position and speed relate to mechanical ones through the number of pole pairs $n_p \in \mathbb{N}$:

$$\theta_e = \text{modulus}(n_p\theta), \ \omega_e = n_p\omega \tag{4}$$

Where $\theta_e \in [0, 2\pi[$ is the electrical angle of the induced voltage by the rotor in the stator windings, $\omega_e \in \mathbb{R}$ is the electrical angular frequency.

Note: the function (modulus) is just to ensure that all angles are within the required ranges.

By applying Clarke forward transformation to the equation that relate the total flux of surface mounted PMSM to its stator phase currents and the generated flic by the permanent magnets, we can find that the total flux in the two-phase frame verifies:

$$\lambda_{\alpha\beta} = Li_{\alpha\beta} + \lambda_m C(\theta) \tag{5}$$

Where $L \in \mathbb{R}_+^*$ is the stator inductance which is the same of the stator windings self-inductance and the inductance that represents the armature reactance phenomena in the motor, λ_m is the constant flux generated by the permanent magnets, and $C(\theta)$ is given by definition:

$$C(\theta) = \begin{bmatrix} \cos(n_p\theta) & \sin(n_p\theta) \end{bmatrix}^T \tag{5}$$

It follows from differentiating (5):

$$\dot{\lambda}_{\alpha\beta} = L\frac{di_{\alpha\beta}}{dt} + \lambda_m \omega C'(\theta) \tag{6}$$

Where:

$$C'(\theta) = \frac{dC}{d\theta}(\theta) = n_p JC(\theta) \tag{7}$$

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{8}$$

Thus, the equation of the first state variable (stator currents $i_{\alpha\beta}$) is obtained by combining (6) and (1):

$$\frac{di_{\alpha\beta}}{dt} = \frac{1}{L}(v_{\alpha\beta} - Ri_{\alpha\beta} - \lambda_m \omega C'(\theta)) \tag{9}$$

The torque of electrical origin generated by the PMSM is calculated as follows:

$$\tau_e = \frac{P_{conv}}{\omega} \tag{10}$$

Where $\tau_e = \frac{P_{conv}}{\omega}$ is the converted electrical energy into a mechanical form, which equal multiplication of the induced voltage by rotor permanent magnets field and the stator currents:

$$P_{conv} = \lambda_m \omega i^T C'(\theta) \tag{11}$$

Thus, it is found:

$$\tau_e = \lambda_m i_{\alpha\beta}^T C'(\theta) = n_p \lambda_m i_{\alpha\beta}^T J C(\theta) \tag{12}$$

Using the fact that there is no power dissipated in the stator inductance the electrical torque of the motor can be found by:

$$\tau_e = n_p i_{\alpha\beta}^T J \lambda_{\alpha\beta} = n_p (\lambda_\alpha i_\beta - \lambda_\beta i_\alpha) \tag{13}$$

Thus, a state–space model of the PMSM motor is given by:

$$\begin{aligned} \frac{di_{\alpha\beta}}{dt} &= \frac{1}{L}(v_{\alpha\beta} - Ri_{\alpha\beta} - \lambda_m \omega C'(\theta)) \\ \dot{\omega} &= \frac{1}{J_m}(-B\omega + \lambda_m i_{\alpha\beta}^T C'(\theta) - \tau_L) \\ \dot{\theta} &= \omega \end{aligned} \tag{14}$$