Control Sensorless controller based on DREM adaptive observer

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1. Introduction

Synchronous motors are synchronous machines used to convert electrical power to mechanical power. Controlling these types of machines requires measurement or observation of some physical variables of them like, rotor position, rotation speed, generated flux, and so on. In the previous part we have considered the adaptive observer with gradient-based estimator. However, we continue in this part with the dynamic regressor extension and mixing (DREM) adaptive observer and sensorless controller using Matlab/Simulink environment.

2. Parameters

In the **Table 1**, the parameters of the PMSM, Initial conditions, external voltages, and external load torque are represented.

Explanation	Symbol [unit]	Value
Variant number	V	2
Input voltage amplitude	$A[V]$	136
Stator resistance	$R[\Omega]$	6.25
Stator inductance	$L[H]$	0.030
The rotor inertia	$J_m[kg.m^2]$	0.00027
The constant flux generated by permanent magnet	$\lambda_m[Wb]$	0.32
Number of poles pairs	n_p	5
Load torque	$\tau_{L1}[N.m]$	0.151
The viscous friction coefficient	$B[N.m.s]$	0
Initial currents at stator phases	$i_{\alpha\beta\_0}[A]$	$\begin{bmatrix}0 & 0\end{bmatrix}^T$
Initial value of mechanical angular frequency	$\omega_0[rad/s]$	0
Initial angle of the rotor	$\theta_0[rad]$	0

Table 1: Parameters of the PMSM, Initial conditions, external voltages, and external load torque.

3. Theoretical part

• PMSM model

Considering Clarke forward transformation, the equivalent model of the unsaturated, non-salient PMSM in stationary two-phase coordinate system associated with stator winding is given by:

$$\dot{\lambda}_{\alpha\beta} = v_{\alpha\beta} - Ri_{\alpha\beta} \tag{1}$$

$$J_m\dot{\omega} = -B\omega + \tau_e - \tau_L \tag{2}$$

$$\dot{\theta} = \omega \tag{3}$$

Where $\lambda_{\alpha\beta} \in \mathbb{R}^2$ is the total flux, $i_{\alpha\beta} \in \mathbb{R}^2$ is the stator current, $v_{\alpha\beta} \in \mathbb{R}^2$ is the stator voltages, $R \in \mathbb{R}_+^*$ is the stator windings resistance, $J_m \in \mathbb{R}_+^*$ is the rotor inertia, $\theta \in [0, 2\pi[$ is the rotor angular position , $\omega \in \mathbb{R}$ is the rotor mechanical angular velocity, $B \in \mathbb{R}_+$ is the viscous friction coefficient, $\tau_L \in \mathbb{R}$ is the load torque, and $\tau_e \in \mathbb{R}$ is the generated electrical torque by the motor.

The electric angular position and speed relate to mechanical ones through the number of pole pairs $n_p \in \mathbb{N}$:

$$\theta_e = \text{modulus}(n_p\theta),\ \ \omega_e = n_p\omega \tag{4}$$

Where $\theta_e \in [0, 2\pi[$ is the electrical angle of the induced voltage by the rotor in the stator windings, $\omega_e \in \mathbb{R}$ is the electrical angular frequency.

Note: the function (modulus) is just to ensure that all angles are within the required ranges.

By applying Clarke forward transformation to the equation that relate the total flux of surface mounted PMSM to its stator phase currents and the generated flic by the permanent magnets, we can find that the total flux in the two-phase frame verifies:

$$\lambda_{\alpha\beta} = Li_{\alpha\beta} + \lambda_m C(\theta) \tag{5}$$

Where $L \in \mathbb{R}_+^*$ is the stator inductance which is the same of the stator windings self-inductance and the inductance that represents the armature reactance phenomena in the motor, λ_m is the constant flux generated by the permanent magnets, and $C(\theta)$ is given by definition:

$$C(\theta) = \begin{bmatrix} \cos(n_p\theta) & \sin(n_p\theta) \end{bmatrix}^T \tag{5}$$

It follows from differentiating (5):

$$\dot{\lambda}_{\alpha\beta} = L\frac{di_{\alpha\beta}}{dt} + \lambda_m \omega C'(\theta) \tag{6}$$

Where:

$$C'(\theta) = \frac{dC}{d\theta}(\theta) = n_p JC(\theta) \tag{7}$$

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{8}$$

Thus, the equation of the first state variable (stator currents $i_{\alpha\beta}$) is obtained by combining (6) and (1):

$$\frac{di_{\alpha\beta}}{dt} = \frac{1}{L}(v_{\alpha\beta} - Ri_{\alpha\beta} - \lambda_m \omega C'(\theta)) \tag{9}$$

The torque of electrical origin generated by the PMSM is calculated as follows:

$$\tau_e = \frac{P_{conv}}{\omega} \tag{10}$$

Where $\tau_e = \frac{P_{conv}}{\omega}$ is the converted electrical energy into a mechanical form, which equal multiplication of the induced voltage by rotor permanent magnets field and the stator currents:

$$P_{conv} = \lambda_m \omega \mathbf{i}^T C'(\theta) \tag{11}$$

Thus, it is found:

$$\tau_e = \lambda_m \mathbf{i}_{\alpha\beta}^T C'(\theta) = n_p \lambda_m \mathbf{i}_{\alpha\beta}^T JC(\theta) \tag{12}$$

Using the fact that there is no power dissipated in the stator inductance the electrical torque of the motor can be found by:

$$\tau_e = n_p i_{\alpha\beta}^T J \lambda_{\alpha\beta} = n_p (\lambda_\alpha i_\beta - \lambda_\beta i_\alpha) \tag{13}$$

Thus, a state–space model of the PMSM motor is given by:

$$
\begin{aligned}
\frac{di_{\alpha\beta}}{dt} &= \frac{1}{L}(v_{\alpha\beta} - Ri_{\alpha\beta} - \lambda_m \omega C'(\theta)) \\
\dot{\omega} &= \frac{1}{J_m}(-B\omega + \lambda_m i_{\alpha\beta}^T C'(\theta) - \tau_L) \\
\dot{\theta} &= \omega
\end{aligned}
\tag{14}
$$

- **problem statement**

The goal is to estimate the motor state variables – the total flux $\lambda_{\alpha\beta}$, the rotor position $\theta(t)$ and the rotor speed $\omega(t)$ ensuring the fulfillment of the following conditions:

$$
\begin{aligned}
\lim_{t\to\infty}(\hat{\lambda}_{\alpha\beta}(t) - \lambda_{\alpha\beta}(t)) &= 0 \\
\lim_{t\to\infty}(\hat{\omega}(t) - \omega(t)) &= 0 \\
\lim_{t\to\infty}(\hat{\theta}(t) - \theta(t)) &= 0
\end{aligned}
\tag{15}
$$

And to use the estimates $\hat{\lambda}_{\alpha\beta}(t)$, $\hat{\omega}(t)$, and $\hat{\theta}(t)$ to control the motor that has no speed and position sensors. It is necessary to construct the state observer in $\alpha\beta$ stationary frame since the dq coordinate system is unavailable.

The task is to develop an observer is developed under the following assumptions.

A1- The currents $i_{\alpha\beta}$, voltages $v_{\alpha\beta}$ are measurable variables, while angular position $\theta(t)$, and speed $\omega(t)$ are immeasurable variables.

A2- The known parameters of the PMSM are the stator inductance L, the stator resistance R, and number of pair poles, while the unknown parameters are the rotor inertia J_m, the constant flux λ_m, and the viscous friction coefficient B.

A3- The load torque $\tau_L(t)$ is unknown.

A4- Signals $v_{\alpha\beta}(t)$ and $\tau_L(t)$ are such that the PMSM system (14) is forward complete, and all signals are bounded.

A5- Signals $v_{\alpha\beta}(t)$ and $i_{\alpha\beta}(t)$ are integrable functions.

- **Observer design**

In this section we briefly recall the standing equations for the adaptive observer with gradient estimator.

- **Model reparameterization**

First, we denote the component of magnetic flux generated by the rotor permanent magnet as the vector

$$x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}=\lambda_m\begin{bmatrix} \cos(n_p\theta(t)) \\ \sin(n_p\theta(t)) \end{bmatrix} \tag{16}$$

Using (16) the total flux in the motor will take the following form

$$\lambda_{\alpha\beta}=Li_{\alpha\beta}+x(t) \tag{17}$$

differentiating (17) leads to

$$\dot{\lambda}_{\alpha\beta}=L\frac{di_{\alpha\beta}}{dt}+\dot{x}(t) \tag{18}$$

By substituting (18) in the equation that connect between stator input voltages, stator input currents, and the total flux (1) we get

$$L\frac{di_{\alpha\beta}}{dt}+\dot{x}(t)=v_{\alpha\beta}-Ri_{\alpha\beta} \tag{19}$$

We introduce two auxiliary variables $z_1(t)$ and $z_2(t)$ such that

$$\dot{z}_1(t)=v_{\alpha\beta}(t)\ \ ,\ \ \dot{z}_2(t)=i_{\alpha\beta}(t) \tag{20}$$

One can see that (20) leads to the following expression of the variables $z_1(t)$ and $z_2(t)$

$$z_1(t)=\int_0^t v_{\alpha\beta}(t)dt+z_1(0)\ \ ,\ \ z_2(t)=\int_0^t i_{\alpha\beta}(t)dt+z_2(0) \tag{21}$$

Where $z_1(0)$ and $z_2(0)$ are constant parameters which considered to be unknown, as they represent the initial value of the integrators for the voltages and currents variable. Integrating both sides of (19) one gets

$$x(t)=z_1(t)-Rz_2(t)-Li_{\alpha\beta}(t)-z_1(0)+Rz_2(0)+i_{\alpha\beta}(0)+x(0) \tag{22}$$

To simplify notations, we rewrite the latter equation in the form

$$x(t)=m(t)+\eta \tag{22}$$

Where $m(t) = z_1(t) - Rz_2(t) - Li_{\alpha\beta}(t)$ is a known vector, and $\eta = -z_1(0) + Rz_2(0) + i_{\alpha\beta}(0)$ $+x(0)$ is unknown constants, that are to be estimated.

At this step, the task of motor states estimation is translated into another one, that is an identification of two unknown constants $\eta = [\eta_1 \quad \eta_2]^T$. Considering the equation (22), and replacing $x(t)$ by $\lambda_m C(\theta)$, one can find

$$\lambda_m \begin{bmatrix} \cos(n_p\theta) \\ \sin(n_p\theta) \end{bmatrix}^T = m(t) + \eta \tag{23}$$

Squaring both sides of the equation (23) and using the basic trigonometric identity $\cos^2(.) + \sin^2(.) = 1$, one can get

$$g = 2m^T\eta + C \tag{24}$$

Where $g = -m^T m$ and $C = \eta^T\eta - \lambda_m^2$.

To get ride off the constant C in expression (24), a filter $F(p) = \left[\dfrac{\alpha p}{p+\alpha}\right]$ is applied to both sides of the equation (24). As a result, we get the linear regression model

$$y = q^T\eta \tag{25}$$

Where $y = F(p)g$ and $q = 2F(p)m$ are known signals.

- **<u>Regressor extension</u>**

DREM algorithm (Dynamic Regression Extension and Mixing) is used for modification of linear regression model such that n-size regression model transforms into n scalar regression model (n = 2 in this case). DREM algorithm can be divided into two steps: regressor extension and regressor mixing. To extend the regressor, new measurements for both regressor and regressant must be obtained (in regression model (25) vector q is the regressor and scalar y is the regressant). Original and obtained regressors must be linear independent. It can be achieved by using some dynamical operators, such as:

1) A stable linear filter, for example:

$$\begin{cases} \bar{y} = \dfrac{\mu}{s+\mu}[y] \\ \bar{q} = \dfrac{\mu}{s+\mu}[q] \end{cases} \tag{26}$$

Where $\mu > 0$.

2) The delay operator:

$$\begin{cases} \bar{y} = y(t-\tau) \\ \bar{q} = q(t-\tau) \end{cases} \tag{27}$$

Where $\mu > 0$.

The given PMSM model nonlinear parametrization also can be used to apply DREM algorithm in the following way. Values y and q in the equation (25) are obtained by filtering of signals g and $2m$. Therefore, it is possible to obtain new measurements $\bar{y}$ and $\bar{q}$ by applying the filter $F(p,\beta)$ to the signals g and $2m$:

$$\bar{y} = \bar{q}^T \eta \tag{28}$$

Where $\bar{y} = F(p,\beta)[g]$ and $\bar{q} = 2F(p,\beta)m$.

Thus, the models (25) and (28) can be combined and transformed into the following matrix extended model:

$$Y = Q\eta \tag{29}$$

Where $Y = \begin{bmatrix} y & \bar{y} \end{bmatrix}^T$ and $Q = \begin{bmatrix} q & \bar{q} \end{bmatrix}^T$.

- **<u>Regressor mixing</u>**

The next step is regressor mixing. It is possible to express the matrix Q^{-1} in the following way:

$$Q^{-1} = \operatorname{adj}\{Q\}.\det\{Q\}^{-1} \tag{31}$$

Where $\operatorname{adj}\{Q\}$ is adjugate of matrix Q and $\det\{Q\}$ is it's determinant. In this case, these functions can be written in the following way:

$$\operatorname{adj}\{Q\} = \begin{pmatrix} \bar{q}_2 & -q_2 \\ -\bar{q}_1 & q_1 \end{pmatrix} \quad , \quad \det\{Q\} = q_1\bar{q}_2 - q_2\bar{q}_1 \tag{32}$$

Multiplying both sides of the equation (29) by $\text{adj}\{Q\}$ we get:

$$L = \Delta\eta \tag{33}$$

Where $L = \text{adj}\{Q\}Y = \begin{bmatrix} y\bar{q}_2 - q_2\bar{y} \\ q_1\bar{y} - y\bar{q}_1 \end{bmatrix}$ and $\Delta = \det\{Q\}$.

- **<u>Application of the gradient algorithm to the linear regression models</u>**

Thus, to estimate η we use the classical gradient algorithm

$$\dot{\hat{\eta}} = \Gamma\Delta(L - \Delta\hat{\eta}) \tag{34}$$

Where Γ is a design matrix that is symmetric positive definite.

For simplicity, Γ usually is chosen γI with $\gamma > 0$ is a design parameter (adaptation coefficient). Then, equation (25) is represented in the following form

$$\begin{bmatrix} \dot{\hat{\eta}}_1 \\ \dot{\hat{\eta}}_2 \end{bmatrix} = \begin{bmatrix} \gamma\Delta(l_1 - \Delta\hat{\eta}_1) \\ \gamma\Delta(l_2 - \Delta\hat{\eta}_2) \end{bmatrix} \tag{35}$$

- **<u>Flux, position, and speed observer</u>**

The estimate of the total flux $\lambda_{\alpha\beta}$ is calculated from equation (17) using expression (22) and $\hat{\eta}$

$$\hat{x}(t) = m(t) + \hat{\eta}(t) \qquad , \qquad \hat{\lambda}_{\alpha\beta} = Li_{\alpha\beta}(t) + \hat{x}(t) \tag{36}$$

The constant flux generated by the permanent magnet in the rotor can be estimated using (16) as follows

$$\hat{\lambda}_m(t) = \sqrt{\hat{x}_1^2(t) + \hat{x}_2^2(t)} \tag{38}$$

The unmeasurable angle θ is reconstructed from (22) and the estimate of η using trigonometry

$$\hat{\theta} = \frac{1}{n_p}\arctan(\frac{\hat{x}_2}{\hat{x}_1}) \tag{39}$$

The velocity is estimated with the help of a standard phase-locked loop (PLL) speed estimator

$$\begin{aligned}\dot{\chi}_1 &= K_p(\hat{\theta}-\chi_1)+K_i\chi_2\\ \dot{\chi}_2 &= \hat{\theta}-\chi_1\\ \hat{\omega} &= K_p(\hat{\theta}-\chi_1)+K_i\chi_2\end{aligned} \tag{40}$$

where $K_p > 0$ and $K_i > 0$ are proportional and integral gains, respectively.

This PLL-type speed estimator is constructed from angle error $\hat{\theta}-\chi_1$. The estimator uses PLL angle χ_1 and PLL angle error $\dot{\chi}_2$. Using these two new variables we form a tracking controller consisting of a PI regulator and an integrator. The PI gains of the controller are $K_p > 0$ and $K_i > 0$. Motivation of the PI regulator application stems from the fact that PI controllers have the ability of suppressing error under the presence of a disturbance.