Vector control of PMSM

Conducted by:

Sokrat ALDARMINI

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1. Introduction

Synchronous motors are synchronous machines used to convert electrical power to mechanical power. It is well-known that there is a need for control methods that provide some required characteristics for the operation of motors. As PMSM is an AC motor, the development of these control methods requires some special considerations that must be considered. In this part, we implement one control method for permanent magnet synchronous motor, which is the field-oriented control (vector control) method, using Matlab/Simulink environment.

2. Parameters

In the **Table 1**, the parameters of the PMSM, Initial conditions, external voltages, and external load torque are represented.

Explanation	Symbol [unit]	Value
Variant number	V	2
Input voltage amplitude	$A[V]$	136
Stator resistance	$R[\Omega]$	6.25
Stator inductance	$L[H]$	0.030
The rotor inertia	$J_m[kg.m^2]$	0.00027
The constant flux generated by permanent	$\lambda_m[Wb]$	0.32
Number of poles pairs	n_p	5
Load torque	$\tau_{L1}[N.m]$	0.151
The viscous friction coefficient	$B[N.m.s]$	0
Initial currents at stator phases	$i_{\alpha\beta\_0}[A]$	$\begin{bmatrix}0 & 0\end{bmatrix}^T$
Initial value of mechanical angular frequency	$\omega_0[rad/s]$	0
Initial angle of the rotor	$\theta_0[rad]$	0

Table 1: Parameters of the PMSM, Initial conditions, external voltages, and external load torque.

3. Theoretical part

- **PMSM model**

Considering Clarke forward transformation, the equivalent model of the unsaturated, non-salient PMSM in stationary two-phase coordinate system associated with stator winding is given by:

$$\dot{\lambda}_{\alpha\beta} = v_{\alpha\beta} - Ri_{\alpha\beta} \quad (1)$$

$$J_m\dot{\omega} = -B\omega + \tau_e - \tau_L \quad (2)$$

$$\dot{\theta} = \omega \tag{3}$$

Where $\lambda_{\alpha\beta} \in \mathbb{R}^2$ is the total flux, $i_{\alpha\beta} \in \mathbb{R}^2$ is the stator current, $v_{\alpha\beta} \in \mathbb{R}^2$ is the stator voltages, $R \in \mathbb{R}_+^*$ is the stator windings resistance, $J_m \in \mathbb{R}_+^*$ is the rotor inertia, $\theta \in [0, 2\pi[$ is the rotor angular position , $\omega \in \mathbb{R}$ is the rotor mechanical angular velocity, $B \in \mathbb{R}_+$ is the viscous friction coefficient, $\tau_L \in \mathbb{R}$ is the load torque, and $\tau_e \in \mathbb{R}$ is the generated electrical torque by the motor.

The electric angular position and speed relate to mechanical ones through the number of pole pairs $n_p \in \mathbb{N}$:

$$\theta_e = \text{modulus}(n_p\theta),\ \ \omega_e = n_p\omega \tag{4}$$

Where $\theta_e \in [0, 2\pi[$ is the electrical angle of the induced voltage by the rotor in the stator windings, $\omega_e \in \mathbb{R}$ is the electrical angular frequency.

Note: the function (modulus) is just to ensure that all angles are within the required ranges.

By applying Clarke forward transformation to the equation that relate the total flux of surface mounted PMSM to its stator phase currents and the generated flic by the permanent magnets, we can find that the total flux in the two-phase frame verifies:

$$\lambda_{\alpha\beta} = Li_{\alpha\beta} + \lambda_m C(\theta) \tag{5}$$

Where $L \in \mathbb{R}_+^*$ is the stator inductance which is the same of the stator windings self-inductance and the inductance that represents the armature reactance phenomena in the motor, λ_m is the constant flux generated by the permanent magnets, and $C(\theta)$ is given by definition:

$$C(\theta) = \left[\cos(n_p\theta) \quad \sin(n_p\theta)\right]^T \tag{5}$$

It follows from differentiating (5):

$$\dot{\lambda}_{\alpha\beta} = L\frac{di_{\alpha\beta}}{dt} + \tag{6}$$

Where:

$$C'(\theta) = \frac{dC}{d\theta}(\theta) = n_p JC(\theta) \tag{7}$$

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{8}$$

Thus, the equation of the first state variable (stator currents $i_{\alpha\beta}$) is obtained by combining (6) and (1):

$$\frac{di_{\alpha\beta}}{dt} = \frac{1}{L}(v_{\alpha\beta} - Ri_{\alpha\beta} - \lambda_m \omega C'(\theta)) \tag{9}$$

The torque of electrical origin generated by the PMSM is calculated as follows:

$$\tau_e = \frac{P_{conv}}{\omega} \tag{10}$$

Where $\tau_e = \frac{P_{conv}}{\omega}$ is the converted electrical energy into a mechanical form, which equal multiplication of the induced voltage by rotor permanent magnets field and the stator currents:

$$P_{conv} = \lambda_m \omega i^T C'(\theta) \tag{11}$$

Thus, it is found:

$$\tau_e = \lambda_m i_{\alpha\beta}^T C'(\theta) = n_p \lambda_m i_{\alpha\beta}^T JC(\theta) \tag{12}$$

Using the fact that there is no power dissipated in the stator inductance the electrical torque of the motor can be found by:

$$\tau_e = n_p i_{\alpha\beta}^T J \lambda_{\alpha\beta} = n_p (\lambda_\alpha i_\beta - \lambda_\beta i_\alpha) \tag{13}$$

Thus, a state–space model of the PMSM motor is given by:

$$\begin{gathered} \frac{di_{\alpha\beta}}{dt} = \frac{1}{L}(v_{\alpha\beta} - Ri_{\alpha\beta} - \lambda_m \omega C'(\theta)) \\ \dot{\omega} = \frac{1}{J_m}(-B\omega + \lambda_m i_{\alpha\beta}^T C'(\theta) - \tau_L) \\ \dot{\theta} = \omega \end{gathered} \tag{14}$$

- **problem statement**

The task is to develop a control system for the PMSM model, which provides tracking of a given speed $\omega^*(t)$ under the following assumptions.

A1- The currents $i_{\alpha\beta}$, voltages $v_{\alpha\beta}$, angular position θ, and speed ω are measurable.

A2- All parameters of PMSM, namely, the stator inductance L, the stator resistance R, the rotor inertia J_m, the constant flux λ_m are known.

A3- The load torque $\tau_L(t)$ is unknown.

A4- Signals $v_{\alpha\beta}(t)$ and $\tau_L(t)$ are such that the PMSM system (14) is forward complete, and all signals are bounded.

- **Vector control**

Basic techniques required for vector control:

1) Forward three-to-two phase (a,b,c)-to-(α,β) and backward (α,β)-to-(a,b,c) projection using the Clarke and inverse Clarke transformations, respectively.
2) Forward and backward two-to-two phase (α,β)-to-(d,q) and (d,q)-to-(α,β) projections using the Park and inverse Park transformations, respectively.
3) Current controllers in non-stationary (d,q) system.
4) Rotor speed controller.

The usual choice for the current and speed controllers is proportional-integral (PI) controllers.

Clarke transformation is used to simplify the analysis of three-phase circuits. In this work we omit this transformation and immediately use the PMSM model presented in two phases (α,β) stationary frame. Park transformation converts vectors in balanced two-phase orthogonal stationary system (α,β) into orthogonal rotating reference frame (d,q). It is used to simplify analysis and calculations for nonlinear PMSM motor model. The key feature of this transformation is to eliminate sine and cosine functions that depend on the rotor position θ from the motor model. Forward Park transformation is

$$\begin{pmatrix} f_d \\ f_q \end{pmatrix} = \begin{pmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{pmatrix} \begin{pmatrix} f_\alpha \\ f_\beta \end{pmatrix} \tag{15}$$

and inverse Park transformation is given by

$$\begin{pmatrix} f_\alpha \\ f_\beta \end{pmatrix} = \begin{pmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \sin(\theta_e) & \cos(\theta_e) \end{pmatrix} \begin{pmatrix} f_d \\ f_q \end{pmatrix} \tag{16}$$

Multiplying both sides of (5) with the matrix associated with Direct Park transformation we get

$$\lambda_{dq} = Li_{dq} + \lambda_m \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{17}$$

The derivative of total flux $\dot{\lambda}_{\alpha\beta}$ in (α,β) frame is found using the derivative of total flux in (d,q) frame $\dot{\lambda}_{dq}$ as follows

$$\dot{\lambda}_{\alpha\beta} = \frac{d\begin{pmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \sin(\theta_e) & \cos(\theta_e) \end{pmatrix}}{dt} \lambda_{dq} + \begin{pmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \sin(\theta_e) & \cos(\theta_e) \end{pmatrix} \frac{d\lambda_{dq}}{dt} \tag{18}$$

The derivative of rotational Matrix is given by:

$$\frac{d\begin{pmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{pmatrix}}{dt} = n_p \omega J \begin{pmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \sin(\theta_e) & \cos(\theta_e) \end{pmatrix} \tag{19}$$

Thus, using (19) in (18) we get

$$\dot{\lambda}_{\alpha\beta} = n_p \omega J \begin{pmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \sin(\theta_e) & \cos(\theta_e) \end{pmatrix} \lambda_{dq} + \begin{pmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \sin(\theta_e) & \cos(\theta_e) \end{pmatrix} \dot{\lambda}_{dq} \tag{20}$$

By multiplying both sides of the equation that relates the voltages which applied on the stator windings, the currents, and total flux with the matrix associated with Direct Park transformation we get

$$\begin{pmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{pmatrix} \dot{\lambda}_{\alpha\beta} = v_{dq} - Ri_{dq} \tag{21}$$

The left side of (21) simplified using (20) as follows

$$\begin{pmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{pmatrix} \dot{\lambda}_{\alpha\beta} = n_p \omega J \lambda_{dq} + \dot{\lambda}_{dq} \tag{22}$$

Using (22) in (21) we get

$$n_p \omega J \lambda_{dq} + \dot{\lambda}_{dq} = v_{dq} - Ri_{dq} \tag{23}$$

Using (17) to calculate the derivative of the total flux in (d,q) frame we get

$$v_{dq} = Ri_{dq} + L\frac{di_{dq}}{dt} + n_p \omega J \lambda_{dq} = Ri_{dq} + L\frac{di_{dq}}{dt} + \begin{pmatrix} -n_p \omega L i_q \\ n_p \omega (L i_d + \lambda_m) \end{pmatrix} \tag{24}$$

The torque generated by motor in (d,q) frame can be delivered from (13) as follows

$$\tau_e = n_p i_{dq}^T J \lambda_{dq} = n_p(\lambda_d i_q - \lambda_q i_d) \tag{25}$$

One can see that the torque is determined only by the current component i_q. The current i_d is parasitic and creates additional losses in the motor. Thus, the torque τ_e is controlled only by desired i_q^* and the second desired current component is set to zero $i_d^* = 0$. The torque equation in this case will satisfy the following

$$\tau_e = n_p \lambda_m i_q^* \tag{26}$$

The equation (24) contains coupling links along the d and q axes. To ensure independent control of stator voltages, the PI controller for current is supplemented by compensation for counter-emf and decoupling components.

$$\begin{aligned}\begin{pmatrix} v_d^* \\ v_q^* \end{pmatrix} &= PI\begin{pmatrix} i_d^* - i_d \\ i_q^* - i_q \end{pmatrix} + \begin{pmatrix} -n_p \omega L i_q \\ n_p \omega (L i_d + \lambda_m) \end{pmatrix} = \\ &= \begin{pmatrix} K_{pi}(i_d^* - i_d) + K_{ii}\int (i_d^* - i_d)dt \\ K_{pi}(i_q^* - i_q) + K_{ii}\int (i_q^* - i_q)dt \end{pmatrix} + \begin{pmatrix} -n_p \omega L i_q \\ n_p \omega (L i_d + \lambda_m) \end{pmatrix}\end{aligned} \tag{27}$$

where K_{pi} and K_{ii} are proportional and integral gains for current controller, respectively. The errors for the current controllers

$$e_d = i_d^* - i_d \;\;,\;\; e_q = i_q^* - i_q \tag{28}$$

Transition $v_{dq}^* \to v_{\alpha\beta}^*$ is performed by the inverse Park transformation. To track the desired speed ω^* we use PI speed controller

$$\tau_e^* = n_p \lambda_m i_q^* = PI(\omega^* - \omega) = K_{p\omega}(\omega^* - \omega) + K_{i\omega}\int (\omega^* - \omega)dt \tag{29}$$

where $K_{p\omega}$, and $K_{i\omega}$ are proportional and integral gains for the speed controller, respectively.

Note: the first equality in (29) is satisfied since the torque τ_e in (25) is controlled only by desired i_q^* and the second desired current component is set to zero $i_d^* = 0$, then (26) is satisfied.

The speed error is calculated by

$$e_\omega = \omega^* - \omega \tag{30}$$

Finally, the structure of FOC controller with speed regulation is shown in **Figure 1**. The output variables of the motor model are the currents i_α and i_β, as well as angular position θ_e and the speed ω_e of the rotor flux. The inputs for the PMSM model are the voltages v_α and v_β. Projections of currents in the rotating frame i_{dq} are calculated using Park transformation (15) that requires measurement of the rotor position θ.

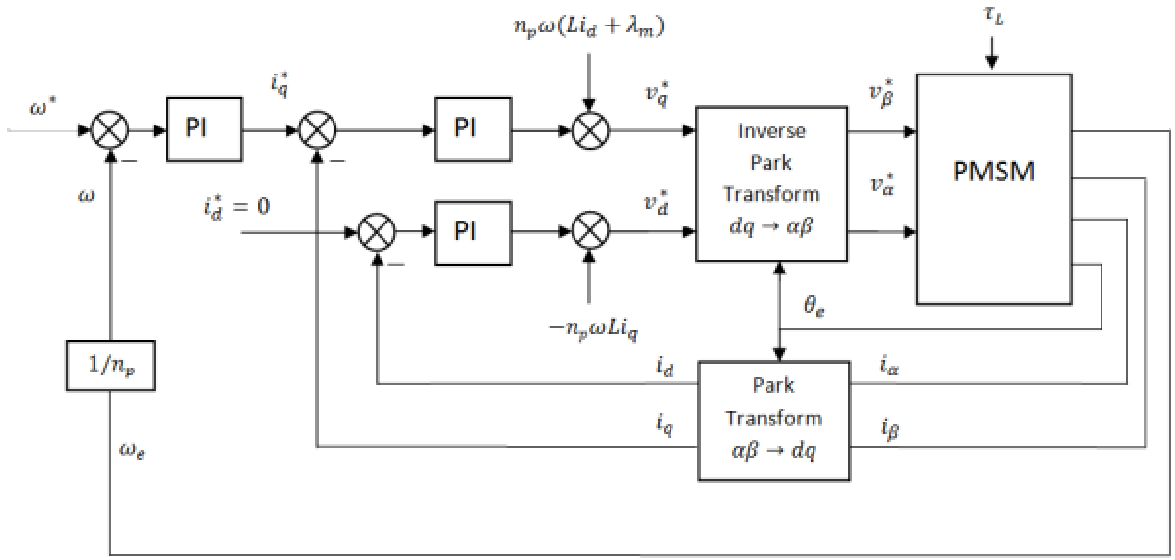

Figure 1: FOC with speed regulation

Control scheme consists of d- and q-loops. The d-loop controls the total flux, and the q-loop is used to control the motor torque and speed. The d-loop includes PI controller for the current i_d with zero reference. The q-loop comprises two controllers connected in series. The inner controller is the PI controller for the current i_q and the outer is the PI controller for the speed ω.

- **Controllers Parameters Design**

First the design of the currents controller is curried out considering the compensation for counter-emf and decoupling components in equation (24), which result in RL system with the following transfer function

$$G_i(s) = \frac{1}{Ls + R} \tag{31}$$

Then the closed loop system with the controller is become

$$G_{ic}(s)=\frac{\frac{(K_{pi}s+K_{ii})}{s(Ls+R)}}{\frac{(K_{pi}s+K_{ii})}{s(Ls+R)}+1} \tag{32}$$

Then by chosen the ratio between the proportional and integral gains such that

$$\frac{K_{ii}}{K_{pi}}=\frac{R}{L} \tag{33}$$

the pole of the open loop system will be deleted, and the closed loop system will take the following form

$$G_{ic}(s)=\frac{K_{pi}}{Ls+K_{pi}} \tag{34}$$

As the system shown in (34) is a first order system the settling time for it is given as follows

$$T_{is}=\frac{3L}{K_{pi}} \tag{35}$$

Then it is quite reasonable from (35), and (33) that the choice of the settling time for the currents, leads to coefficients K_{pi}, K_{ii}.

The design of the velocity controller is curried out considering the mechanical dynamic of the system which is

$$\dot{\omega}=\frac{1}{J_m}(-B\omega+\tau_e-\tau_L) \tag{36}$$

Once the currents are controlled, the torque of the motor is considered controlled, and it can be given by the equation (26). To make the approximation $\tau_e^*=\tau_e$, which the torque reference equals the actual generated torque the dynamic of the controlled velocity of the motor must be much slower than the dynamic of the currents after we controlled them.

Thus, the first requirements to design the speed controller is to have a settling time 10 times the settling time of the currents in the motor.

$$T_{s\omega}=T_{si} \tag{37}$$

The second requirement is to make the closed loop system for the velocity with damping coefficient that result in an admissible overshoot (less than 10%).

Neglecting the load torque in (36) and assuming satisfying the condition that the settling time of the speed is much higher than the settling time of the controlled torque, we get the following linear system

$$J_m \dot{\omega} = -B\omega + \tau_e \tag{38}$$

Thus, the transfer function of the system (38) is

$$G_\omega(s) = \frac{1}{J_m s + B} \tag{39}$$

Then the closed loop with the PI controller in (29) has the following form

$$G_{\omega c}(s) = \frac{K_{p\omega} s + K_{i\omega}}{J_m s^2 + (B + K_{p\omega})s + K_{i\omega}} = \frac{\frac{K_{p\omega}}{J_m} s + \frac{K_{i\omega}}{J_m}}{s^2 + \frac{(B + K_{p\omega})}{J_m} s + \frac{K_{i\omega}}{J_m}} \tag{40}$$

Comparing with the will-known form of the second order system which has the following denominator

$$D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{41}$$

Where ζ is the damping coefficient of the system, and ω_n is its natural frequency.

We can see that the controller coefficients can be found with the following expression

$$K_{p\omega} = 2\xi J_m \omega_n - B \;\; , \;\; K_{i\omega} = J_m \omega_n^2 \tag{42}$$

The only thing that is left is to choose the appropriate coefficients for the second order system that we need to use to calculate the controller parameters. The choice of these parameters is done considering the required settling time for the speed and the overshoot which are given for the second order system as follow

$$\xi = \sqrt{\frac{\ln^2(\frac{\sigma}{100})}{\pi^2 + \ln^2(\frac{\sigma}{100})}} \quad , \quad \omega_n = \frac{4}{\xi T_{s\omega}} \tag{43}$$

Where σ is the value of the overshoot given as a percentage of the steady state value.

It is seen from (42) and (42) that by choosing the value of speed settling time, which is determined by (37) with respect to settling time of the currents, and the value of the overshoot, the controller coefficients can be found.