

# CSC3022H: Machine Learning: Concept Learning

Geoff Nitschke

Department of Computer Science  
University of Cape Town, South Africa

# Examples of Machine Learning Types

- **Supervised Learning:**

- **Classification.**
- Regression.

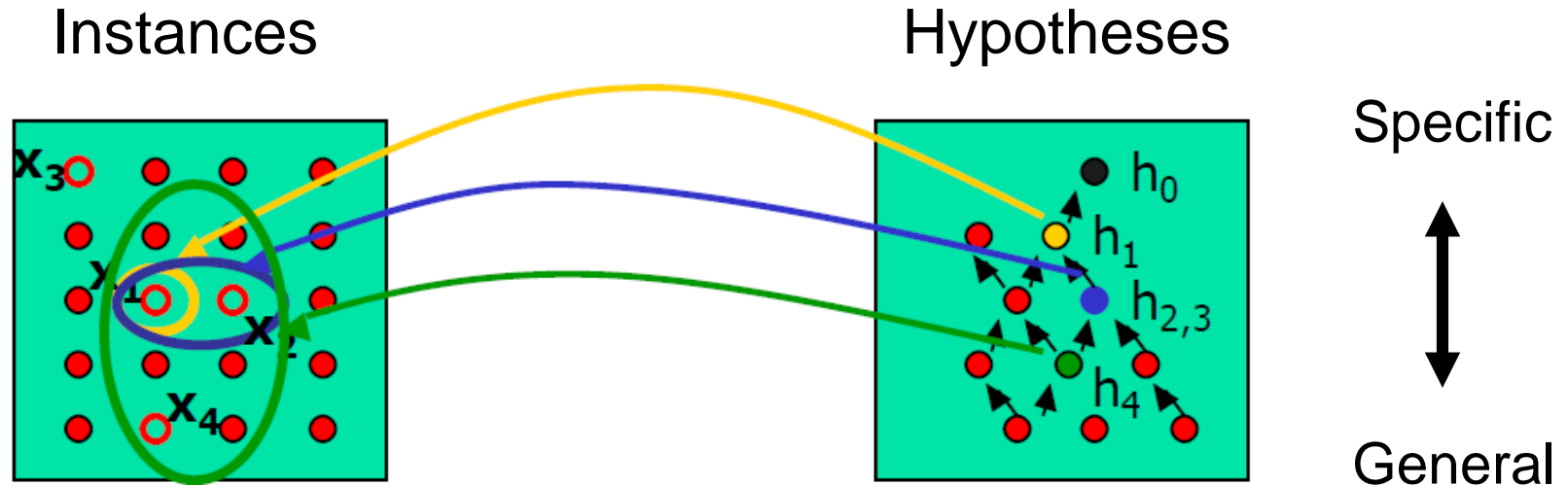
- **Unsupervised Learning:**

- Clustering.
- Dimensionality reduction.

- **Reinforcement Learning:**

- Value and policy iteration.
  - Q Learning.
-

# Recap: Hypothesis Space Search by Find-S



$h_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$

$x_1 = \langle \text{Sunny, Warm, Normal, Strong, Warm, Same} \rangle, +$

$h_1 = \langle \text{Sunny, Warm, Normal, Strong, Warm, Same} \rangle$

$x_2 = \langle \text{Sunny, Warm, High, Strong, Warm, Same} \rangle, +$

$h_2 = \langle \text{Sunny, Warm, ?, Strong, Warm, Same} \rangle$

$x_3 = \langle \text{Rainy, Cold, High, Strong, Warm, Change} \rangle, -$

$h_3 = \langle \text{Sunny, Warm, ?, Strong, Warm, Same} \rangle$

$x_4 = \langle \text{Sunny, Warm, High, Strong, Cool, Change} \rangle, +$

$h_4 = \langle \text{Sunny, Warm, ?, Strong, ?, ?} \rangle$

# Version Spaces

## ■ Hypothesis $h$ :

- Is **consistent** with a set of training examples  $D$  of target concept  $c$  if and only if  $h(x) = c(x)$  for each training example  $\langle x, c(x) \rangle$  in  $D$ .

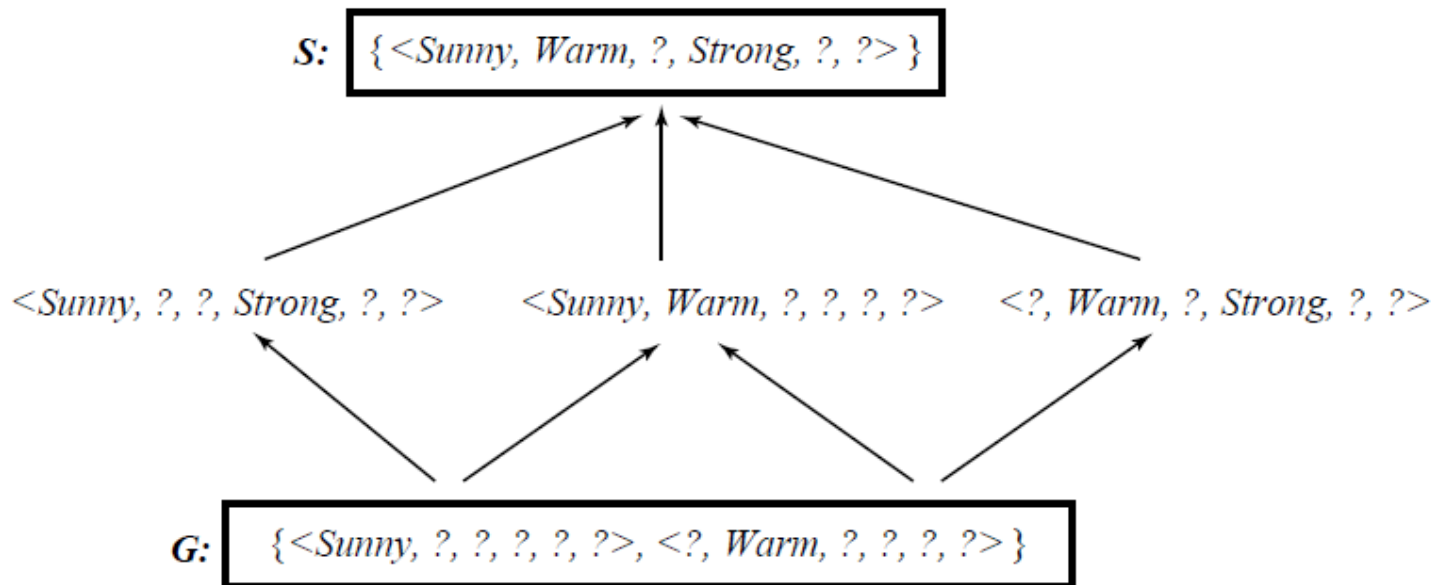
$$Consistent(h, D) \equiv (\forall \langle x, c(x) \rangle \in D) h(x) = c(x)$$

## ■ Version space $VS_{H,D}$ :

- With respect to hypothesis space  $H$ , and training set  $D$ ,  $VS_{H,D}$  is the subset of hypotheses from  $H$  that are consistent with all training examples (in  $D$ ):

$$VS_{H,D} \equiv \{h \in H \mid Consistent(h, D)\}$$

# Example Version Space



# List-Then Eliminate Algorithm

- *VersionSpace*  $\rightarrow$  List containing every hypothesis in  $H$
- For each training example  $\langle x, c(x) \rangle$ 
  - Remove from *VersionSpace* any hypothesis that is inconsistent with the training example  $h(x) \neq c(x)$ .
- Output the list of hypotheses in *VersionSpace*.
- Advantages? – Disadvantages?

# Representing Version Spaces

- **General boundary  $G$ :** Of version space  $VS_{H,D}$  is the set of *maximally general members*.
- **Specific boundary  $S$ :** Of version space  $VS_{H,D}$  is the set of *maximally specific members*.
- Every member of the version space lies between these boundaries:

$$VS_{H,D} = \{h \in H \mid (\exists s \in S)(\exists g \in G)(g \geq h \geq s)\}$$

- **Where:**  $\mathbf{x} \geq \mathbf{y}$  means  $\mathbf{x}$  is more general or equal to  $\mathbf{y}$ .

# Candidate Elimination Algorithm

$G \leftarrow$  Maximally general hypotheses in  $H$

$S \leftarrow$  Maximally specific hypotheses in  $H$

**FOR** each training example  $d$

**IF**  $d$  is a +ve example **THEN**

Remove from  $G$  any hypothesis inconsistent with  $d$

**FOR** each hypothesis  $s$  in  $S$  that is not consistent with  $d$

Remove  $s$  from  $S$

**Add to S:** All minimal generalisations  $h$  of  $s$  such that:

$h$  is consistent with  $d$  **and** a member of  $G$  is more general than  $h$

**Remove from S:** Any hypothesis that is *more general than* another hypothesis in  $S$

**END FOR**

**END IF**

**END FOR**



# Candidate Elimination Algorithm

**IF**  $d$  is a -ve example **THEN**:

Remove from  $S$  any hypothesis that is inconsistent with  $d$

**FOR** each hypothesis  $g$  in  $G$  that is not consistent with  $d$ :

Remove  $g$  from  $G$

Add to  $G$  all minimal specializations  $h$  of  $g$  such that:

$h$  is consistent with  $d$  **and**

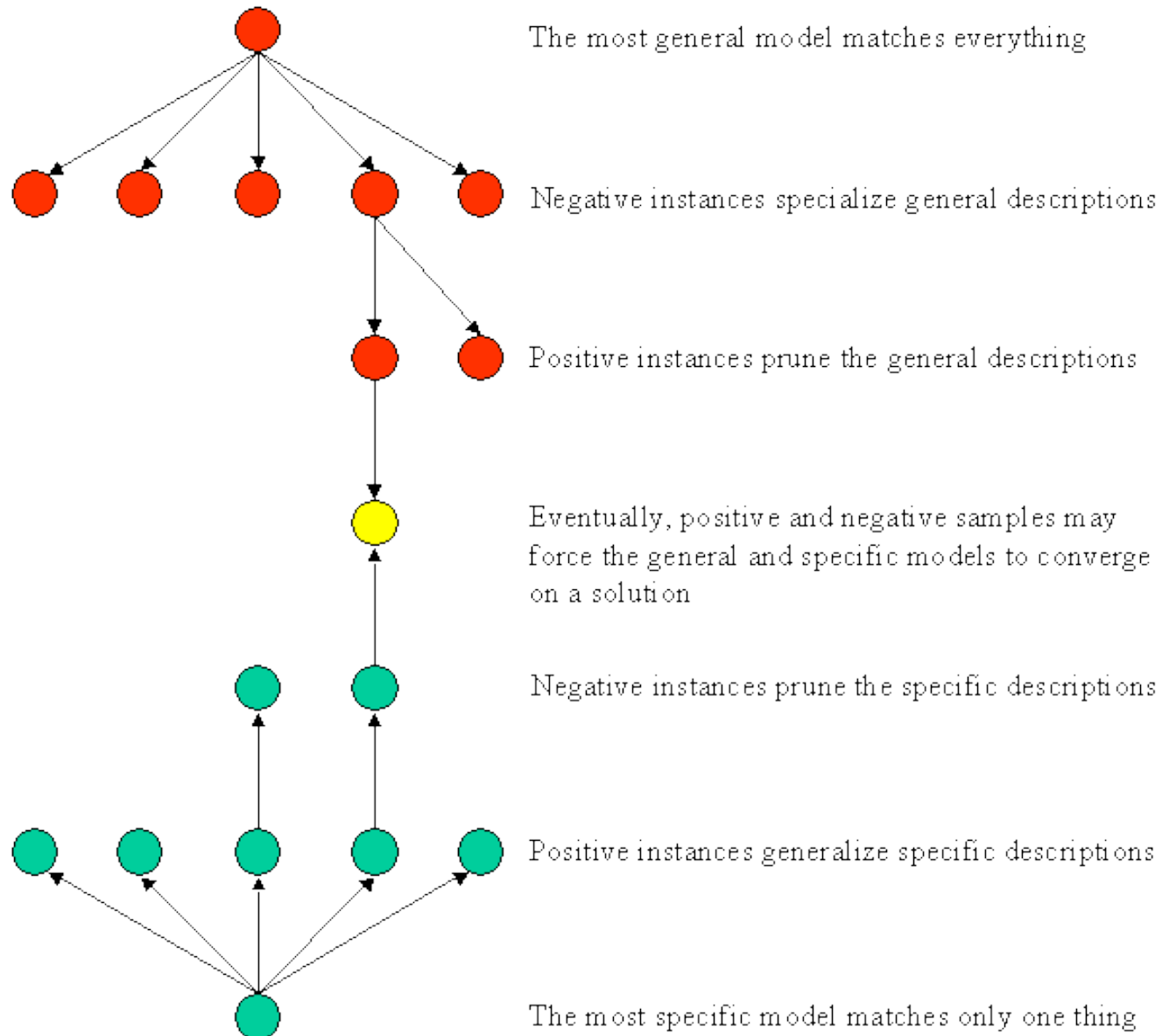
some member of  $S$  is more specific than  $h$

Remove from  $G$  any hypothesis that is less general than another hypothesis in  $G$

**END FOR**

**END IF**

# Version Space Diagram



# Learn concept: "Japanese Economy Car"

**Features:** ( Country of Origin, Manufacturer, Color, Decade, Type )

<i>Origin</i>	<i>Manufacturer</i>	<i>Colour</i>	<i>Decade</i>	<i>Type</i>	<i>Example Type</i>
Japan	Honda	Blue	1980	Economy	Positive
Japan	Toyota	Green	1970	Sports	Negative
Japan	Toyota	Blue	1990	Economy	Positive
USA	Chrysler	Red	1980	Economy	Negative
Japan	Honda	White	1980	Economy	Positive
Japan	Toyota	Green	1980	Economy	Positive
Japan	Honda	Red	1990	Economy	Negative

# Learn concept: "Japanese Economy Car"

## 1. **Positive Example:** ( Japan, Honda, Blue, 1980, Economy ):


Initialise G to a singleton set that includes everything.

$$G = \{ (?, ?, ?, ?, ?) \}$$

Initialise S to a singleton set that includes the first positive example.

$$S = \{ (\text{Japan, Honda, Blue, 1980, Economy}) \}$$

 ( ?, ?, ?, ?, ? )

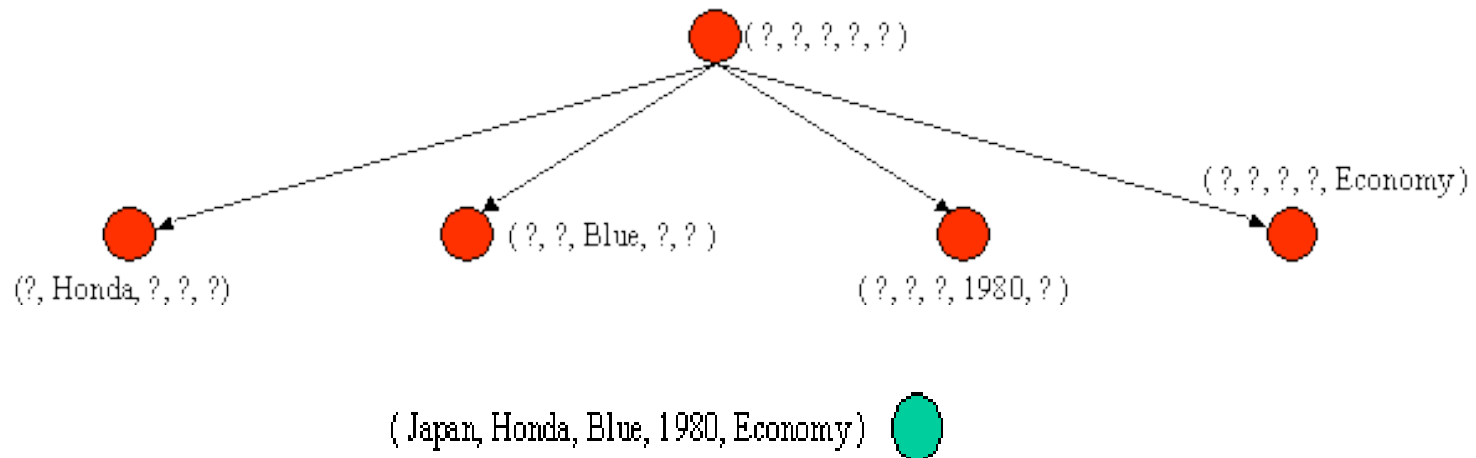
( Japan, Honda, Blue, 1980, Economy ) 

# Learn the concept: "Japanese Economy Car"

**2. Negative Example:** ( Japan, Toyota, Green, 1970, Sports ):

Specialise G to exclude the -ve example.

G =	$\{$ ( ?, Honda, ?, ?, ? ), ( ?, ?, Blue, ?, ? ), ( ?, ?, ?, 1980, ? ), ( ?, ?, ?, ?, Economy ) $\}$
S =	$\{$ ( Japan, Honda, Blue, 1980, Economy ) $\}$



# Learn concept: "Japanese Economy Car"

**Features:** ( Country of Origin, Manufacturer, Color, Decade, Type )

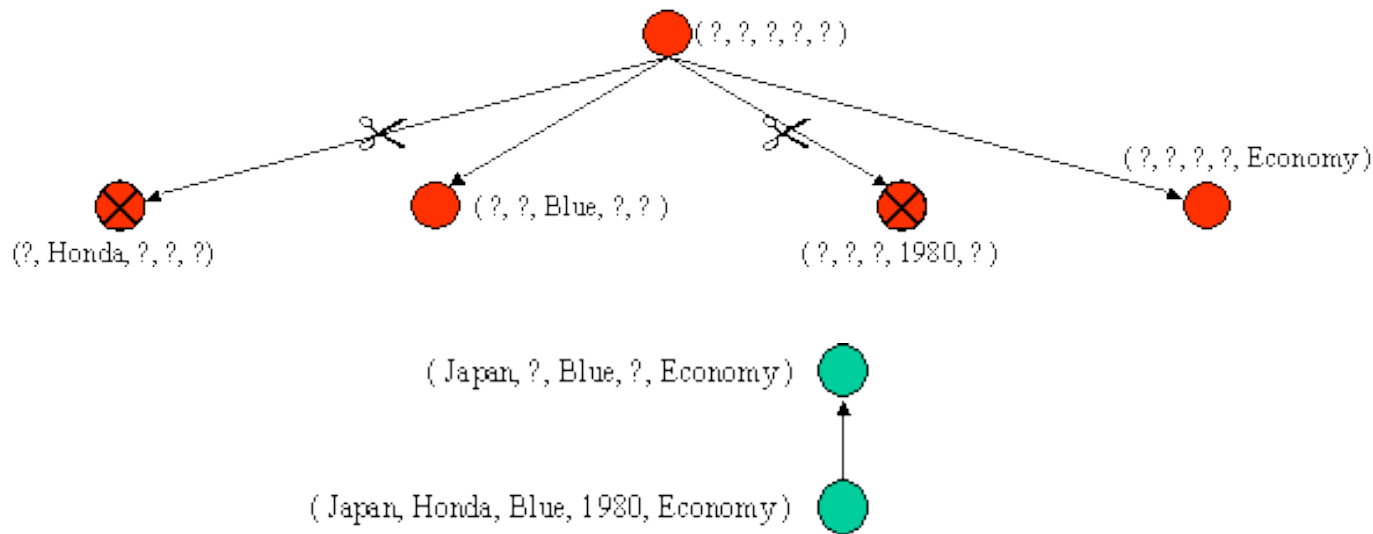
<i>Origin</i>	<i>Manufacturer</i>	<i>Colour</i>	<i>Decade</i>	<i>Type</i>	<i>Example Type</i>
Japan	Honda	Blue	1980	Economy	Positive
Japan	Toyota	Green	1970	Sports	Negative
Japan	Toyota	Blue	1990	Economy	Positive
USA	Chrysler	Red	1980	Economy	Negative
Japan	Honda	White	1980	Economy	Positive
Japan	Toyota	Green	1980	Economy	Positive
Japan	Honda	Red	1990	Economy	Negative

# Learn concept: "Japanese Economy Car"

## 3. Positive Example: ( Japan, Toyota, Blue, 1990, Economy ):

- ❑ Prune G to exclude descriptions inconsistent with +ve example.
- ❑ Generalise S to include the positive example.

G =	$\{ (?, ?, \text{Blue}, ?, ?),$ $(?, ?, ?, ?, \text{Economy}) \}$
S =	$\{ (\text{Japan}, ?, \text{Blue}, ?, \text{Economy}) \}$



# Learn the concept: "Japanese Economy Car"

**Features:** ( Country of Origin, Manufacturer, Color, Decade, Type )

<i>Origin</i>	<i>Manufacturer</i>	<i>Colour</i>	<i>Decade</i>	<i>Type</i>	<i>Example Type</i>
Japan	Honda	Blue	1980	Economy	Positive
Japan	Toyota	Green	1970	Sports	Negative
Japan	Toyota	Blue	1990	Economy	Positive
USA	Chrysler	Red	1980	Economy	Negative
Japan	Honda	White	1980	Economy	Positive
Japan	Toyota	Green	1980	Economy	Positive
Japan	Honda	Red	1990	Economy	Negative

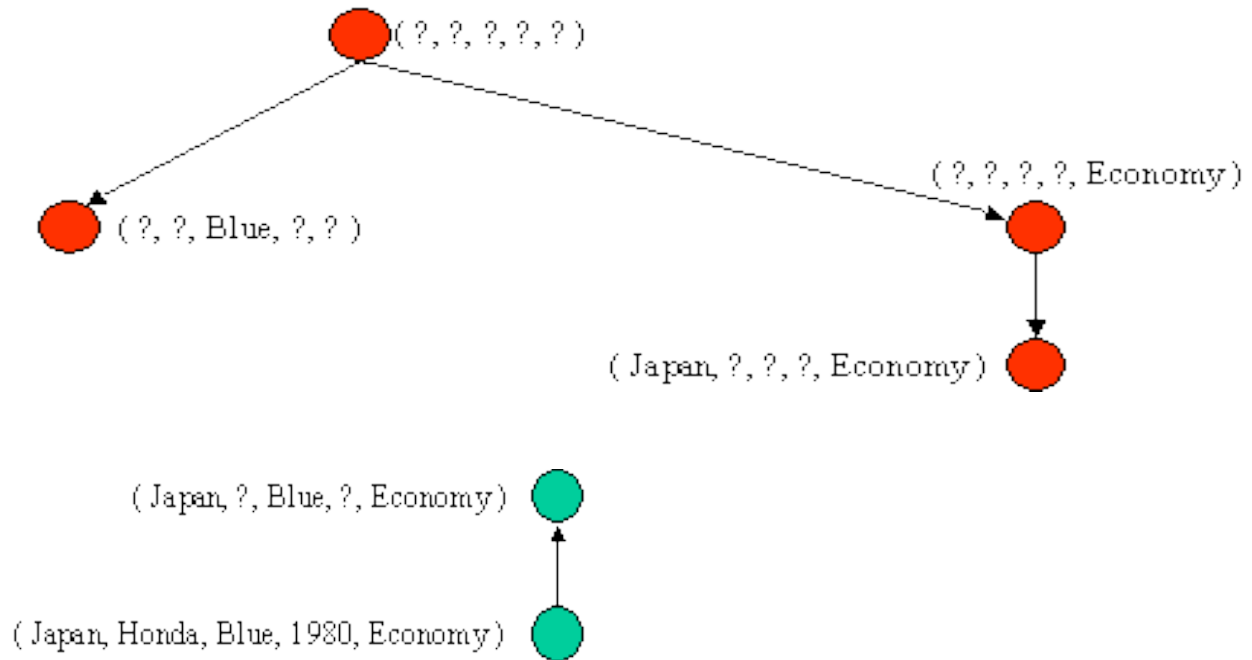


# Learn the concept: "Japanese Economy Car"

**4. Negative Example:** ( USA, Chrysler, Red, 1980, Economy ):

- Specialise  $G$  to exclude -ve example ( but stay consistent with  $S$  ).

$G =$	$\{ (?, ?, \text{Blue}, ?, ?),$ $(\text{Japan}, ?, ?, ?, \text{Economy}) \}$
$S =$	$\{ (\text{Japan}, ?, \text{Blue}, ?, \text{Economy}) \}$



# Learn the concept: "Japanese Economy Car"

**Features:** ( Country of Origin, Manufacturer, Color, Decade, Type )

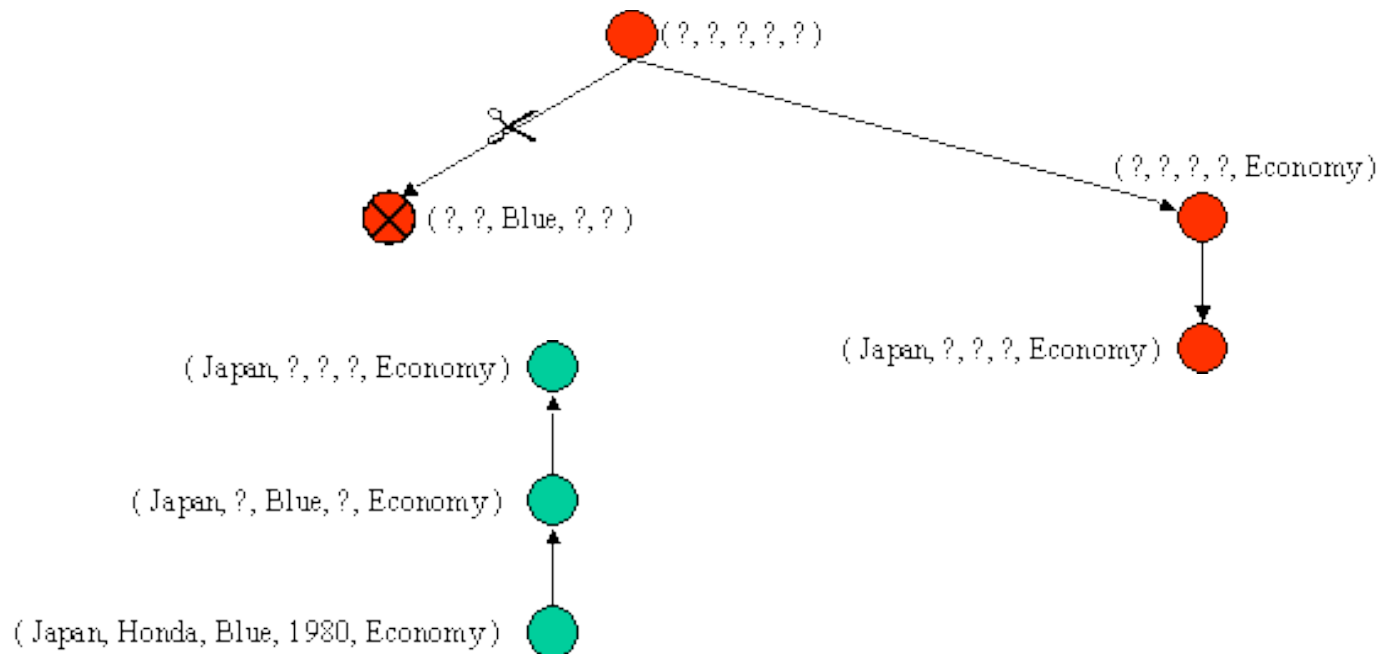
<i>Origin</i>	<i>Manufacturer</i>	<i>Colour</i>	<i>Decade</i>	<i>Type</i>	<i>Example Type</i>
Japan	Honda	Blue	1980	Economy	Positive
Japan	Toyota	Green	1970	Sports	Negative
Japan	Toyota	Blue	1990	Economy	Positive
USA	Chrysler	Red	1980	Economy	Negative
Japan	Honda	White	1980	Economy	Positive
Japan	Toyota	Green	1980	Economy	Positive
Japan	Honda	Red	1990	Economy	Negative

# Learn the concept: "Japanese Economy Car"

**5. Positive Example:** ( Japan, Honda, White, 1980, Economy ):

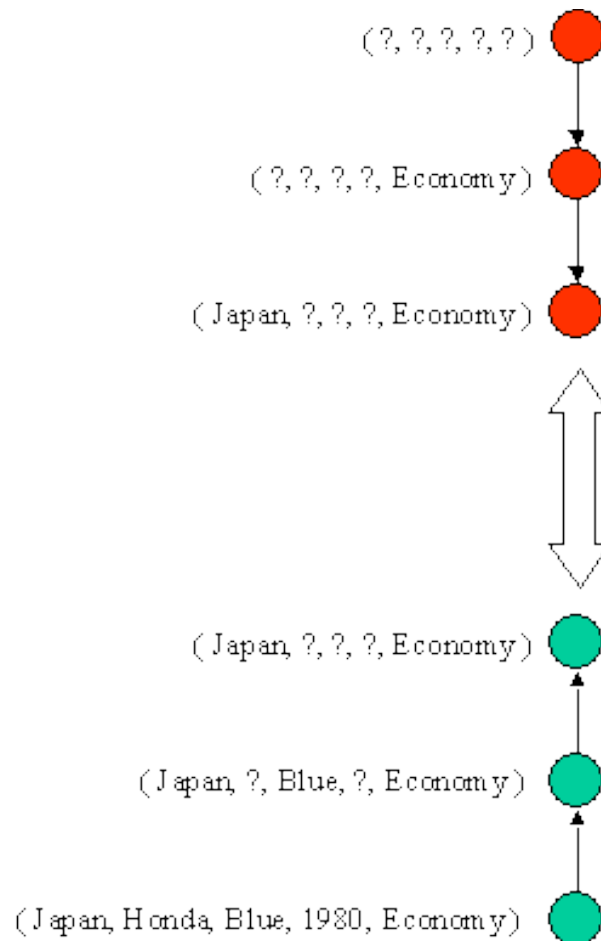
- Prune  $G$  to exclude descriptions inconsistent with +ve example.
- Generalise  $S$  to include positive example.

$G =$	(Japan, ?, ?, ?, Economy) }
$S =$	{ (Japan, ?, ?, ?, Economy) }



# Learn concept: "Japanese Economy Car"

- $G$  and  $S$  are singleton sets and  $S = G$ .

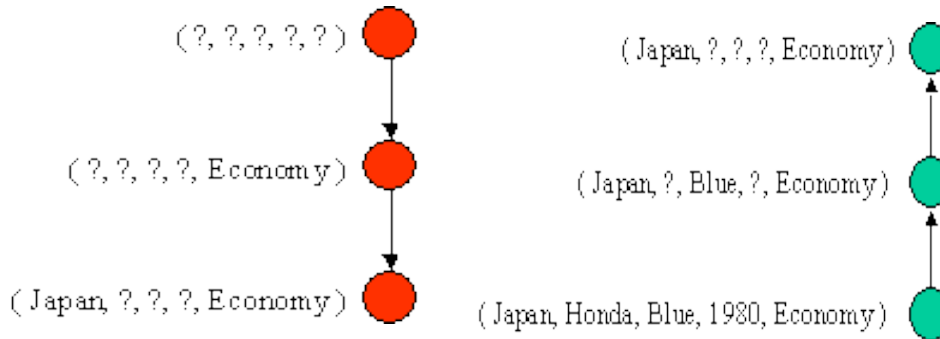


# Learn concept: "Japanese Economy Car"

**6. Positive Example:** ( Japan, Toyota, Green, 1980, Economy ):

New example is consistent with version-space, so no change is made.

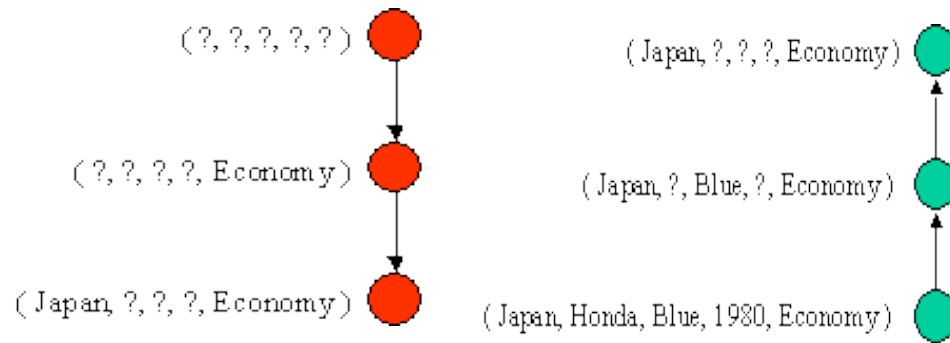
$G = \{ (\text{Japan}, ?, ?, ?, \text{Economy}) \}$   
 $S = \{ (\text{Japan}, ?, ?, ?, \text{Economy}) \}$



**7. Negative Example:** ( Japan, Honda, Red, 1990, Economy ):

???

# Learn concept: "Japanese Economy Car"

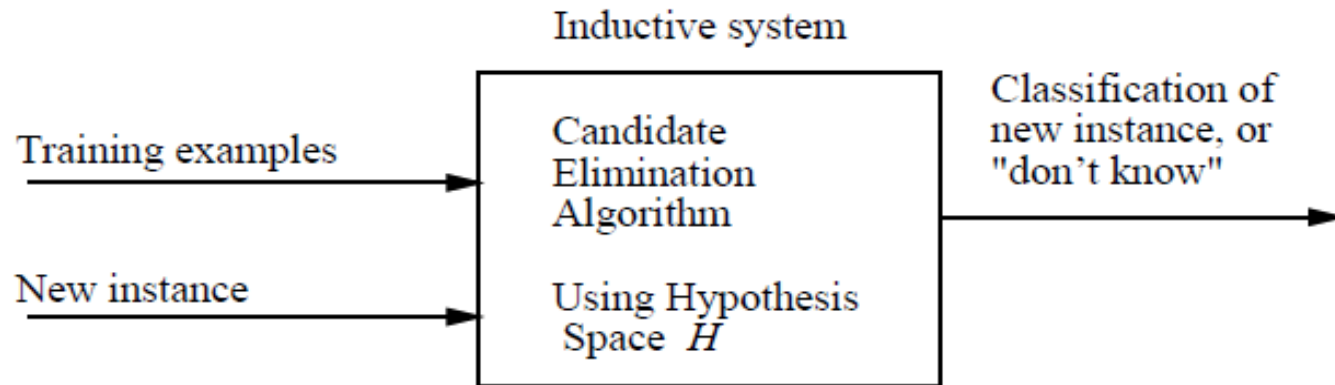


## 7. Negative Example: ( Japan, Honda, Red, 1990, Economy ):

Example is inconsistent with the version-space.

G cannot be specialised.  
S cannot be generalised.

# Problems with Candidate Elimination Algorithm?



# CEA Learner Properties

- If there is a consistent hypothesis then the algorithm will converge to  $S = G = \{ h \}$  when enough examples are provided.
- “Noisy” examples may cause the removal of the correct  $h$ .
- If too many examples are inconsistent,  $S$  and  $G$  become empty.
- This can also happen, when concept to be learned is not in  $H$ .
- **Assumes:**
  - There are no errors in the training examples.
  - There is some  $h$  in  $H$  that correctly describes the target concept.



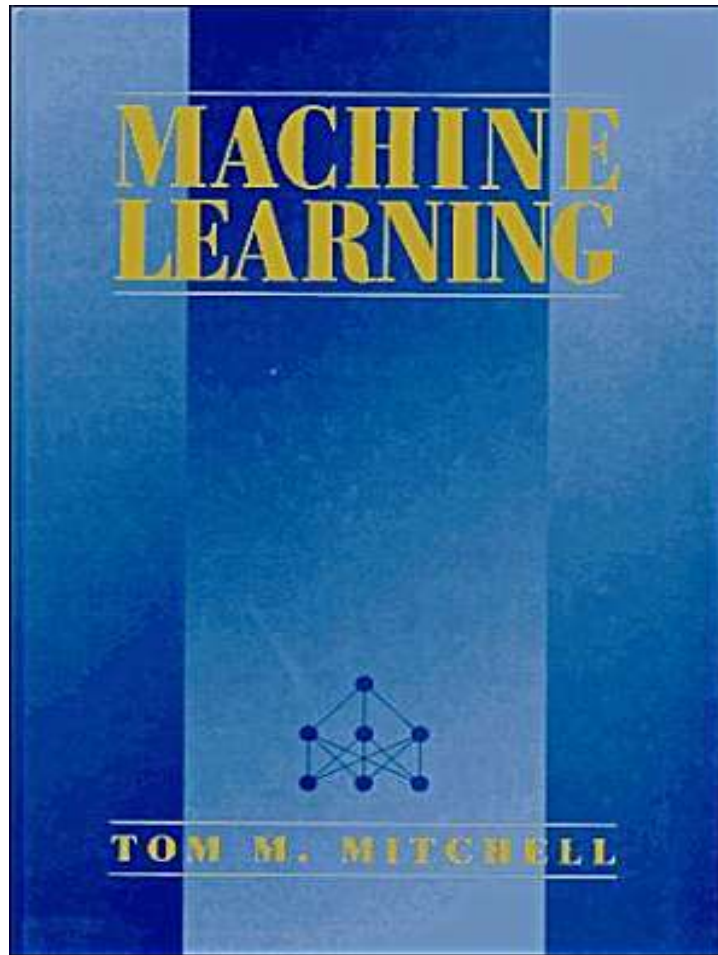
# Inductive Bias

- Concept learners classify unseen examples because of implicit inductive bias.
- Bias is that the target concept can be found in the hypothesis space (  $c \in H$  ).
- More bias implies more generality in classifying unseen examples.
- If there is a hypothesis corresponding to every possible instance, this removes inductive bias from *Candidate-Elimination*.
- This also removes the ability to classify any instance beyond the observed training examples.
- Unbiased learner cannot classify unseen examples (no induction!).

# Concept Learners → Bias !

- Inductive bias of:
  - **Rote learner:** Store examples, Classify  $x$  *iff* it matches previously observed example.
    - No inductive bias ( → no generalisation! )
  - **Candidate Elimination Algorithm:**
    - $c$  is in  $H$ .
  - **Find-S:**
    - $c$  is in  $H$ .
- *Inductive leaps possible only if learner is biased.*

# ML: Reading



## ***Chapter 2: Concept Learning***