**CS390 Computational Game Theory and Mechanism Design** July 10, 2013

# Problem Set 4

*Yiqing Hua*

**Problem 1** We'll first prove that $\forall \varepsilon$, $\sigma =$

**Problem 2** (Collaborated with YYY and ZZZ.)

The root is player 6, and all the other players are his children and the terminated nodes. For $\forall i$, $i \neq 6$, suppose player 6 choose the strategy $\sigma = aH + bT$, $0 \leq a, b \leq 1$. Then player $i$ choose the strategy $\sigma' = a'H + b'T$, $0 \leq a', b' \leq 1$. We have

$$u_i = (a - b)(a' - b')$$

Suppose that $a > b$, to maximize $u_i$, we'll make $a' = 1, b' = 0$. Similarly when $b > a$, $a' = 0, b' = 1$.
And for $a = b = \frac{1}{2}$, $a', b'$ will take arbitrary value. Therefore,

$$T(aH + bT, H) = 1\text{a ¿ b}$$
$$T(aH + bT, T) = 1\text{a ¡ b}$$
$$T(aH + bT, a'H + b'T) = 1\text{a = b, } 0 \leq a', b' \leq 1$$
$$T(X, Y) = 0\text{otherwise}$$

For player 6, when $a \neq b$, it's obvious that $\sigma$ is not the best response to his children's choices.
So we'll only consider $a = b = \frac{1}{2}$. to find a $z_1, z_2 \ldots z_5$ that $\sigma$ being the best response to them.
Clearly, if the opponents of player 6 has higher probability to choose Head or Tail. He'll choose the other one, so he can get more utility, then a pure strategy is his best response. So only when all his opponents have the same probability to choose either sides of the coin, $\sigma$ is the best choice for him.
Therefore, $T(\frac{1}{2}H + \frac{1}{2}T) = 1$. The witness list is all $(z_1, z_2, z_2, z_3, z_4, z_5)$, sum of probability that all the children choose Head is equal to sum of probability that all the children choose Tail.
Then we'll find the Nash Equilibrium top-down.

$$\sigma_i = a_iH + b_iT, 0 \leq a_i, b_i \leq 1, i \neq 6$$
$$\sigma_6 = \frac{1}{2}H + \frac{1}{2}T$$
$$\Sigma_{i=1}^{5} a_i = \Sigma_{i=1}^{5} b_i$$

**Problem 3** (No collaborator.)

Your solution goes here.

## References


[1] M. J. Osborne and A. Rubinstein. *A course in game theory.* MIT Press, 1994.

[2] N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani (eds). *Algorithmic game theory.* Cambridge University Press, 2007. (Available at `http://www.cambridge.org/journals/nisan/downloads/Nisan_Non-printable.pdf`.)