Lecture 1, Part 2

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In this class we talked about iterated elimination.
In a strategic game, if one pure strategy strictly dominates the others then it is obvious the strategy we seek for.
We'll talk about the notion of strictly dominence, and the process to eliminate the strategies strictly dominated by the others so that none of the players may choose them.

1 Definitions

Iterated elimination is defined as follows.

Definition 1 *In a strategic game $\langle N, S, u \rangle$, a set $X \subseteq S$ survives iterated elimination of strictly dominated strategy of $X = X_1 \times X_2 \cdots \times X_n$, if $\exists$ a finite sequence $S^0, S^1 \ldots S^K$ s.t*

- $S^0 = S$
- $S^k = S_1^k \times S_2^k \ldots S_n^k$
- $S^K = X$
- $\forall 0 \leq k \leq K$, $S_i^{k+1} \subseteq S_i^k$, *and* $S^k \backslash S^{k+1} \neq \emptyset$
- $\forall i$, $s_i \in S_i^k \backslash S_i^{k+1}$, *if* $\exists \sigma_i \in \delta(S_i^k)$, *s.t* $u_i(\sigma_i, S_{-i}) > u_i(s_i, S_{-i})$ $\forall S_{-i} \in S_{-i}^k$. *For* $G^k = \langle N, S^k, u \rangle$, *we say that* s_i *is strictly dominated by* σ_i *in* G^k, *that* $s_i \prec \sigma_i$.
- $\forall i$, S_i^K *contains no strategy that is strictly dominated over* S^K.

Remark. Note that σ_i can be either pure strategy or mixed strategy while $s_i \in S_i^k$ are all pure strategies.

2 Example

Given the game $\langle \{1, 2\}, \{T, M, B\} \times \{L, R\}, u \rangle$. The utility is given as the following table. The iterated elimination acts as follows.

	L	R
T	(3, 0)	(0, 1)
M	(0, 0)	(3, 1)
B	(1, 1)	(1, 0)

Table 1: The utility of the given game.

- First we have $S^0 = S = \{T, M, B\} \times \{L, R\}$
  Since no pure strategy can dominate any strategy of each player, we should look for some mixed strategy to do our elimination.
  We choose $\sigma_1 = \frac{1}{2}M + \frac{1}{2}T$. Then we have

  $$u_1(\sigma_1, L) = 1.5 > 1 = u_1(B, L)$$
  $$u_1(\sigma_1, R) = 1.5 > 1 = u_1(B, R)$$

  Therefore, B is strictly dominated by σ_1. We can get S^1.

- $S^1 = \{T, M\} \times \{L, R\}$
  Consider player 2, we have

  $$u_2(T, R) = 1 > 0 = u_2(T, L)$$
  $$u_2(M, R) = 1 > 0 = u_2(M, L)$$

  Therefore, L is strictly dominated by R. We now have S^2.

- $S^2 = \{T, M\} \times \{R\}$
  Since $u_1(M, R) = 3 > 0 = u_1(T, R)$, we can now eliminate strategy T.

- And now we have the final state $S^3 = \{M\} \times \{R\}$.

References

[1] M. J. Osborne and A. Rubinstein. *A course in game theory.* MIT Press, 1994.

[2] N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani (eds). *Algorithmic game theory.* Cambridge University Press, 2007. (Available at `http://www.cambridge.org/journals/nisan/downloads/Nisan_Non-printable.pdf`.)