Laboratory Instructions.

Advanced Control Engineering 1

Non-Linear Control of a Water Tank System

Lab 2

Study program: Master Mechatronics & Smart Technologies

Semester: 1

Group: 2023

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Contents

1 learning target

The learning goal of this laboratory is to give students the opportunity to apply advanced non-linear control techniques using **state linearization** to a relatively well-behaved system. More in particular the following topics will be addressed:

- Modeling of the double tank system as non-linear **affine system**.
- Simulation in MATLAB/Simulink using the **MCI-models** library.
- Controller design and simulation in Simulink.
- Controller rapid prototyping via Simulink Real-Time Target.
- Comparison of simulated values with real-world measurements.

The class time dedicated to this Laboratory is 5 units. Preparation in advance of the laboratory is of utmost importance and part of the grading.

2 modeling

The hydraulic system used in this laboratory consists of a water basin, a pump and three sequential water reservoirs (see Fig. 1). The pump transfers the water from the basin to the uppermost reservoir (reservoir 1) where it exits through an output pipe in the bottom to reservoir 2 and so on. Each reservoir has an emergency drain if the liquid level is approaching the maximum height. control value.

Figure 1: Setup of the hydraulic exercise in the lab.

Note that reservoir 3 is not considered as a plant, since the water level of reservoir 2 is the desired.

2.1 MODELING OF THE PUMP

Use the data depicted in Fig. 2 to find the behavior of the pump. Be aware that the measured data only shows the volumetric flow rate. Therefore, you need to calculate the corresponding mass flow (in $\frac{\mathrm{kg}}{\mathrm{s}}$). Assume that the pump is a linear system and neglect the missing data close to the origin of the graph.

$$\dot{m} = K_{pump} u_{in}$$

where $\dot{m}$ is the mass flow and u_{in} is the input voltage of the pump.

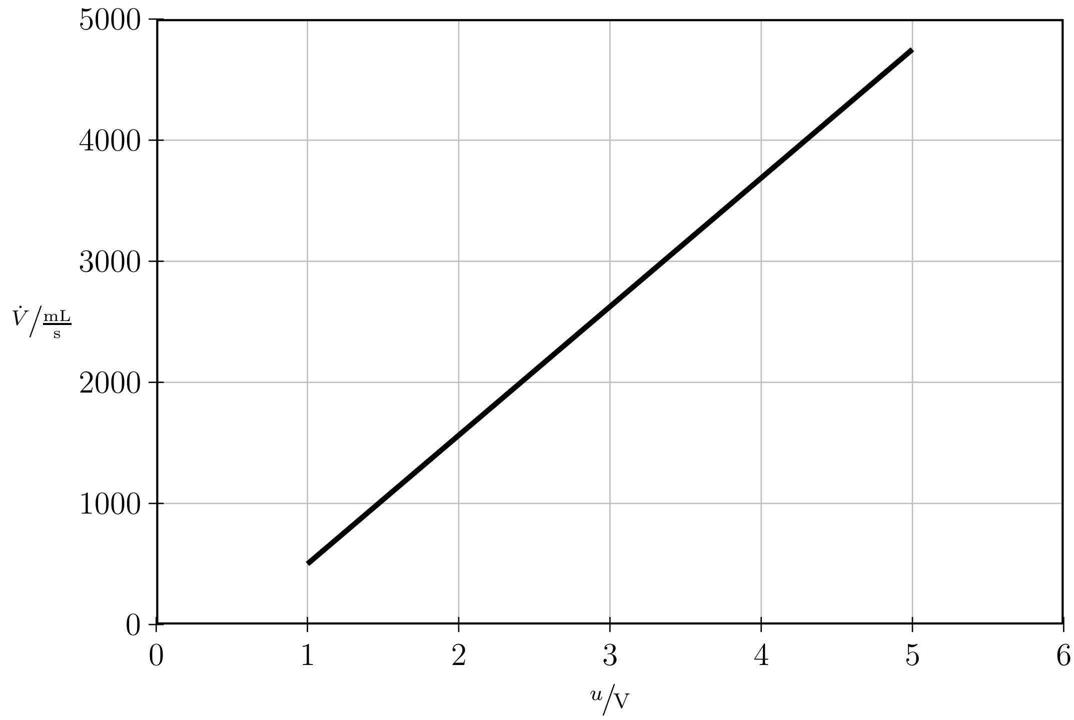


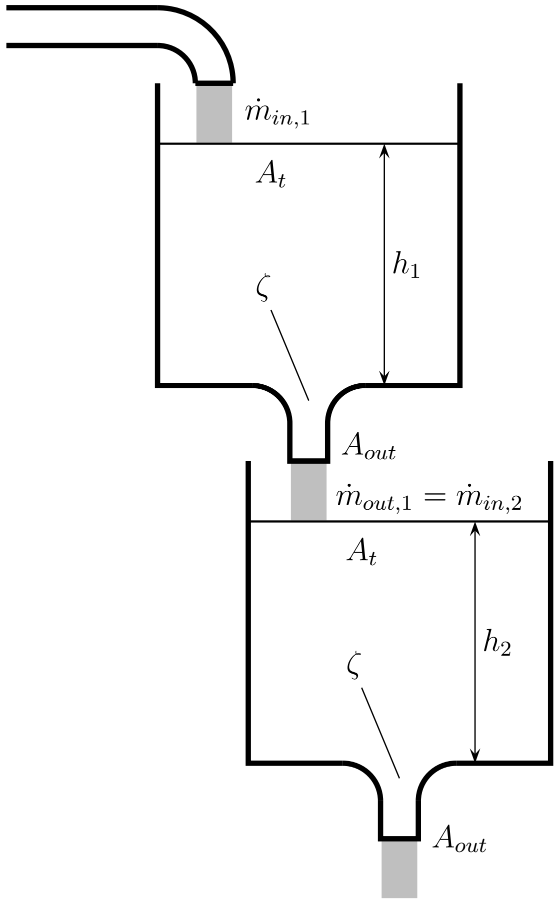
Figure 2: Pump characteristics.

2.2 MODELING OF THE RESERVOIRS

As shown in the reader [1] the model of the double tank system can be derived from elementary fluid mechanics. The continuity equation of a container with in- and outlet is

$$\dot{m} = \dot{m}_{in} - \dot{m}_{out},$$

see upper tank of the figure below.



Schematics of the double tank problem.

Replacing the change of mass on the left side with the change of fill level and the mass outflow by Torricelli's expression for the outflow velocity

$$\dot{m}_{out,1} = \rho A_{out} v_{out,1} = \rho A_{out} \sqrt{\frac{2gh_1}{1+\zeta}}$$

yields

$$\rho A_t \dot{h}_1 = \dot{m}_{in,1} - \rho A_{out} \sqrt{\frac{2gh_1}{1+\zeta}}$$

or

$$\dot{h}_1 = \frac{\dot{m}_{in,1}}{\rho A_t} - \sqrt{\frac{2gh_1}{1+\zeta}} \frac{A_{out}}{A_t}$$

with following quantities.

Symbol	Name
h	fill level
$\dot{m}_{in}$	mass inflow
ρ	density
g	gravitational acceleration
ζ	dimensionless pressure loss
A_t	tank base area
A_{out}	outflow cross section

If the outflow of the first tank is connected to the inflow of a (geometrically) identical subsequent one, the latter is governed by

$$\rho A_t \dot{h}_2 = \rho A_{out}\sqrt{\frac{2gh_1}{1+\zeta}} - \rho A_{out}\sqrt{\frac{2gh_1}{1+\zeta}}$$

or

$$\dot{h}_2 = \sqrt{\frac{2gh_1}{1+\zeta}}\frac{A_{out}}{A_t} - \sqrt{\frac{2gh_2}{1+\zeta}}\frac{A_{out}}{A_t}.$$

Introducing the states

$$\vec{x} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

as well as the input

$$u = \dot{m}_{in,1}$$

yields

$$\dot{\vec{x}} = \begin{bmatrix} -a\sqrt{x_1} \\ a(\sqrt{x_1} - \sqrt{x_2}) \end{bmatrix} + \begin{bmatrix} \frac{1}{\rho A_t} \\ 0 \end{bmatrix} u$$

with

$$a = \sqrt{\frac{2g}{1+\zeta}}\frac{A_{out}}{A_t}.$$

2.3 MODELING OF THE PRESSURE SENSOR

Each of the reservoirs is equipped with a pressure sensor that measures the pressure, which is proportional to the water level. As only the water level of reservoir 2 has to be controlled, only one sensor is required to be modeled. Figure 5 represents the sensor characteristics as a relation between the water level h and the output voltage u_{out}.

3 software

MATLAB/Simulink provides a tool chain for automatic code generation for various hardware. On the computers in the lab all necessary drivers and packages are already installed.

1. In order to access the I/O's you first need to place the correspondent blocks into your Simulink model. To this end in the Library Browser go to **Simulink Desktop Real-Time** and drag out an **Analog Input** and **Analog Input**

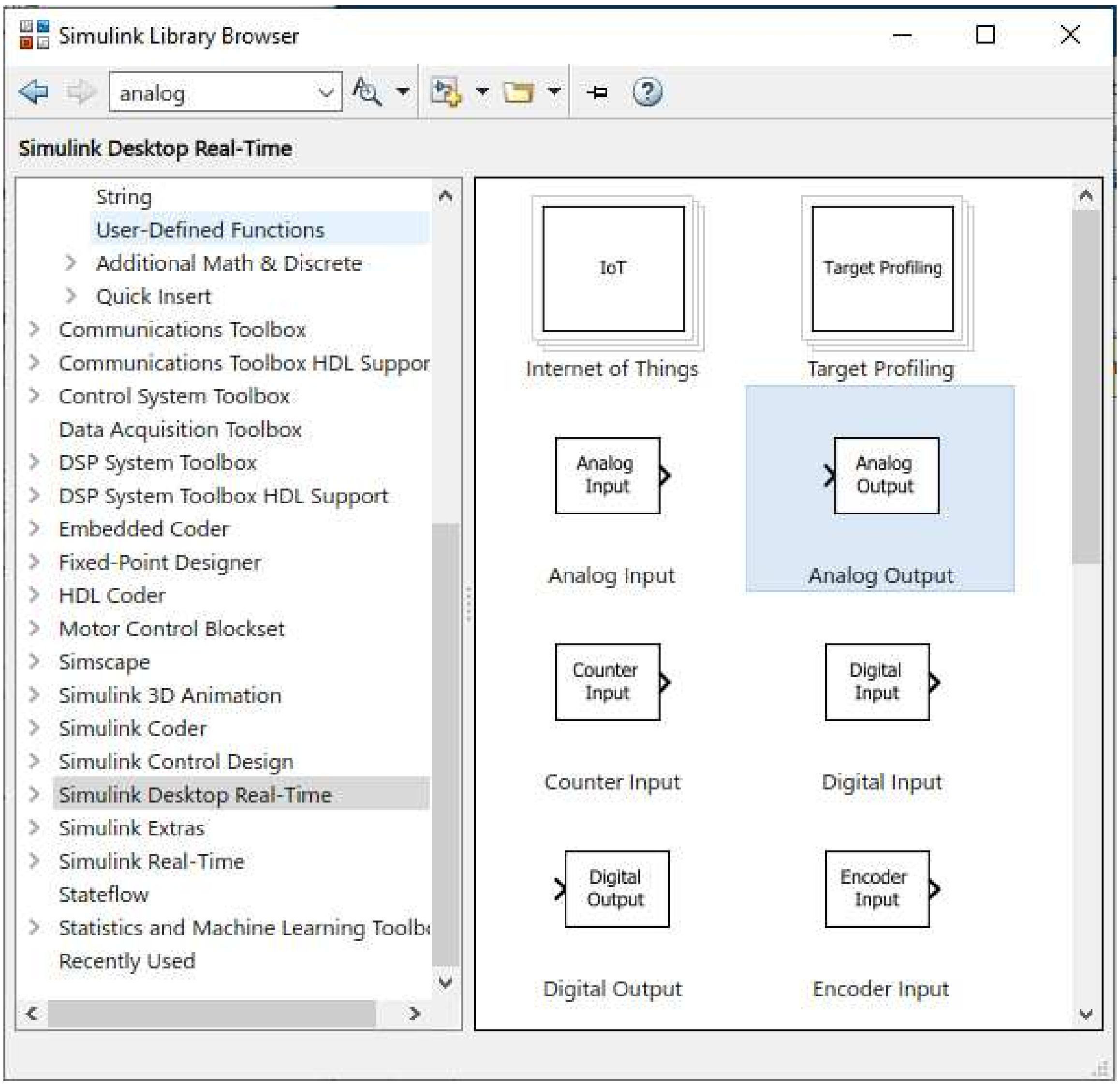


Figure 3: Simulink Library Browser

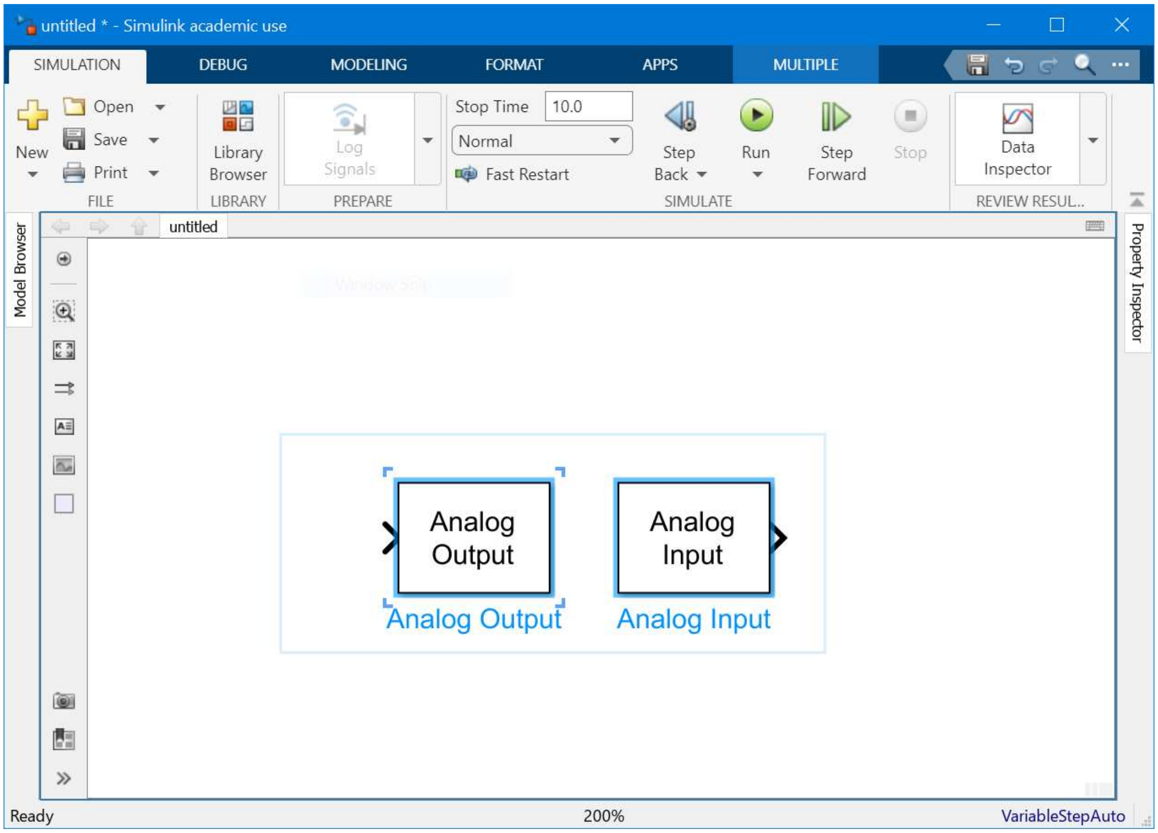


Figure 4: Simulink model file with I/O's

2. In the input and output blocks select the installed National Instruments PCI-6221 cards.

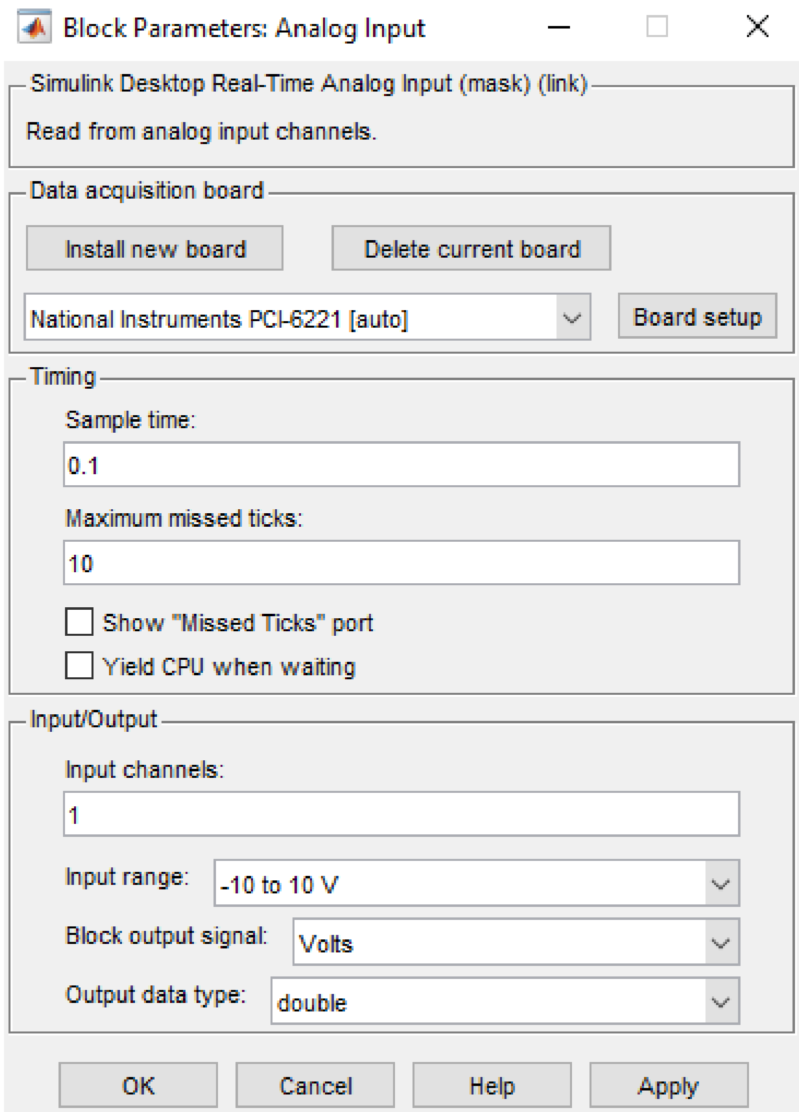


Figure 5: Simulink Library Browser

Make sure that

- Input/Output channels: 1
- Input/Output range: -10 to 10 V
- Block output signal: Volts

3. In **Board setup** the selected hardware can be tested. Click the respective button and wait for the green arrow to appear.

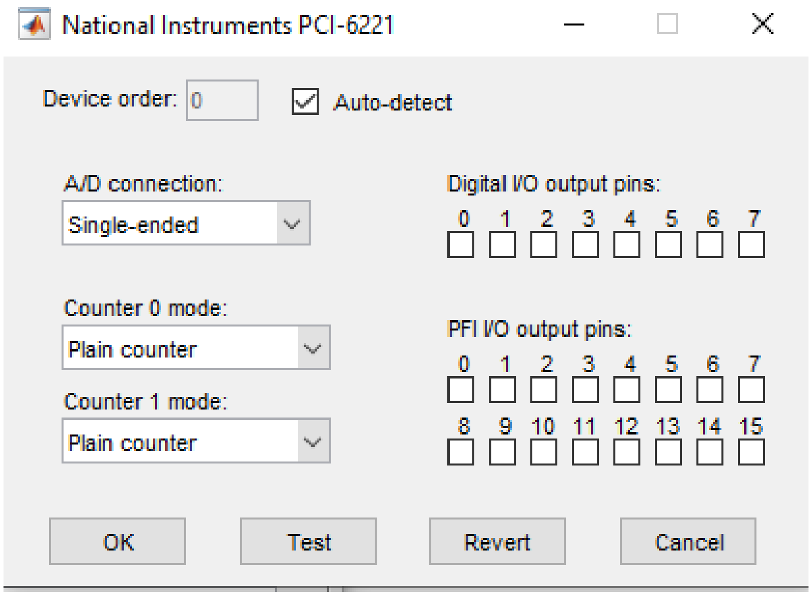


Figure 6: Simulink Library Browser

4. The I/O pins correspond to the respective connectors on the NI board

Figure 7: Simulink Library Browser

4 preparatory work

- State linearize the non-linear double tank model.
- Generate a Simulink model (and a MATLAB initialization file) which state linearizes the model. Test it by creating a virtual double tank model (either by yourself or use the blocks from the **MCI models** library) using following parameters
  - $\rho = 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}$
  - $A_t = 50\,\mathrm{cm}^2$
  - $A_{out} = 25\,\mathrm{mm}^2$
  - $\zeta = 0.05$
  - $g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$
- Design a fixed-point and a trajectory controller for the linearized system. Test both of them with the help of your model. Make sure you take physical limits of your system into account.
- All parameters of the system as well as of the controller should **not** be hard-coded but parameters in your scripts which may be easily updated with different values during the lab.

5 lab tasks

First put in operation hard- and software and try to pump water into the system. Try to identify the physical limits of the system.

Subsequently generate a scale of 5 to 10 voltage levels within the linear range of the system. Wait for each input until the water level in tank 2 reached steady-state and measure height manually as well as the voltage output of the sensor. Use this data to calibrate the sensor. In particular make sure the voltage offset is correct. Add a block to your Simulink sheet converting the sensor voltage output into height.

Measure the tank cross sections (in- and output). Generate a random order of input steps and measure the height in tank 2. Adjust a (respectively the outflow section A_{out} if you use the library) in your simulation model that it fits the measured data.

Recalculate your controllers for the updated parameters and rerun the simulation. Once you are sure your controllers do what they should, apply them to the real system. Compare the results with the simulations.

6 report

Prepare a report with a **maximum of 10** pages (not including: cover, index, list of figures, appendixes,...). The following is a suggested structure of the main chapters:

1. Definition of the learning target of the laboratory experiment and description of the structure of the document
2. Summary of theoretical aspects addressed both in preparatory work as well as in class activities (among others: state linearization, parameter identification, ...)
3. Description of laboratory set-up
4. Simulations (**Remark:** this chapter contains mainly the block diagrams and relevant comments related to the implemented simulations)
5. Results and interpretation (**Remark:** in this chapter the measurements shall be compared with results out of the simulations possibly showing simulation and measurements on the same plots or with side-by-side figures)
6. Conclusions: summarize the main achievements and problems/solutions encountered during the laboratory experience
7. Appendixes: provide any schematic, datasheet, document, MATLAB script, ... relevant to what is summarized in the report

A single **.zip file** containing the following:

- Report in pdf
- Latex or .doc of the Report
- Folders with simulations and measurements

shall be submitted only in digital form on SAKAI Assignments (please upload **one copy** per group only) within the date reported on the announcements on SAKAI. Please mark on the cover page at least the following information:

- Title of the activity
- Date of the lab
- Lecturer(s)
- Group members

The report shall be compliant to the **MCI laboratory guidelines** [2].

References

[1] A. Mehrle, "Control Engineering 1 - Modeling," pp. 76–77, 2022, Management Center Innsbruck.

[2] T. Kofler, B. Massow, M. Pillei, M. Spruck, and W. Stadlmayr, "Laboratory Tutorial Guide," 2015, Management Center Innsbruck.

List of Figures