

**Problem1.** (*Percolation.java*)

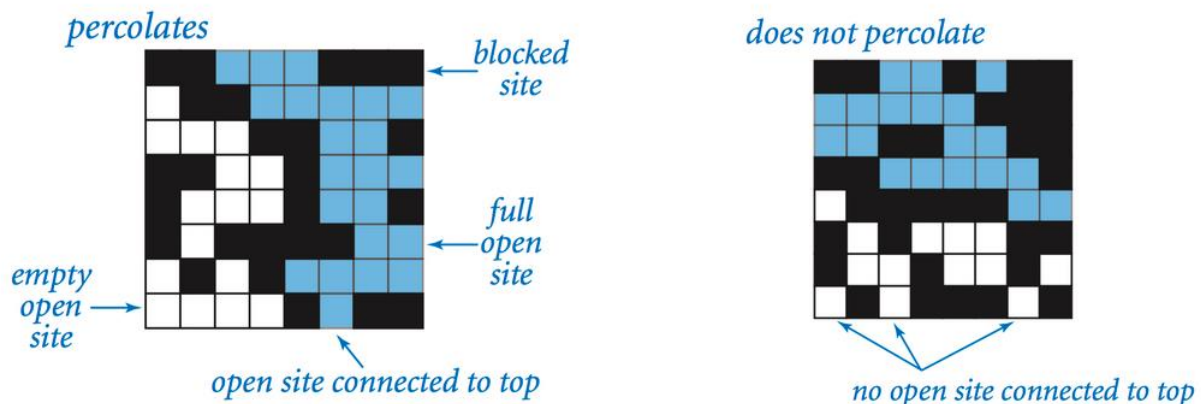
1. Write a program to estimate the value of the *percolation threshold* via Monte Carlo simulation.
2. Use the .java files attached
3. To access a class in `algs4.jar`, you need an `import` statement, such as the ones below:

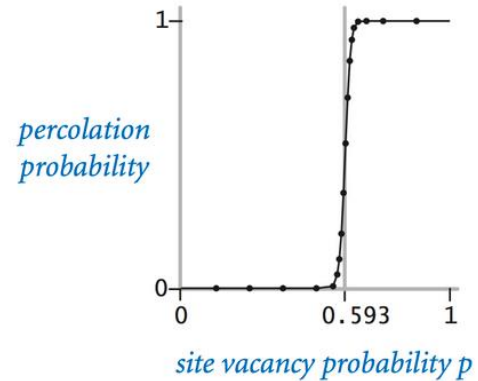
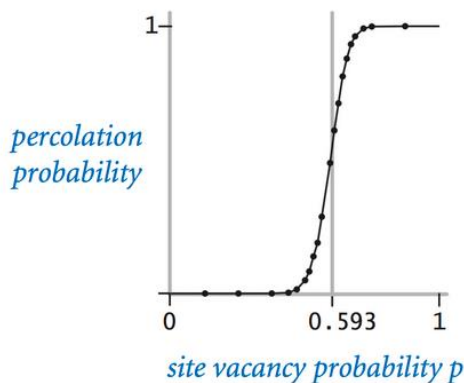
```
import edu.princeton.cs.algs4.StdRandom;
import edu.princeton.cs.algs4.StdStats;
import edu.princeton.cs.algs4.WeightedQuickUnionUF;
```

4. To run the files (Windows, Mac), use `java-algs4`

**Percolation.** Given a composite systems comprised of randomly distributed insulating and metallic materials: what fraction of the materials need to be metallic so that the composite system is an electrical conductor? Given a porous landscape with water on the surface (or oil below), under what conditions will the water be able to drain through to the bottom (or the oil to gush through to the surface)? Scientists have defined an abstract process known as *percolation* to model such situations.

**The model.** We model a percolation system using an  $n$ -by- $n$  grid of *sites*. Each site is either *open* or *blocked*. A *full* site is an open site that can be connected to an open site in the top row via a chain of neighboring (left, right, up, down) open sites. We say the system *percolates* if there is a full site in the bottom row. In other words, a system percolates if we fill all open sites connected to the top row and that process fills some open site on the bottom row. (For the insulating/metallic materials example, the open sites correspond to metallic materials, so that a system that percolates has a metallic path from top to bottom, with full sites conducting. For the porous substance example, the open sites correspond to empty space through which water might flow, so that a system that percolates lets water fill open sites, flowing from top to bottom.)





In a famous scientific problem, researchers are interested in the following question: if sites are independently set to be open with probability  $p$  (and therefore blocked with probability  $1 - p$ ), what is the probability that the system percolates? When  $p$  equals 0, the system does not percolate; when  $p$  equals 1, the system percolates. The plots below show the site vacancy probability  $p$  versus the percolation probability for 20-by-20 random grid (left) and 100-by-100 random grid (right).

When  $n$  is sufficiently large, there is a *threshold* value  $p^*$  such that when  $p < p^*$  a random  $n$ -by- $n$  grid almost never percolates, and when  $p > p^*$ , a random  $n$ -by- $n$  grid almost always percolates. No mathematical solution for determining the percolation threshold  $p^*$  has yet been derived. Your task is to write a computer program to estimate  $p^*$ .

**Percolation data type.** To model a percolation system, create a data type `Percolation` with the following API:

```
public class Percolation {
    // create n-by-n grid, with all sites blocked
    public Percolation(int n)
    // open site (row, col) if it is not open already
    public void open(int row, int col)
    public boolean isOpen(int row, int col) // is site (row, col) open?
    public boolean isFull(int row, int col) // is site (row, col) full?
    public int numberOfOpenSites() // number of open sites
    public boolean percolates() // does the system percolate?

    public static void main(String[] args) // test client (optional)
}
```

**Corner cases.** By convention, the row and column indices are integers between 1 and  $n$ , where (1, 1) is the upper-left site:

Throw a `java.lang.IllegalArgumentException` if any argument to `open()`, `isOpen()`, or `isFull()` is outside its prescribed range.

The constructor should throw a `java.lang.IllegalArgumentException` if  $n \leq 0$ .

**Performance requirements.** The constructor should take time proportional to  $n^2$ ; all methods should take constant time plus a constant number of calls to the union–find methods `union()`, `find()`, `connected()`, and `count()`.

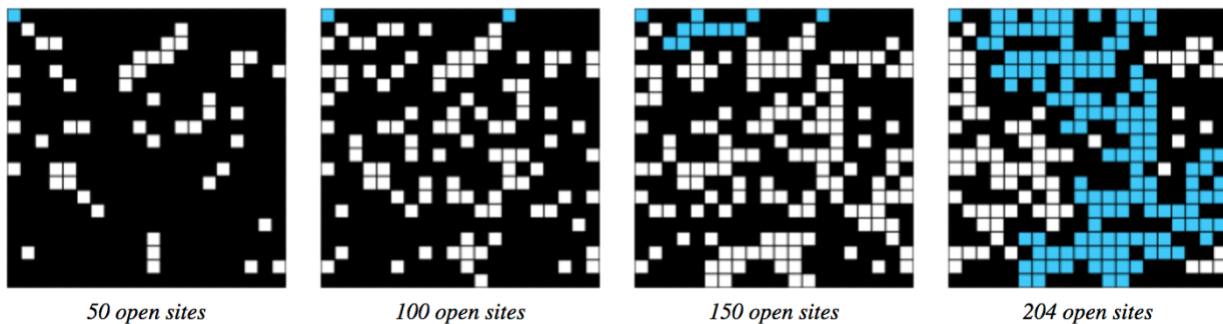
```
$java-algs4 Percolation < input10.txt
56 open sites
percolates
$java-algs4 Percolation < input10-no.txt
55 open sites
does not percolate
```

## Problem2. (*PercolationStats.java*)

**Monte Carlo simulation.** To estimate the percolation threshold, consider the following computational experiment:

- Initialize all sites to be blocked.
- Repeat the following until the system percolates:
  - Choose a site uniformly at random among all blocked sites.
  - Open the site.
- The fraction of sites that are opened when the system percolates provides an estimate of the percolation threshold.

For example, if sites are opened in a 20-by-20 lattice according to the snapshots below, then our estimate of the percolation threshold is  $204/400 = 0.51$  because the system percolates when the 204th site is opened.



By repeating this computation experiment  $T$  times and averaging the results, we obtain a more accurate estimate of the percolation threshold. Let  $x_t$  be the fraction of open sites in computational experiment  $t$ . The sample mean  $\bar{x}$  provides an estimate of the percolation threshold; the sample standard deviation  $s$ ; measures the sharpness of the threshold.

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_T}{T}, \quad s^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_T - \bar{x})^2}{T - 1}$$

Assuming  $T$  is sufficiently large (say, at least 30), the following provides a 95% confidence interval for the percolation threshold:

$$\left[ \bar{x} - \frac{1.96s}{\sqrt{T}}, \bar{x} + \frac{1.96s}{\sqrt{T}} \right]$$

To perform a series of computational experiments, create a data type `PercolationStats` with the following API.

```
public class PercolationStats {
    // perform trials independent experiments on an n-by-n grid
    public PercolationStats(int n, int trials)
    // sample mean of percolation threshold
    public double mean()
    // sample standard deviation of percolation threshold
    public double stddev()
    public double confidenceLo() // low endpoint of 95% confidence interval
    public double confidenceHi() // high endpoint of 95% confidence interval

    public static void main(String[] args) // test client (described below)
}
```

The constructor should throw a `java.lang.IllegalArgumentException` if either  $n \leq 0$  or  $trials \leq 0$ .

Also, the `main()` method that takes two *command-line arguments*  $n$  and  $T$ , performs  $T$  independent computational experiments (discussed above) on an  $n$ -by- $n$  grid, and prints the sample mean, sample standard deviation, and the *95% confidence interval* for the percolation threshold.

Use [StdRandom](#) to generate random numbers; use [StdStats](#) to compute the sample mean and sample standard deviation.

Corner cases. The constructor should throw a `java.lang.IllegalArgumentException` if either  $N \leq 0$  or  $T \leq 0$ .

```
% java-algs4 PercolationStats 200 100
mean                = 0.5929934999999997
stddev              = 0.00876990421552567
95% confidence interval = [0.5912745987737567, 0.5947124012262428]

% java-algs4 PercolationStats 200 100
mean                = 0.592877
stddev              = 0.009990523717073799
95% confidence interval = [0.5909188573514536, 0.5948351426485464]

% java-algs4 PercolationStats 2 10000
mean                = 0.666925
stddev              = 0.11776536521033558
95% confidence interval = [0.6646167988418774, 0.6692332011581226]
```

*Acknowledgements This project is an adaptation of the Percolation assignment developed at Princeton University by Robert Sedgewick and Kevin Wayne.*

**Submitting Information:**

- Use the code I provided for each problem. DON'T DELETE ANY FUNCTION
- You should write your code in (Percolation.java, PercolationStats.java)
- Submit your work on Canvas.
- The deadline is Thursday, Sep 25<sup>th</sup> at 6:00PM
- Follow the guidelines in homework rubric P1