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Multidisciplinary Design Optimization of a UAV Wing using Kriging based Multi-Objective Genetic Algorithm

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This paper investigates the preliminary wing design of Unmanned Aerial Vehicle (UAV) using a two step optimization approach. The first step is a single objective aerodynamic optimization whereas the second step is a coupled dual objective aerodynamic and structural optimization. In the single objective case, airfoil geometry is optimized to get maximum endurance parameter at a 2D level with maximum thickness to chord ratio and maximum camber as design variables. Constraints are imposed on the leading edge curvature, trailing edge radius, zero lift drag coefficient and zero lift moment coefficient. After arriving at the optimized airfoil geometry, the wing planform parameters are optimized with minimization of wing weight and maximization of endurance parameter corresponding to the wing and four more design variables from the aerodynamics discipline namely taper ratio, aspect ratio, wing loading and wing twist are added in the second step. Also, four more design variables from the structures discipline namely the upper and lower skin thicknesses at root and tip of the wing are added with stall speed, maximum speed, rate of climb, strength and stiffness as constraints. The 2D airfoil and 3D wing aerodynamic analysis is performed by the XFLR5 code and the structural analysis is performed by the MSC-NASTRAN software. In the optimization process, a relatively newly developed multi-objective evolutionary algorithm named NSGA-II (non-dominated sorting genetic algorithm) is used to capture the full Pareto front for the dual objective problem. In the second step, in order to reduce the time of computation, the analysis tools are replaced by a Kriging meta-model. For this dual objective design optimization problem, numerical results show that several useful Pareto optimal designs exist for the preliminary design of the UAV wing.

Nomenclature

UAV	=	Unmanned Aerial Vehicle
AR	=	Wing Aspect Ratio
C_L	=	3D Lift coefficient for wing
C_D	=	3D drag coefficient for wing
C_l	=	2D Lift coefficient for airfoil
C_d	=	2D drag coefficient for airfoil
$\overline{g}$	=	Inequality constraint
HCR	=	Loiter Altitude in m
ROC	=	Rate of Climb in m/s
t	=	Endurance in hrs
TC	=	Thickness to Chord ratio of wing in percentage
TR	=	Wing Taper Ratio
UAV	=	Unmanned Aerial Vehicle
VMAX	=	Maximum Speed in kmph

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VS	=	Stall Speed in kmph
WL	=	Wing Loading in kg/m^2
WW	=	Wing Weight in kg
$\bar{x}$	=	Design variable

I. Introduction

Aircraft design is an ideal candidate for Multidisciplinary Design Optimization (MDO). This is because of the fact that aircraft design is governed by more than one discipline like aerodynamics, structure, control and propulsion. A conventional aircraft design process starts with aerodynamic design to satisfy the performance requirements and is followed by design iterations and checks to satisfy requirements from other disciplines. In contrast, a good optimal design handles all the inputs from various disciplines and performs an interdisciplinary trade off. Aircraft designers are looking for such an optimal solution through MDO.

In the aircraft design field, many optimization works have been carried out over the last 30 years. These works primarily focus on obtaining the best aerodynamic or structural design. Over the last decade, designers have applied MDO to aircraft design[1,2,3] . The survey of developments by Sobieski and Haftka[1] reports that MDO methodology has transcended its structural optimization roots and is growing in scope and depth toward encompassing the complete set of disciplines required by applications. According to that survey, the two major obstacles in realizing the full potential of MDO technology appear to be the high computational demands and complexities arising from organization of the MDO task.

Bartholomew[2] provides a definition of MDO incorporating state of the art analysis tools and discusses the function of MDO as a key tool in the context of concurrent engineering. He also says that MDO permits the constraints of a diverse range of disciplines to be addressed from an early stage of the design process. Kroo[3] highlights some important aspects of MDO applications in the preliminary design phase and summarizes how the field is evolving. Kroo also covers the evolution of computational tools, strategies and challenges. Most of the researchers focus mainly on applying MDO to the complete aircraft design[4-5] during its conceptual design stage and to the design of wing[6-14] during the preliminary design phase. Sobester and Keane[4] studied and constructed a multidisciplinary analysis for UAV airframes. They consider a blended wing body design and illustrate optimization of the geometry using a constraint analysis. Rajagopal et al[5] formulated the conceptual design of an UAV as an optimization problem and performed the initial aircraft sizing through the optimization approach. During the conceptual design phase, the different disciplines involve low fidelity analysis tools like empirical relations and hence the MDO approach does not demand large computing power and time. On the other hand, during the preliminary design phase, the design is maturing and hence the involved disciplines deploy high fidelity analysis tools like Computational Fluid Dynamics (CFD), Finite Element Method (FEM) etc. that demand enormous computing power and time. Also, during this stage, the coupling between the disciplines increases thereby increasing the time and power of computation.

Much research in the field of aircraft design using the MDO approach has focused on applying MDO for conventional commercial transport aircraft and current generation fighters. Grossman et al[6] integrate aerodynamic and structural design of a subsonic transport wing for minimum weight subject to a constraint on range. They recommend two methods to alleviate the computational burden. One is to reduce the cost of sensitivity derivatives that is called as a modular sensitivity method which allows the usage of black box disciplinary software packages. They have shown in the study that derivatives of the aeroelastic response and divergence speed can be calculated without the costly computation of derivatives of aerodynamic influence coefficient and structural stiffness matrices. Apart from this, in order to reduce the computational cost, a sequential approximate optimization is used. Dovi and Wrenn[7] provide a new technique KSOPT, which is an envelope function formulation that converts a constrained optimization problem into an unconstrained one. The primary benefit from this new method for multi-objective optimization is the elimination of separate optimization for each objective, which is required by some optimization methods. A typical wide body transport aircraft is used for comparison studies. This method is compared with the other two classic multi-objective optimization methods namely the Penalty function method and Global Criterion method. Wakayama et al[8] presents the basic results from wing planform optimization for minimum drag with constraints on structural weight and maximum lift. The study gives the basic influence of drag, weight and maximum lift on optimal wing planform. It clearly brings out that induced drag and structural considerations strongly favor highly tapered wings to attain large spans while parasite and compressibility drag have limited effect on wing taper, making maximum lift constraints necessary for generating realistic tip chords. Martins et al[9] focus on demonstrating a new integrated aerodynamic structural design method for aerospace vehicles. They employ high fidelity models for both aerodynamic and structural disciplines and also use a high fidelity coupling procedure. They employ Euler equations for aerodynamic analysis and a detailed FEM model for the primary structure. Carrier[10] describes the MDO system implemented at ONERA. It contains different optimization algorithms including a gradient based optimizer and a GA. Here the two disciplines of aerodynamics and structure are analyzed with high fidelity methods whereas the other disciplines such as engine performance and flight mechanics are evaluated with simpler methods. This system is applied for optimizing the performance of a high-speed civil transport aircraft. The overall objective is to maximize the aircraft range while multiple design constraints are considered. Kumano et al[11] describe the MDO system for a small jet aircraft design by integrating the CFD codes and NASTRAN based aeroelastic structural interface code. They employ a kriging model to save computational time of objective function evaluation in the multi-objective genetic algorithm (MOGA). Several non-dominated solutions indicating the trade off among the drag, structural weight, drag divergence and pitching moment are found by Obayashi and his co-workers[11]. Kim et al[12] provide an aerodynamic/structural multidisciplinary design with multiple objectives for a supersonic fighter wing using response surface methodology. Nine wing and airfoil parameters were chosen for the aerodynamic design variables and four structural variables were added to determine the wing skin thickness. To consider various flight conditions, multipoint design optimization was performed on the three representative design points.

In recent years, UAV's have gained the attention of aerospace engineers due to their possibility in reconnaissance roles related to counter terrorism. Some unique opportunities are provided by UAV design as compared to the conventional manned aircraft design. UAV's are not governed by the strict airworthiness requirements of manned aircraft and provide more flexibility in the selection of wing design parameters. Some recent research activity has focused on UAV applications[13-14]. Gonzalez et al[13] discuss the use of evolutionary algorithms (EA) for a single and multi-objective airfoil optimization. They bring out the demerits of applying the gradient based approach for problems involving multi-objective, multi-modal and non-differentiable functions. Also, they show that EA's have the capability to find global optima and can be executed in parallel by adapting to arbitrary solver codes. In another work, Gonzalez et al[14] highlight the difficulties in the design of UAV's arising due to the varied and non-intuitive nature of the configurations and missions that can be performed by these vehicles. An MDO framework[14] is applied and two case studies are performed using high fidelity analysis codes. The first case study involves dual objective UAV airfoil section optimization. Detailed design of a single element airfoil for a small UAV application similar to RQ-7A Shadow 200 tactical UAV is performed with the two fitness functions defined as minimization of drag at two different flight conditions. Three constraints for maximum thickness, maximum thickness location and pitching moment are used. In the second case study, multi-criteria wing design optimization for a UAV with the two fitness functions defined as minimization of wave drag and minimization of the spanwise cap weight is performed. Constraints are imposed on minimum thickness and position of maximum thickness.

We see that some works have been done on the design optimization of UAV's. However, the use of MDO in UAV designs is much less compared to its use in conventional manned aircraft. This is especially true for the application of evolutionary algorithm to UAV design. Also, most works use simple GA for the optimization problem and have not exploited the power of evolving multi-objective GAs. The advantages of evolutionary methods over classical algorithms in single and multi-objective optimization problems are well highlighted by Goldberg[15] and Deb[16,17]. This paper investigates the preliminary design of an UAV wing as an optimization problem by taking into account the aerodynamics and structural concepts. Evolutionary algorithms, which capture the full Pareto Front for multi-objective problems are used in conjunction with a Kriging meta-model of the analysis. An example of preliminary design of a low speed, long endurance UAV is illustrated.

II. Problem Formulation

The preliminary design of the UAV under consideration focuses on achieving its main goal i.e. the long endurance and minimum structural weight. The fuselage design is usually governed by the amount of the fuel to be carried and the volume of systems like payload and equipment. Therefore, only the wing design is considered for the optimization problem. In the optimization process, the design aims at maximizing the endurance, which is an aerodynamic aspect and minimizing the wing weight, which is a structural aspect. The optimization problem involves objective functions, design variables and constraints.

A. Objective functions

The choice of the objective function in any aircraft optimization problem is dictated by the design mission of the aircraft. Since the design mission of the UAV under consideration is long endurance, the main objective is chosen as endurance while formulating the optimization problem. Also, achieving minimum structural weight is a general challenge for aircraft designers. Therefore, minimization of wing weight is also considered as another objective for the optimization problem. Since the aerodynamic analysis of the wing is performed in two steps, the optimization methodology is also performed in two parts namely the 2D airfoil optimization and 3D wing optimization. In the 2D airfoil optimization, the airfoil geometry is optimized with a single objective of maximizing the endurance parameter ($C_l^{3/2} / C_d$). The lift and drag characteristics correspond to the 2D characteristics of the airfoil. In the 3D wing optimization, the wing planform parameters are optimized with dual objectives. The first objective function is the maximization of the endurance parameter ($C_L^{3/2} / C_D$), which reflects the aerodynamic discipline. Here the lift and drag characteristics corresponds to that of the wing. The second objective function is minimization of the wing weight that reflects the structures discipline.

B. Design Variables

The design space is chosen to reflect the effect of aerodynamic and structural discipline. In this study, the wing parameters related to planform, the airfoil shapes and the structural skin thicknesses are identified as the design variables, as summarized in Table 1. The first two parameters described in Table 1 namely the wing thickness to chord ratio and the maximum camber are the design variables for the 2D airfoil optimization. The rest of the parameters are additional design variables included for the 3D wing optimization. The 2D design optimization uses 2 design variables while the 3D design optimization uses 8 additional design variables.

Table 1. Design Variables – Upper and Lower Bounds

No	Design Variables	Notation	Usage	Lower Bound	Upper Bound
1	Wing Thickness to chord Ratio	TCR	2D	0.727	1
2	Maximum camber	MCR	2D	0	1
3	Wing Aspect Ratio	AR	3D	0.727	1
4	Wing Loading	WS	3D	0.6	1
5	Wing Taper Ratio	TR	3D	0.25	1
6	Wing twist angle	θ	3D	0	1
7	Upper skin thickness at root	$\Delta t1$	3D	0	1
8	Upper skin thickness at tip	$\Delta t2$	3D	0	1
9	Lower skin thickness at root	$\Delta t3$	3D	0	1
10	Lower skin thickness at tip	$\Delta t4$	3D	0	1

Throughout this paper, the design variables are normalized and presented as ratios with respect to the upper bound value.

C. Constraints

Aerodynamic constraints are imposed on the performance parameters of the UAV and on the airfoil shape. For the 2D airfoil optimization, the aerodynamic constraints are imposed on the leading edge curvature (LEC), trailing edge radius (TER), zero drag and zero moment coefficient. Moreover, for the 3D wing optimization, the aerodynamic constraints are imposed on (1) rate of climb (ROC) at that altitude, (2) stall speed (VS) and (3) maximum speed (VMAX) at sea level condition, which arise from the requirements. The structural constraints imposed are based on the strength and stiffness of the wing.

D. Mathematical Representation of 2D airfoil Design Problem

The 2D airfoil design problem can be written as a standard optimization problem:

$$\text{Maximize } f(\bar{x}) = \text{Maximize} \left[\frac{C_l^{3/2}}{C_d} \right]$$

$$\begin{aligned}
&\text{Subject to} \\
&\bar{g}_1(\bar{x}) = \left[LEC(\bar{x}) - LEC^* \right] \le 0 \\
&\bar{g}_2(\bar{x}) = \left[-TER(\bar{x}) + TER^* \right] \le 0 \\
&\bar{g}_3(\bar{x}) = \left[C_{d_0}(\bar{x}) - C_{d_0}{}^* \right] \le 0 \\
&\bar{g}_4(\bar{x}) = \left[C_{m_0}(\bar{x}) - C_{m_0}{}^* \right] \le 0 \\
&x_l \le \bar{x} \le \bar{x}_u \\
&\bar{x} = [TC \quad MC]^T
\end{aligned} \tag{1}$$

where C_l is the 2D lift coefficient of the airfoil, C_d is the 2D drag coefficient of the airfoil, LEC^* is the maximum allowable leading edge curvature in the airfoil geometry, TER is the minimum trailing edge angle allowed, $C_{d_0}{}^*$ is the maximum allowable zero lift drag coefficient and $C_{m_0}{}^*$ is the maximum allowable zero lift moment coefficient. The 2D optimization problem has one objective function, four constraints and two design variables. Only the aerodynamics discipline is involved in this problem.

E. Mathematical Representation of 2D airfoil Design Problem

The 3D wing design problem can be written as a standard optimization problem:

$$\text{Maximize } f(\bar{x}) = \text{Maximize} \begin{bmatrix} \dfrac{C_L^{3/2}}{C_D} \\ -WW(\bar{x}) \end{bmatrix}$$

Subject to

$$\begin{aligned}
&\bar{g}_1(\bar{x}) = \left[-ROC(\bar{x}) + ROC^* \right] \le 0 \\
&\bar{g}_2(\bar{x}) = \left[VS(\bar{x}) - VS^* \right] \le 0 \\
&\bar{g}_3(\bar{x}) = \left[-VMAX(\bar{x}) + VMAX^* \right] \le 0 \\
&\bar{g}_4(\bar{x}) = \left[LEC(\bar{x}) - LEC^* \right] \le 0 \\
&\bar{g}_5(\bar{x}) = \left[-TER(\bar{x}) + TER^* \right] \le 0 \\
&\bar{g}_6(\bar{x}) = \left[\sigma(\bar{x}) - \sigma^* \right] \le 0 \\
&\bar{g}_7(\bar{x}) = \left[\delta(\bar{x}) - \delta^* \right] \le 0 \\
&\bar{x}_l \le \bar{x} \le \bar{x}_u \\
&\bar{x} = \left[AR \quad WS \quad TR \quad \theta \quad \Delta t1 \quad \Delta t2 \quad \Delta t3 \quad \Delta t4 \right]^T
\end{aligned} \tag{2}$$

where C_L is the 3D lift coefficient of the wing, C_D is the 3D drag coefficient of the wing, WW is the wing weight, σ is the maximum stress, δ is the maximum deflection of the wing, ROC* is the minimum allowable rate of climb at that altitude, VS* is the maximum allowable stall speed, VMAX* is the minimum allowable maximum speed at sea level, σ^* is the maximum allowable stress and δ^* is the maximum allowable deflection of the wing. The 3D optimization problem has two objective function, seven constraints and eight design variables. It involves multi-objective optimization. Both aerodynamic and structural disciplines are involved in this problem.

III. Analysis

As the optimization problem involves analysis in the aerodynamics and structures discipline, the mathematical model consists of two key components namely (i) Aerodynamic analysis and (ii) Structural analysis. These analysis procedures are briefly described below.

A. Aerodynamic Analysis

The panel method code XFLR5[19] is considered for the aerodynamic analysis. Basically, XFLR5 is a user friendly interface for the XFOIL[20] code. The XFOIL code uses a higher order panel method with coupled integral boundary layer. The algorithms for foil analysis implemented in XFLR5 are exactly the same as those of the original XFOIL code, except for the translation from FORTRAN to C++.

The inviscid formulation of XFOIL is a simple linear-vorticity stream function panel method. A finite trailing edge base thickness is modeled with a source panel. The equations are closed with an explicit Kutta condition. A Karman-Tsien compressibility correction is incorporated, allowing good compressible predictions all the way to sonic conditions. The theoretical foundation of the Karman-Tsien correction breaks down in supersonic flow, and as a result accuracy rapidly degrades as the transonic regime is entered. Of course, shocked flows cannot be predicted with any certainty.

As far as the viscous formulation is concerned, the boundary layers and wake are described with a two-equation lagged dissipation integral BL formulation and an envelope e^n transition criterion, both taken from the transonic analysis/design ISES code. The entire viscous solution (boundary layers and wake) is strongly interacted with the incompressible potential flow via the surface transpiration model (the alternative displacement body model is used in ISES). This permits proper calculation of limited separation regions. The drag is determined from the wake momentum thickness far downstream. A special treatment is used for a blunt trailing edge which fairly accurately accounts for base drag. The total velocity at each point on the airfoil surface and wake, with contributions from the freestream, the airfoil surface vorticity, and the equivalent viscous source distribution, is obtained from the panel solution with the Karman-Tsien correction added. This is incorporated into the viscous equations, yielding a nonlinear elliptic system which is readily solved by a full-Newton method as in the ISES code.

If lift is specified, then the wake trajectory for a viscous calculation is taken from an inviscid solution at the specified lift. If alpha is specified, then the wake trajectory is taken from an inviscid solution at that alpha. This is not strictly correct, since viscous effects will in general decrease lift and change the trajectory. This secondary correction is not performed, since a new source influence matrix would have to be calculated each time the wake trajectory is changed. This would result in unreasonably long calculation times. The effect of this approximation on the overall accuracy is small, and will be felt mainly near or past stall, where accuracy tends to degrade anyway. In attached cases, the effect of the incorrect wake trajectory is imperceptible.

Also, wing analysis capabilities using the Vortex Lattice Method (VLM) are added in the XFLR5 code. The wing is defined as a set of panels. Each panel is defined by its length, its root and tip chords, by its dihedral angle and by its mesh for VLM analysis. Twist is processed as a modification of the angle of attack[19]. As discussed in Ref 19, the principle of a VLM is to assimilate the perturbation generated by the wing to that of a sum of vortices distributed over the wing's planform. The strength of each vortex is calculated to meet the appropriate boundary conditions, i.e. non-penetration conditions on the surface of the panels. The induced drag is calculated by integration of surface forces at the ¾ point of the VLM panels. The viscous drag is estimated by interpolation of XFOIL pre-generated polars from the C_l value resulting from the linear VLM analysis.

The aerodynamic analysis is performed in two parts namely the 2D airfoil analysis and the 3D wing planform analysis. Both these analysis are performed using the XFLR5 code where the 2D airfoil analysis is exactly same as that of the XFOIL code and the 3D wing analysis is done using the VLM method.

The performance constraints are evaluated using the commercially available and well-validated aircraft design software RDS based on the book by D. Raymer[21] on the aircraft design, which is introduced after the XFLR5 code. The sizing and synthesis analysis of this software is well-validated for many aircraft conceptual design stage applications. The aerodynamic characteristics evaluated by the XFLR5 are used by the performance module of RDS that evaluates the performance constraints.

B. Structural Analysis

In this research, the commercially available and well validated FEM code MSC-NASTRAN[22] is used for the structural analysis of the wing. This software is used to perform the analysis and estimate the wing weight. For carrying out the structural analysis using the FEM code MSC-NASTRAN, the geometry is modeled using commercially available and well validated CAD software CATIA[23]. This software is used to generate the model geometry in the CAD environment and this geometric model is read by the commercially available and well validated mesh generation code MSC-PATRAN[24]. The mesh generated by this software is analyzed using the FEM solver MSC-NASTRAN.

The structural model is developed by dividing the entire structure into a number of discrete elements. The basic steps required to perform the structural analysis using FEM solver MSC-NASTRAN are as follows,

a) The continuous structure is represented as a collection of nodal points connected by discrete elements

b) From the given element properties, material properties and geometry, the elemental stiffness matrices are formulated

c) The global stiffness matrix corresponding to the full structure is assembled from the elemental stiffness matrices

d) The boundary conditions are applied to constrain the model and the load vectors are generated

e) The static equilibrium equation {f}=[K]{d} where K is the system stiffness, f is the load vector and d is the nodal displacement vector is solved. The unknowns are the nodal displacements which are evaluated by inverting the stiffness matrix and multiplying by the force vector.

f) Other required outputs like strains and stresses can be derived from the nodal displacements.

The geometry, loads and boundary conditions, material properties are captured in the pre-processor MSC-PATRAN. The meshing is also carried out in the pre-processor itself.

IV. Optimization

The optimization process is performed in two steps. First the airfoil is finalized through a single objective optimization problem. Once the airfoil is frozen, the wing planform parameters are arrived at through a dual objective problem. The optimization model is illustrated in Fig. 1 and is composed of different modules namely the mesh module, analysis module, meta-model module and optimization module. For the first step, the analysis module includes only the aerodynamic analysis which is performed by the XFLR5 code. For the second step, the analysis module includes both the aerodynamics and structural analysis.

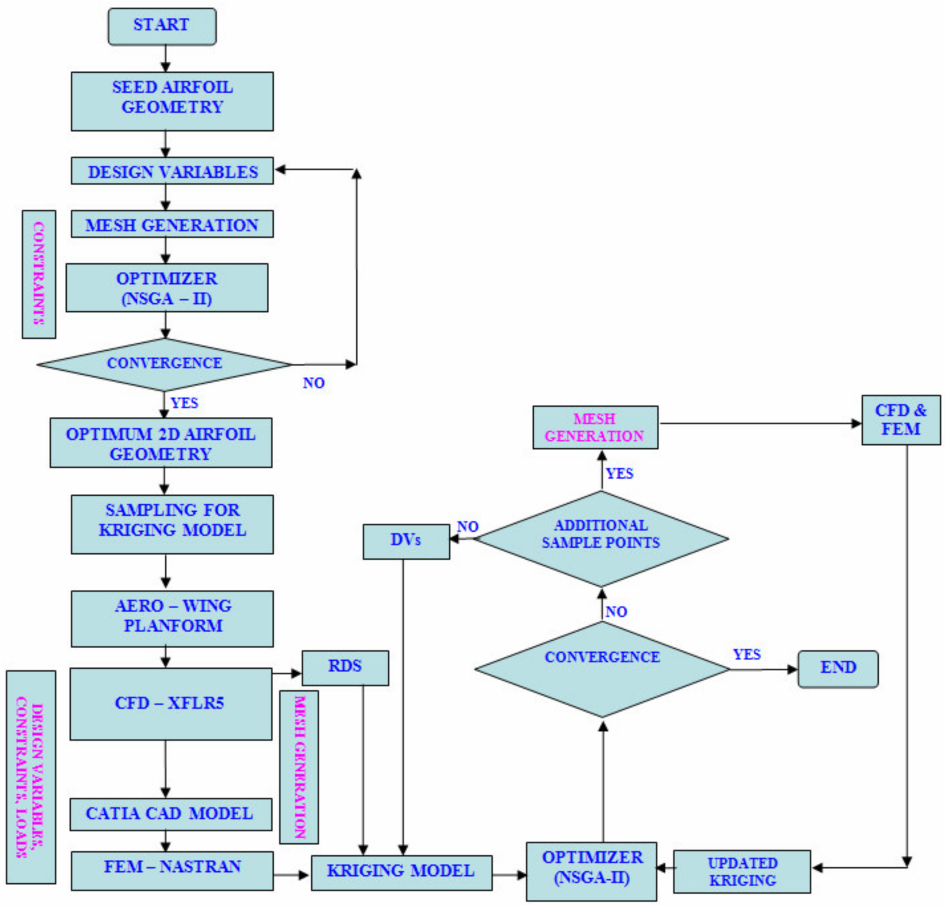

Figure 1. Optimization Model

Large computer time remains the biggest challenge in solving any MDO problem. The computation time increases because of the strong interactions between the disciplines and the use of high fidelity codes for aerodynamic and structural analysis. Such codes typically solve partial differential equations through discretization methods in a procedure which is computationally intensive. Fortunately, the computation time can be drastically reduced if accurate approximate models to replace the analysis tools can be created. Such models of models, or meta-models, are very useful in optimization. For example, the response surface method[12] and Kriging model[5,11] have been successfully applied for MDO problems. While response surface methods use polynomial expressions which are locally valid, Kriging is suited to approximate highly nonlinear functions and can be used to create globally valid meta-models. In the present study, Kriging model is employed for design of the UAV wing. The meta-model module generates the Kriging model and validates for sample points.

A. Kriging Model

Kriging model has its original roots in the field of geostatistics which is a mixed discipline of mining, engineering, geology, mathematics and statistics. In this field, this meta-model is used to predict temporal and spatial correlated data. A wide range of correlation functions can be chosen for building the meta-model thereby making the Kriging meta-models extremely flexible.

The analysis tool is replaced with the Kriging model in the objective function evaluation process of MOGA. As the Kriging model introduces uncertainty at the prediction point, the biggest advantage of GA in obtaining the global optimization may be lost[25]. In order to retain this advantage, both the prediction value and its uncertainty are to be considered at the same time[25]. This is captured by updating the Kriging model during the optimization. If the optimization algorithm is not converged, additional random designs are initialized and the Kriging model is reconstructed. The Gaussian correlation function is used in the meta-model interpolates all data points exactly. Kriging model takes a combination of a polynomial model and a statistical function and is written as follows,

$$\mathrm{y}(\mathbf{x}) = \mathrm{f}(\mathbf{x}) + \mathrm{z}(\mathbf{X})$$

where $\mathrm{y}(\mathbf{x})$ is the unknown function, $\mathrm{f}(\mathbf{x})$ is the known polynomial function and $\mathrm{z}(\mathbf{X})$ is the function from a stochastic process with mean zero, variance and non zero covariance. The polynomial function $\mathrm{f}(\mathbf{x})$ approximates the design space globally and the localized deviations are created by the function $\mathrm{z}(\mathbf{X})$. The polynomial function is taken as a constant term for this study. The covariance matrix of $\mathrm{z}(\mathbf{X})$ that is responsible for the local deviations is given below,

$$\mathrm{Cov}[\mathrm{z}(\mathrm{x}^{\mathrm{i}}), \mathrm{z}(\mathrm{x}^{\mathrm{j}})] = \sigma^2 \mathbf{R}([\mathrm{R}(\mathrm{x}^{\mathrm{i}}, \mathrm{x}^{\mathrm{j}})])$$

where $\mathbf{R}$ is the correlation matrix, and $\mathrm{R}(\mathrm{x}^{\mathrm{i}}, \mathrm{x}^{\mathrm{j}})$ is the correlation function between any two of the sample data points x^{i} and x^{j}. The correlation matrix considered in this paper is the common Gaussian correlation function as described below,

$$\prod_{k=1}^{n} \exp\left(-\theta_k \left|d_k\right|\right)^2$$

where n is the number of design variables, θ_k is the unknown correlation parameters used to fit the model and d_k is the distance between the k^{th} component of sample points x^{i} and x^{j}.

The optimization algorithm in this approach is as follows,

a) Initially some random designs are initialized. The procedure to identify these designs is to randomly choose the design points available within 50% on both sides of the initial population.

b) The analysis code is run to construct the Kriging model for the two objective functions.

c) The MOGA is performed on the Kriging model.

d) If the convergence is not achieved, additional random designs are chosen and the optimization process is repeated till convergence.

Further details of Kriging model are available in Ref 26.

B. NSGA II

An important aspect of this paper is the use of a multi-objective genetic algorithm NSGA-II[18]. This algorithm can give the full Pareto front for multi-objective problems. The merits and demerits of the classical gradient based approach and evolutionary approach for a given optimization problem are extensively discussed in literature[27-28]. In a nutshell, the difficulties encountered by the classical optimization algorithm for general optimization problem occur in situations with (1) non-smooth variables, (2) nonlinear and discontinuous constraints, (3) noisy functions and (4) multiple minima. However, there are disadvantages in evolutionary optimization such as (1) no clearly defined convergence criteria, (2) parameter tuning mostly by trial and error, (3) computationally expensive population-based approach and (4) slow convergence to optimum. The main difference between classical optimization and evolutionary algorithm (EA) is that EA uses a population of solutions in each of the iterations, instead of a single solution. Since a population of solutions is processed in each of the iterations, the outcome of an EA is also a population of solutions.

When the optimization problem involves more than one objective function, the task of finding one or more optimum solutions is termed as multi-objective optimization[16]. If the objective functions are conflicting in nature, each objective corresponds to a different optimal solution. Thus, there exist a set of optimal solutions where a gain in one objective calls for a sacrifice in some other objective. For a designer, knowing a number of optimal solutions becomes important and it also gives considerable insight into the design thereby providing several feasible and useful design solutions. Therefore, as far as the designer is concerned, the ideal multi-objective optimization procedure is to find the multiple trade-off optimal solutions with a wide range of values for the objectives and choose one of the obtained solutions using higher-level information. Often this higher-level information would be non-technical, qualitative and experience driven. Most of the multi-objective optimization algorithms use the concept of domination. This concept of domination is described in detail in Ref 5 and Ref 16.

It is well known that there exist multiple Pareto optimal solutions in a problem only if the objectives are conflicting with each other. If the objectives are not conflicting with each other, the cardinality of the Pareto optimal set is one as shown in an example described in Ref. 16. We need to find out whether the dual objectives considered in the conceptual design of UAV are conflicting in nature. To check for this conflict, a design space exploration is done by performing a parametric study. Each of the design variables is allowed to vary and the variation in both the objectives and constraints is observed. This exercise is already performed in the conceptual design of the UAV under consideration. The results of that work are given in Ref 5.

The optimization model workflow is created in such a way that for a given starting value of design variables, it calculates the aerodynamic characteristics namely the lift, drag and moment coefficient. Using these coefficients, it maximizes the endurance of the UAV. Simultaneously, it generates the wing weight and checks for the constraint violation of strength and stiffness.

Most of the multi-objective optimization algorithms use the concept of domination. In these algorithms, two solutions are compared on the basis of whether one dominates the other solution or not. A solution $x_{(1)}$ is said to dominate the other solutions $x_{(2)}$ (in other words, $x_{(1)}$ is non-dominated by $x_{(2)}$), if both the following conditions are true[17],

(1) The solution $x_{(1)}$ is no worse than $x_{(2)}$ in all objectives

(2) The solution $x_{(1)}$ is strictly better than $x_{(2)}$ in at least one objective

The non-dominated set of the entire feasible search space S is the globally Pareto optimal set and these solutions are known as non-dominated solutions (Pareto optimal solutions). The rest are called dominated solutions. Since none of the solutions in the non-dominated set is absolutely better than any other, any one of them is an acceptable solution. Since the NSGA-II is based on sorting the non-dominated solutions, it is referred to as non-dominated sorting GA. There are three approaches for determining the non-dominated set[17] namely naive and slow approach, continuously updated approach and Kung et al's efficient approach.

The steps involved in the non-dominated sorting of a population are listed below[17],

- Step 1: Set all non-dominated sets $P_j, (j = 1,2,...)$ as empty sets. Set non-domination level counter $j = 1$
- Step 2: Use any one approaches to find the non-dominated set $P^{'}$ of population P
- Step 3: Update $P_j = P^{'}$ and $P = \frac{P}{P^{'}}$
- Step 4: If $P \neq 0$ increment j by one and go to Step 2. Otherwise, stop and declare all non-dominated sets P_j for $i = 1,2,..j$

The binary coded GAs has the following difficulties:

(1) Search space becomes discrete,

(2) Hamming cliffs problem,

(3) Search restricted with variable boundaries,

(4) Arbitrary precision impossible due to fixed length coding.

To address these difficulties with binary GA, the NSGA-II uses the real coded GAs and the decision variables are coded directly. The idea behind the NSGA-II algorithm is that a ranking selection method is used to emphasize good points and a niche method is used to maintain stable sub populations of good points. NSGA-II is different from the simple GA only in the way selection operation works. The mutation operations for NSGA-II are similar to simple GA. The "Simulated Binary Crossover (SBX)" operator used in NSGA-II is slightly different from the cross over operator of the simple GA and details are given in Ref. 22. The NSGA-II procedure is outlined as follows,

- Step 1: Combine parent P_t and offspring Q_t populations and create $R_t = P_t \cup Q_t$. Perform a non dominated sorting to R_t and identify different fronts: $F_i, i = 1,2....$ etc
- Step 2: Set new population $P_{t+1} = 0$. Set a counter $i = 1$. Until $|P_{t+1}| + |F_i| < N$, perform $P_{t+1} = P_{t+1} \cup F_i$ and $i = i + 1$.
- Step 3: Perform the crowding sort procedure and include most widely spread solutions by using the crowding distance values in the sorted F_i to P_{t+1}
- Step 4: Create offspring population Q_{t+1} from P_{t+1} by using the crowded tournament selection, cross over and mutation operators.

The NSGA-II algorithm for finding the multiple Pareto optimal solution in a multi objective optimization problem has the three key features as discussed in Ref. 22: (a) use of an elitist principle (b) an explicit diversity preserving mechanism and (c) emphasis on non dominated solution in a population. The NSGA-II requires a non-dominated sorting of a population of size $2N$. This requires at most $O\left(MN^2\right)$ computations, where N is the number of solutions and M is the number of function evaluations. Thus the computational complexity of this algorithm is at most $O\left(MN^2\right)$. The greatest advantage of NSGA-II is that the diversity among the non-dominated solutions is introduced by using the crowding comparison procedure, which is used with the tournament selection and during the population reduction phase. Since the solutions compete with their crowding distances, no extra niching parameter is required here. The elitism mechanism does not allow an already found Pareto optimal solution to be deleted.

V. Results and Discussions

The low speed UAV under consideration has to loiter over the target area for many hours at a medium altitude. From the preliminary weight estimation, the empty weight fraction is 0.53 and the useful weight (payload weight and fuel weight) fraction is 0.47. For this study, the following parameters are set for simulations in the genetic algorithm: Population size = 50, Number of generations = 100, Crossover probability = 0.9, Crossover distribution index = 20, Mutation probability = 0.1, Mutation distribution index = 100. For the kriging model construction, the number of random designs chosen is 20 and the number of additional random designs is 10. The baseline airfoil for aerodynamic optimization is the NASA/LANGLEY LS (1)-0417 (GA (W)-1). The baseline airfoil has the following characteristics

Maximum thickness to chord ratio	=	17%
Maximum thickness position	=	30.2%c
Maximum camber	=	2.33%c
Maximum camber position	=	20.10%c
Trailing edge gap	=	0.73%c

The airfoil is discretized into 100 panels in the XFLR5 code.

As far as the 2d airfoil design optimization problem is concerned, Fig 2 shows the comparison of the airfoil geometry for the baseline and optimized design. Figs. 3-5 present the lift, drag and endurance parameter. The physical properties of the designed airfoil are compared with the original GA (W)-1. The thickness of the airfoil is increased from 17% to 18.9%. The optimized airfoil is having slightly better characteristics in terms of area enclosed by the airfoil, which is a measure of the total internal volume available for fuel storage between the spars. Optimized airfoil maintains almost same camber of the seed airfoil as 2.51% while the leading edge curvature is reduced from 28.94% to 18.42%. The trailing edge angle is increased slightly from 3.19º to 4.62º in order to reduce the constraint imposed to structural stiffness of the airfoil. The optimized airfoil has the following characteristics

Maximum thickness to chord ratio = 18.9%
Maximum thickness position = 26.7%c
Maximum camber = 2.51%c
Maximum camber position = 21.3%c
Trailing edge gap = 0.67%c

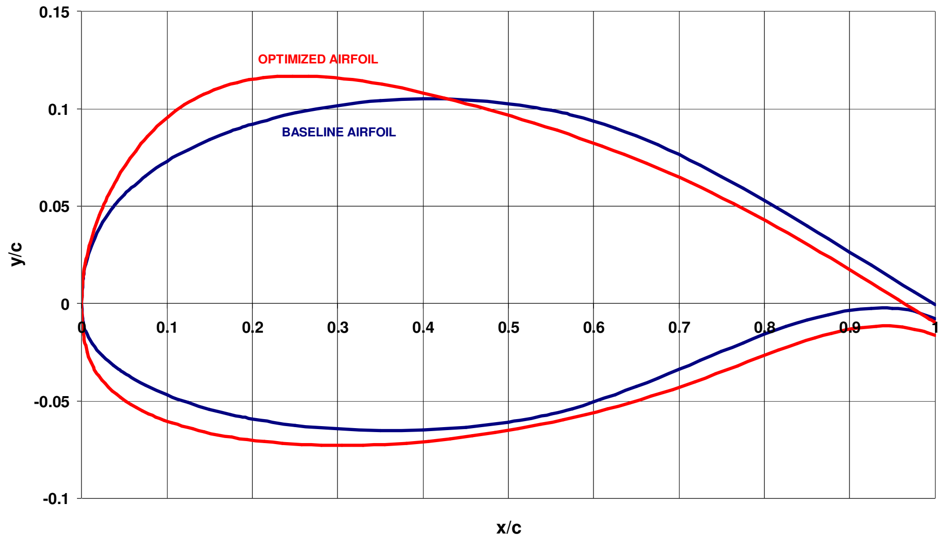

Figure 2. Geometry of baseline and optimized airfoil

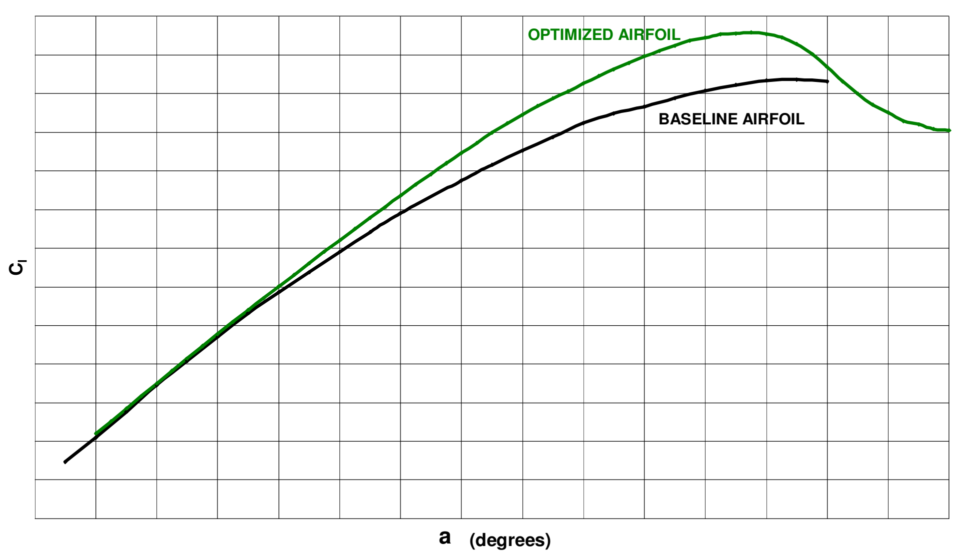

Figure 3. 2D Lift characteristics for baseline and optimized airfoil

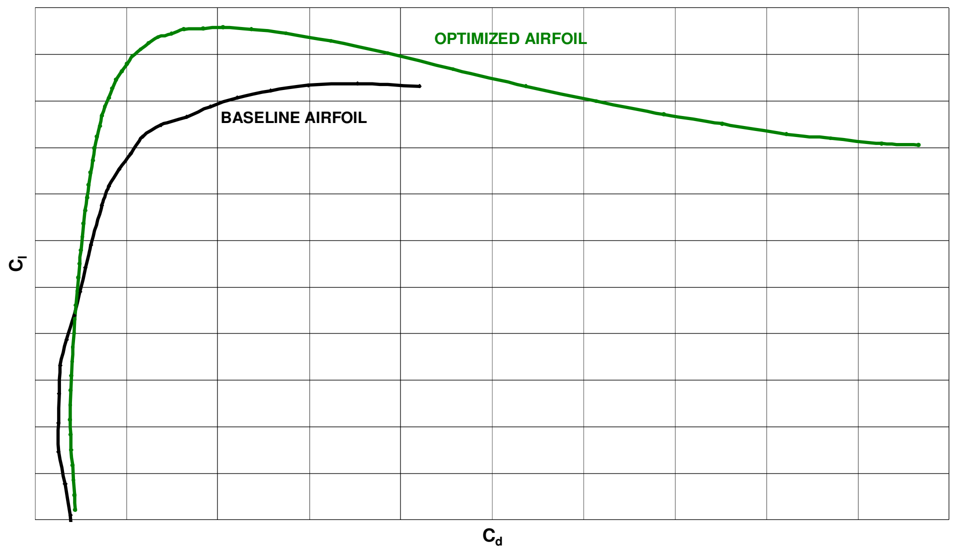

Figure 4. 2D Drag characteristics for baseline and optimized airfoil

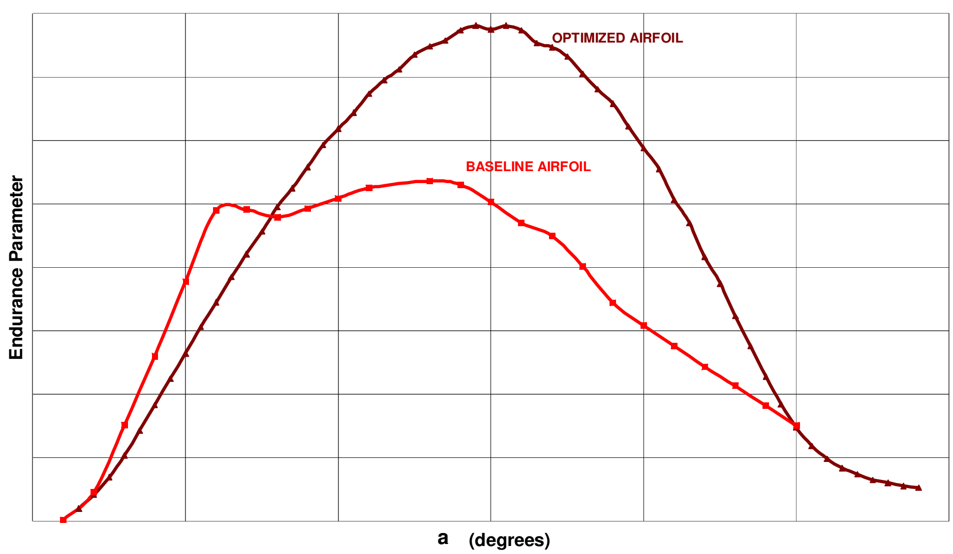

Figure 5. Endurance parameter for baseline and optimized airfoil

Table 2. Comparison of characteristics of baseline and optimized airfoil

PARAMETER	Baseline airfoil	Optimized airfoil
C_{lmax}	1.85	2.1
C_{l0}	0.55	0.55
Alpha – max in deg	19	17
Alpha 0 in deg	-4	-4
C_l-alpha (Linear range) per deg	0.1125	0.1175
C_{d0} at alpha = 0	0.005	0.005
C_{m0}	-0.12	-0.09
Max Endurance Parameter	108	155

It can be clearly seen from Table 2 that the aerodynamic characteristics of the optimized airfoil is far superior compared to the baseline airfoil. The C_{lmax} has increased by 13.5% and the maximum endurance parameter has increased by 43.5%.

For the baseline model in the structural analysis, the wing is made of three parts in the chordwise direction namely the leading edge box, center section and the trailing edge box. For the optimization problem, only the center box is considered. The center box consists of upper and lower skin and front and rear spars. The upper and lower skin thicknesses at root and tip are considered as the design variables. The structural model is created in MSC-PATRAN with loads and boundary conditions. The element chosen for modeling the wing structure is the general shell element. Only one half of the wing is modeled with symmetric boundary conditions at the root of the wing. All the six degrees of freedom of the nodes at the root of the wing takes the values as UZ=0, ROTX=0, ROTY=0. Also, in order to arrest the rigid body motion of the wing, only node at the wing root takes the boundary condition UX=0, UY=0. The air loads are given in each element of the wing. The self weight of the wing structure is also taken into consideration which gives a weight (inertia) relief for the structure. The wing is made up of CFRP skin with Rohacell foam core. The material properties are defined in MSC-PATRAN. The material properties considered for this optimization problem is the quasi-isotropic properties of the composite material under consideration. Fig 6 gives the FEM model created in the MSC-PATRAN.

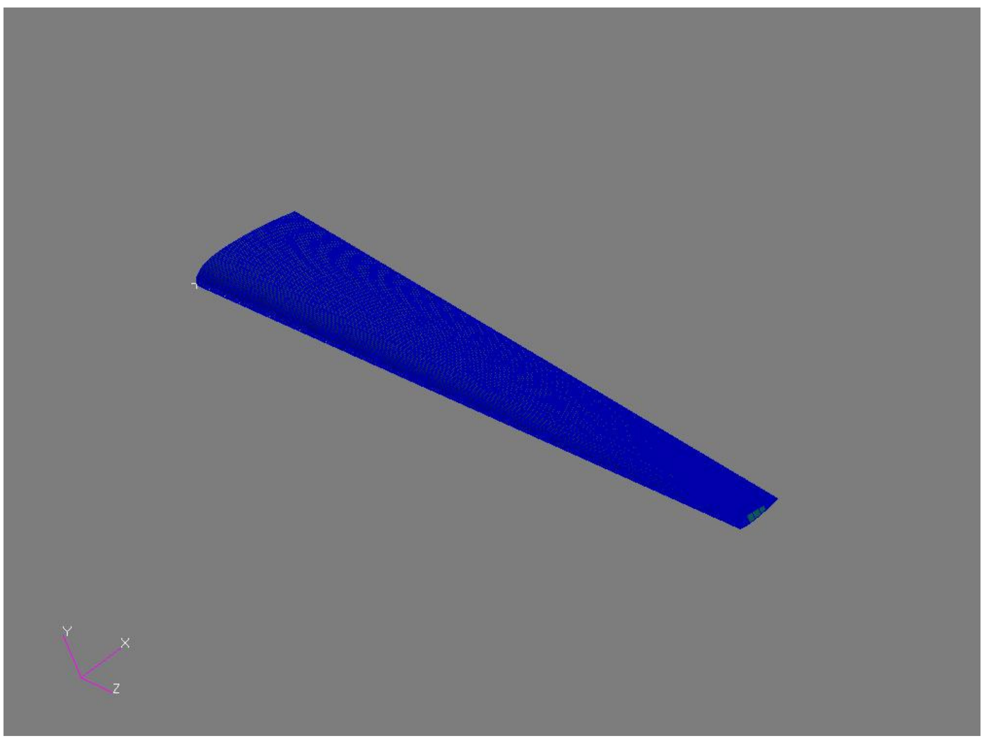

Figure 6. FEM Model of the 3D wing

The structural analysis results are presented in Fig 7-8. The displacement and stress contour of the optimized wing is shown in Fig 7 and Fig 8 respectively.

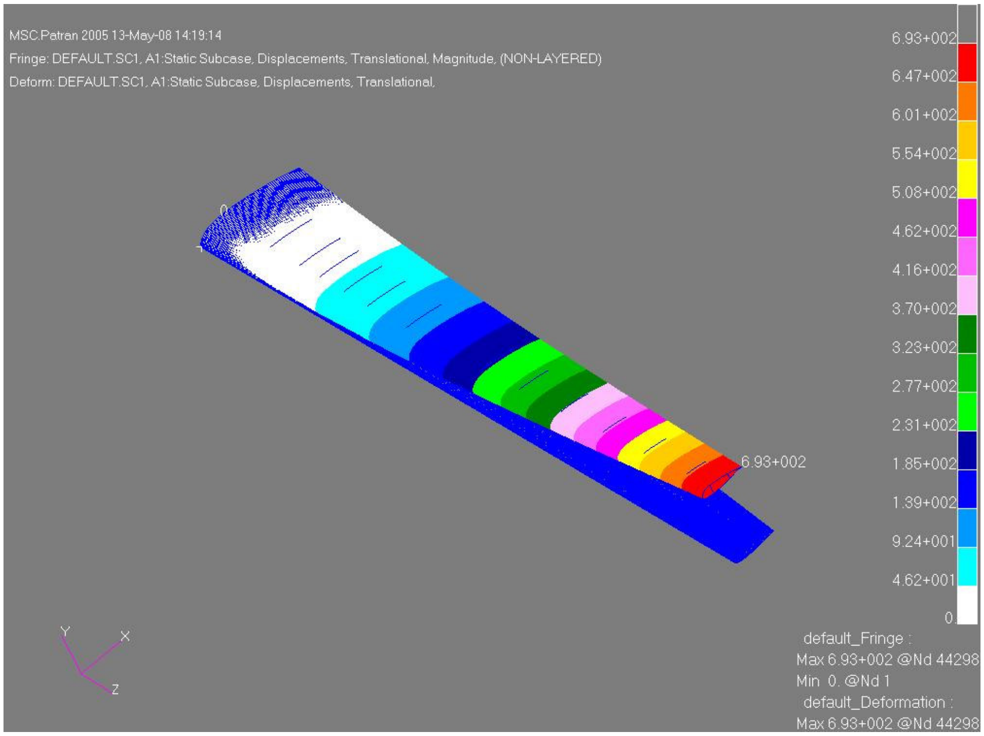

Figure 7. Displacement contour of the optimized 3D wing

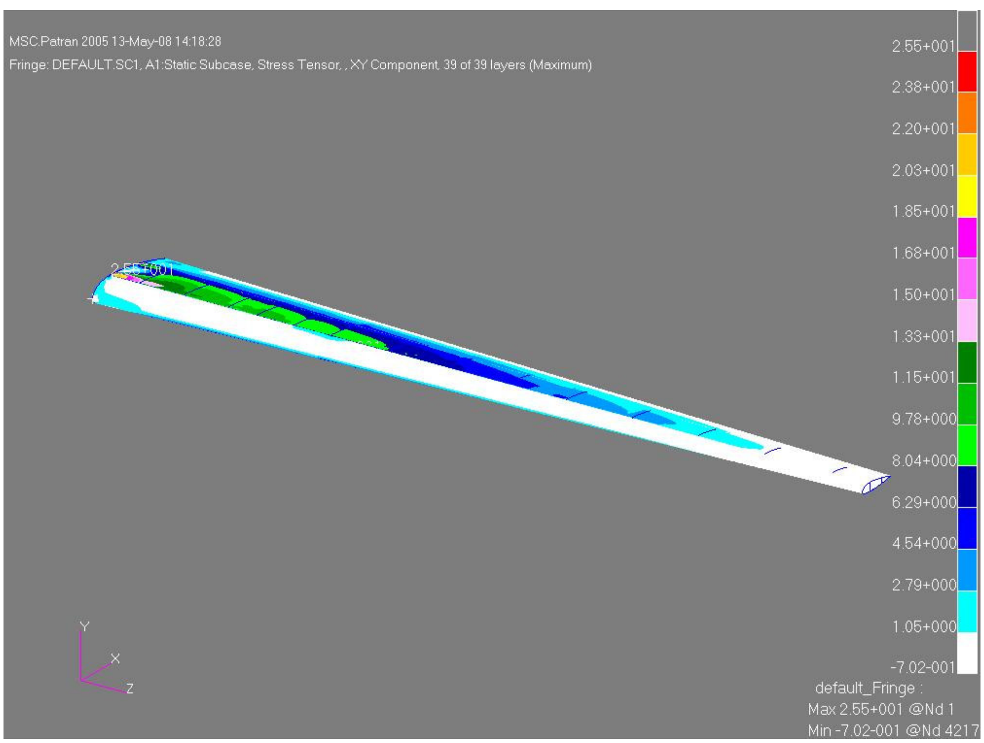

Figure 8. Stress contour of the optimized 3D wing

As far as the 3D wing design optimization problem is concerned, a graph between the two objectives namely the wing weight and endurance parameter are plotted to obtain the Pareto front. The Pareto front is shown in Fig 9. It is a typical Min-Max Pareto front. Five design solutions are possible with optimum solutions lying in the Pareto front. These Pareto points are designated with PP1 to PP5. The design variables corresponding to the two extreme Pareto points PP1 and PP5 are described in Table 3.

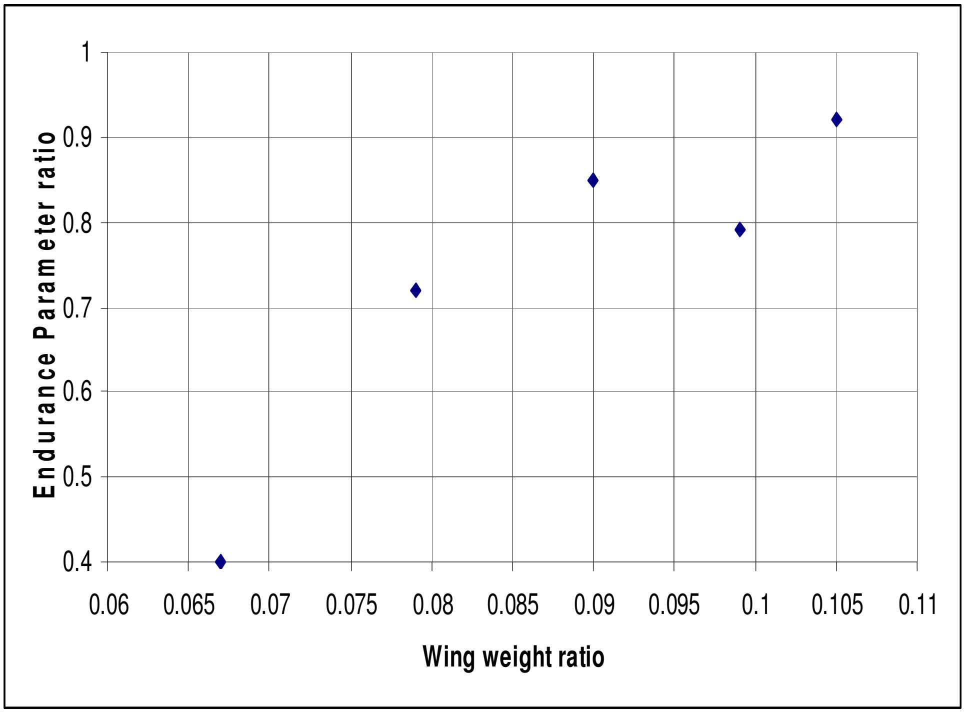

Figure 9. Pareto Front

Table 3. Design Variables corresponding to Pareto points PP1 and PP5

Design Variables	Pareto Point PP1	Pareto Point PP5
Wing Aspect Ratio	0.82	0.98
Wing Loading	0.99	0.91
Wing Taper Ratio	0.3125	0.45
Wing twist angle	0.4	0.93
Upper skin thickness at root	0.73	0.92
Upper skin thickness at tip	0.37	0.44
Lower skin thickness at root	0.73	0.92
Lower skin thickness at tip	0.37	0.44
Wing Weight	0.067	0.105
Endurance parameter	0.4	0.92

VI. Conclusion

An evolutionary optimization algorithm capable of finding the full Pareto front of multi-objective optimization problems is applied for preliminary UAV wing design. The preliminary design problem is formulated as a mathematical optimization problem. A single objective problem of maximizing the endurance parameter and a dual objective problem of maximizing endurance parameter and minimizing wing weight is considered. The formulation and workflow is done in two steps, initially a 2D airfoil optimization where the airfoil geometry is optimized to give a maximum endurance parameter with constraints on geometrical parameters. In the second step, the 3D wing planform is optimized with objective functions from aerodynamics and structures. In the second step, the Kriging approximation model, replaces both the aerodynamic and structural analysis code. Employing the Kriging model reduces the computation time. The robust evolutionary algorithm NSGA-II used to capture the Pareto front is capable of identifying the trade-off between the conflicting objectives thereby providing alternative useful designs for the designer. It is found that the five Pareto designs obtained offer various possibilities to the designer in terms of higher-level requirement and may offer novel and non-traditional solutions to the design problem. The Kriging model employed reduces the computation time without losing the Pareto points. The optimization results confirm the feasibility of the Kriging based MOGA for UAV wing design optimization.

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