Adiabatic Processes

Suppose the frequency ω in a harmonic oscillator drifts slowly:

$$\ddot{x} + \omega(t)^2 x = 0 \tag{1}$$

where $\omega(t)$ is some slowly varying function. The energy of the system is the sum of kinetic and potential energies:

$$E = \frac{\dot{x}^2}{2} + \frac{\omega^2 x^2}{2} \tag{2}$$

It is constant in time for the harmonic oscillator itself. The question is this: What is the effect on E of a slow drift in the frequency ω drifts slowly? Investigate this numerically, using various functions $\omega(t)$. One such function should be of the form $\omega(t) = \omega_0 + \epsilon t$, for a small constant ϵ. Does the effect depend strongly on the trajectory ω follows as it drifts from one value to another?

Does E remain constant, or does it vary systematically with ω? If for example it increases as ω increases, this means that the system absorbs energy as the oscillation is speeded up. This is what would happen for example if you were to slowly shorten the length of a pendulum. How does E vary with ω, if it does? Investigate this in as much detail as you can, and account analytically for as many observed features as you can.