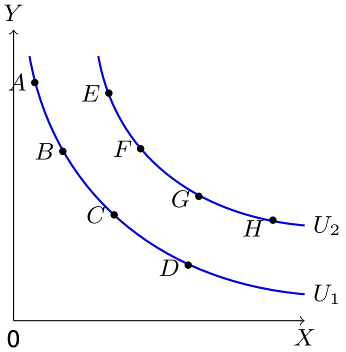
Reference sheet: Indifference Curves

1 Definition

One can create a collection of all the bundles A, B, C, D that all make a consumer indifferent. The line that connects these points is called an indifference curve. All bundles on the same indifference curve give the same level of satisfaction (utility). The same consumer has many indifference curves – there is an indifference curve through any point in the $x - y$ space.

Here x and y are *goods*: the consumer would like as much of each as possible. So the further away from the origin the indifference curve is, the higher the level of utility.



The slope of the indifference curve is called the marginal rate of substitution – the rate at which the consumer is willing to trade y for an extra unit of x: $|MRS_{xy}| = \frac{\Delta y}{\Delta x}$. Along the same indifference curve, we must have $|MU_x \Delta x| = |MU_y \Delta y|$ so we can also write the MRS as $|MRS_{xy}| = \frac{MU_x}{MU_y}$.

2 General shape of Indifference Curves

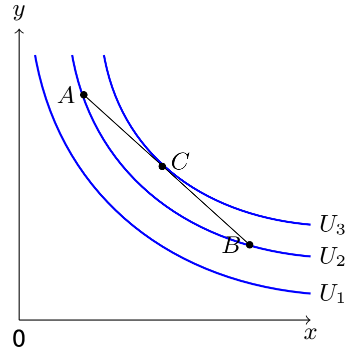
2.1 Decreasing

Because the consumer likes both goods, typically indifference curves are downward sloping: if the consumer receives less y, they need to receive more x in order to remain equally well off.

2.2 Convex

Typically indifference curves are convex (they becomes flatter as one moves along them down to the right): as the consumer has more of x, they value each extra unit of x less and less (diminishing marginal utility) and therefore are willing to give up less and less of y in exchange for an extra unit of x. This property is known as diminishing marginal rate of substitution: as we move down and to the right on an indifference curve, the quantity of x increases and the quantity of y decreases, so MYU_x decreases and MU_y increases, so $MRS_{xy} = \frac{MU_x}{MU_y}$ decreases.

Note that a consumer whose preferences are convex always prefers mixtures of goods to extremes of either good. If we draw a line between two points A and B on the same indifference curve U_1, point C on the line is a mixture of the two end-points. When the indifference curves are convex, C gives higher utility than A and B.

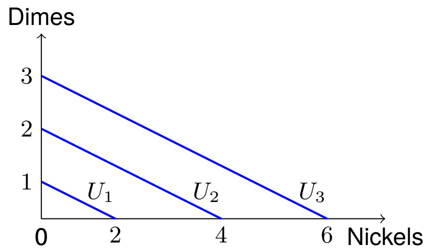


3 Special Cases

3.1 Perfect Substitutes

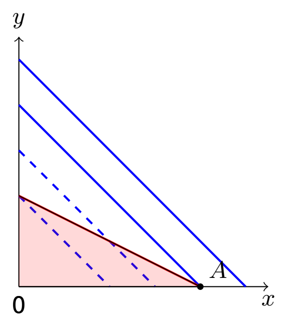
The consumer considers some quantity x as identical to some quantity of y, so they can substitute y for y at a constant rate, irrespective of the point on the indifference curve. The indifference curves have a constant slope (MRS is constant).

For example, x is nickels, y is dimes: for the consumer, 1 dime and 2 nickels are perfectly interchangeable (both amount to 10 cents). In the graph below, at every point along U_2, the consumer gets an equivalent of 20 cents. It is not important how this amount is split between nickels and dimes.

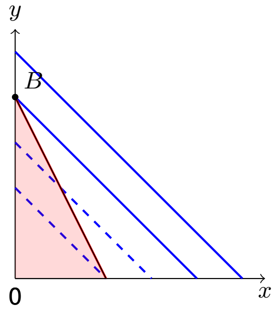


The optimal consumption point will be on the highest indifference curve, but still touching the budget line. Three cases can arise depending on P_x/P_y (slope of the budget constraint):

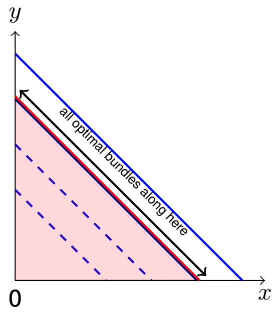
1. $P_x/P_y < |MRSxy|$: the budget constraint is flatter than indifference curves. At the optimal consumption point A the consumer buys only x and no y.



2. $P_x/P_y > |MRSxy|$: the budget constraint is steeper than indifference curves. At the optimal consumption point B the consumer buys only y and no x.



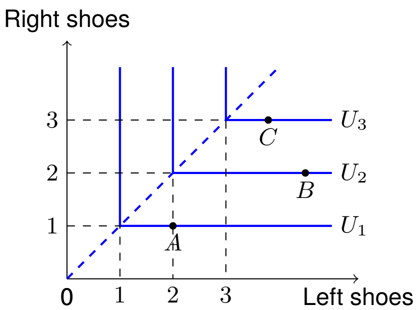
3. $P_x/P_y = |MRSxy|$: the budget constraint has the same slope as indifference curves. Any point on it is optimal.



3.2 Perfect Complements

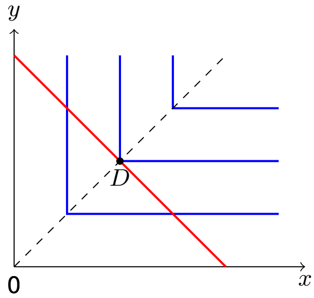
The consumer considers that x and y should be consumed together in fixed proportions, and regards having more of one good without more of the other as being pointless. Indifference curves are L-shaped.

For example x is left shoes, y is right shoes (assuming that the individual has 2 feet). In the graph below, U_2 has a kink at $(2, 2)$ – where the consumer has 2 pairs of shoes. Increasing the number of left shoes while keeping 2 right shoes, or increasing the number of right shoes while keeping 2 left shoes is of no value to the consumer. So the consumer is indifferent between the kink and any point to the right of it, or above it: from this kink the indifference curve is horizontal to the right and vertical above.



Note that $MRS_{xy}(A) = MRS_{xy}(B) = MRS_{xy}(C) = 0$. On the vertical part of the indifference curves and at their kinks, the MRS is undefined.

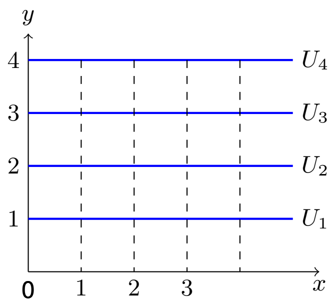
The optimal consumption point will be on the indifference curve furthest out from the origin, but still touching the budget line:



4 For the curious: beyond course material

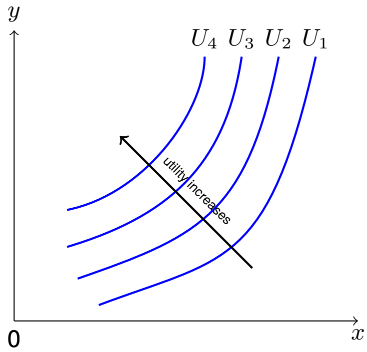
4.1 Neuters

If x is a *neuter* for the consumer, having more or less of x does not affect the utility in any way. Indifference curves are horizontal: utility is constant for a given quantity of y (whatever the quantity of x), and increases as the quantity of y increases. If y was a “neuter” (and x was a *good*), indifference curves would be vertical.



4.2 Bads

If y is a *good* but x is a *bad* (for example: pollution): “less x is better” – the consumer gets more satisfaction by consuming less x. Indifference curves are upward sloping: when x icreases, the consumer needs y to increase in order to remain as well off.



In the graph below x is a *good* but y is a *bad*.