2D Smoke Simulation

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***Abstract*—In this project, I complete a GUI program, which can be used to simulate the motion of 2-D smoke by adding instant or permanent sources to it. It is interactive and can be used to study the basic physics behind the phenomenon we observe.**

I. Introduction

SMOKE simulation is one of the major interests in today's computer graphics.

II. Format

Navier-Stokes Equation[1]:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = \vec{g} + \nu \nabla \cdot \nabla \vec{u} \tag{1}$$

It is extremely difficult for us to obtain the analytic solution to the above equation. So in computer graphics, we use numerical methods and split the original one into several parts, and solve these sub-step one by one[2].

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = 0 \tag{2}$$

$$\frac{\partial \vec{u}}{\partial t} = \nu \nabla \cdot \nabla \vec{u} \tag{3}$$

$$\frac{\partial \vec{u}}{\partial t} = \vec{g} \tag{4}$$

$$\frac{\partial \vec{u}}{\partial t} + \frac{1}{\rho} \nabla p = 0 \tag{5}$$

In my project the exterior force is not taken into account. And by using numerical methods, we create a $(N+2) \times (N+2)$ grid and store density at the center of every cell, store velocity at the edge(see MAC grid[3]).

A. Advection

Advection is the transport of a substance or quantity by bulk motion. We use a method called back-trace to update the properties(density and velocity) of every cell(see Fig.1).

B. Diffusion

Diffusion is the net movement of molecules from a region of higher concentration to a region of lower concentration. It is driven by a gradient in chemical potential of the diffusing species, which makes the concentration tend to be more unifying.

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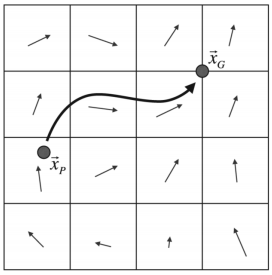

Fig. 1. To find a fluid value at grid point $\vec{x}_G$ at the new time step, we need to know where the fluid at $\vec{x}_G$ was one time step ago, position $\vec{x}_P$, following the velocity field.

C. Projection

Projection is an important step to make the smoke incompressible, in other word, to make the velocity field divergence-free[1]:

$$\nabla \cdot \vec{u} = 0 \tag{6}$$

TABLE I
Simulation Parameters

grid size	302×302
number of iterations	20
time step	$0.033s$
ν	0.1

III. Result

Just show some typical results, for more presentation, see attachment.

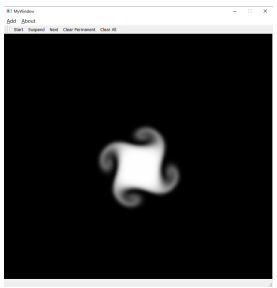

Fig. 2. This is a simple simulation. The initial density is set to be 1.0 inside the circle and 0 outside the circle. The velocity is set to be only have the tangential component.

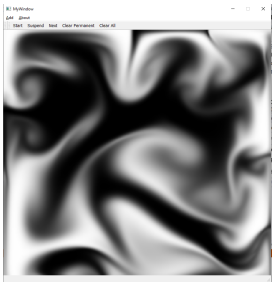

Fig. 3. This is the result of some randomly-set sources.

REFERENCES

[1] Bridson R. *Fluid Simulation for Computer Graphics*[M]. AK Peters/CRC Press, 2015.

[2] Stam J. *Real-Time Fluid Dynamics for Games*[C]//Proceedings of the game developer conference. 2003, 18: 25.

[3] Harlow F H, Welch J E. *Numerical Calculation of TimeDependent Viscous Incompressible Flow of Fluid with Free Surface*[J]. The physics of fluids, 1965, 8(12): 2182-2189.