PH4428: Problem Sheet 1

Solve the problems and bring the solutions by 14 a.m. Monday, 4/02/2017.

1. A wavefunction of a two-level system on the Bloch sphere.
Sketch the Bloch sphere.

(a) Show the following states of the qubit on the Bloch sphere:
$\frac{|0\rangle+|1\rangle}{\sqrt{2}}$, $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$, $\frac{|0\rangle+i|1\rangle}{\sqrt{2}}$ and $\frac{|0\rangle-i|1\rangle}{\sqrt{2}}$.

(b) Prove that two states $\Psi = e^{i\varphi}\cos\frac{\theta}{2}|0\rangle+\sin\frac{\theta}{2}|1\rangle$ and
$\Psi = \cos\frac{\theta}{2}|0\rangle+e^{-i\varphi}\sin\frac{\theta}{2}|1\rangle$ are the same vector on the Bloch sphere.

2. The Cooper pair box and the charge qubit.

(a) Use Mathematica (or Matlab, Python or any other programming language) to calculate and plot energy bands (eigenvalues) of the Cooper pair box with
(i) $E_C = 70$, $E_J = 0$;
(ii) $E_C = 70$, $E_J = 10$;
(iii) $E_C = 20$, $E_J = 10$;
(iv) $E_C = 5$, $E_J = 10$.

Use Cooper pair charging energy definition $E_C = \frac{(2e)^2}{2C}$.

Hints:
- Create the Hamiltonian in a matrix form in the charge basis for charge states from 0 to 5.
- Plot the eigenvalues of the Hamiltonian in the range of gate induced charge n_g from 1 to 4.
- To plot the eigenvalues use Matematica function “Eigenvalues” or equivalent function for finding eigenvalues in other programs.
- Use range in y-axis from -10 to 100.
(In Mathematics “PlotRange->{-10,100}”)

Highlight the part of the plot in (ii), which can be used as a qubit.