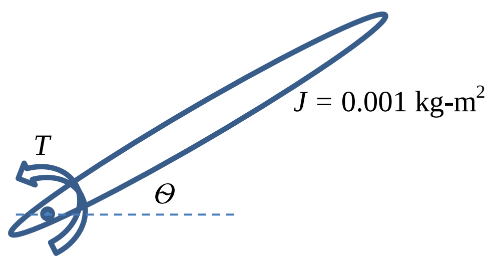
Homework Assignment #8 (ME-190, Fall 2016)

Due date: Tuesday, Nov. 22, 2016, 2:00 pm.

Objective: Analytical approach for feedback loop design

The underlying mechanics of many mechatronic systems is a rotational mass under a control torque input. Applications such as hard disk drives, robotic manipulators, cranes, etc., represent examples of such system. The goal of this homework is to analyze a position control system that leads to the stabilization and accurate positioning of a rotational mass system.



Part 1 (Modeling): Using the Newton's law, write the equation of motion for the above system, and develop a state-space and a transfer function model from the input torque T to angle Θ, neglecting the friction and viscos damping (Note that the moment of intertia, J, is about the axis of rotation).

Part 2 (Stability analysis): Investigate if the system is open-loop stable or not. For this part, simulate the state-space model for a nonzero initial angle and angular velocity (e.g., Θ = 1 rad, and $\dot{\Theta}$ = 1 rad/s). Use Matlab's "ss" and "initial" commands to create and simulate the system response. If all the states approaches zero over time from nonzero initial conditions, the system is called "asymptotically" stable. If both states remain bounded the system is called "Lyapunov "stable (which is a weaker stability condition). If at least one of the states increases continuously over time, the system is unstable.

What would be your guess before checking it through Matlab (no need to respond)?

```
sys = ss(A,B,C,D)
[y,t,x] = initial(sys,x0)
plot(t,x)
```

Part 3 (Control Design):

The goal of this section is to design a controller that moves the position of the system to a desired angle without control saturation. For this part, we will use a simple zero-pole controller:

$$C(s) = k\frac{s+a}{s+b}$$

where k, a, and b are design parameters to be tuned through pole-placement and simulation.

3.1. Calculate the sensitivity transfer function for the above control and plant system obtained in Part 1.

3.2. Apply the Final Value Theorem to the sensitivity transfer function to obtain the steady-state control error value in response to a step input.

3.3. Find the equations for k, a, and b (in terms of parameters J and p) so that the poles of the closed-loop system are all mapped into a single location $s = -p$, where p is a positive real number.

Part 4 (Control tuning):

Create a Simulink model that simulates the closed loop system. Use a step input for the reference trajectory. Choose the value of p (integers only) such that the maximum input torque is just below 0.1 N-m for a step reference input of $\pi/2$ radians. Does the theory (from the Final Value Theorem) match the simulation result?

Include in your submission:

- All the required analysis details
- All Matlab codes
- The Simulink model
- Simulation plots for the stability analysis
- Simulation plots of closed-loop system reference, output, and control input for the final control gain values.