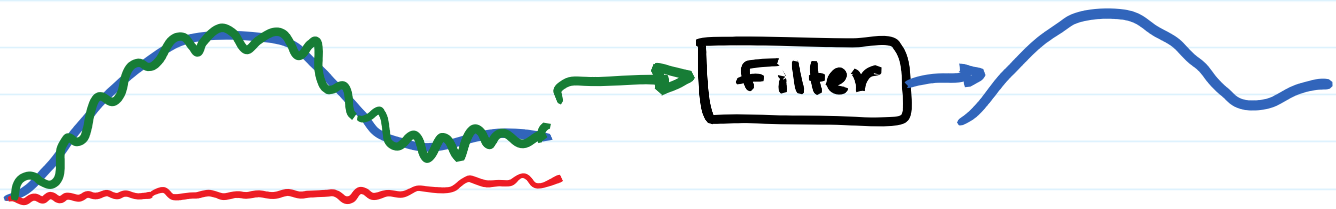
ME-190 Noise Filtering Basics

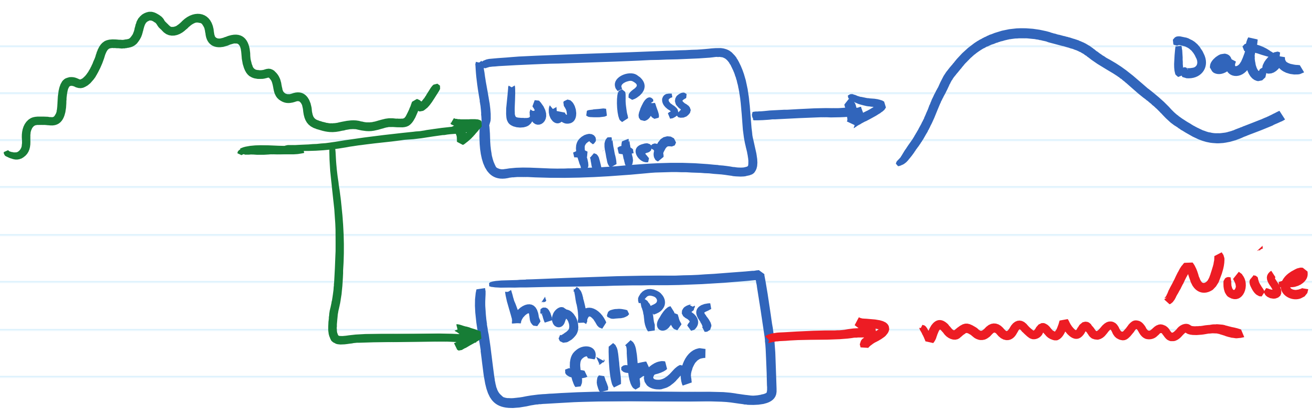
Real world measurements come with noise



signals : Low-freq + high freq

Data   noise

Two types of filters in general :



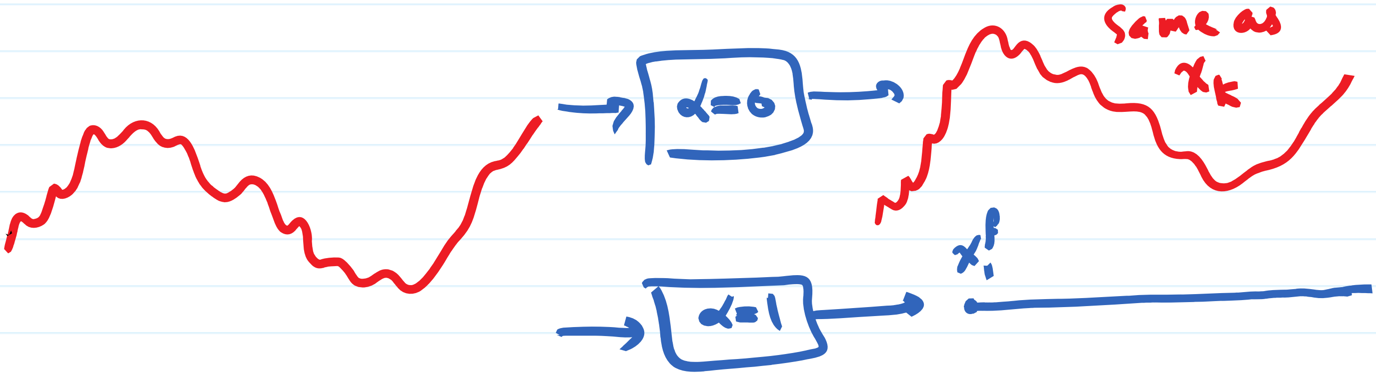
* Low-pass filters:

Let's take $\underline{x}$ as a sequence of measured signal : $x_1, x_2, \ldots, x_k, \ldots$

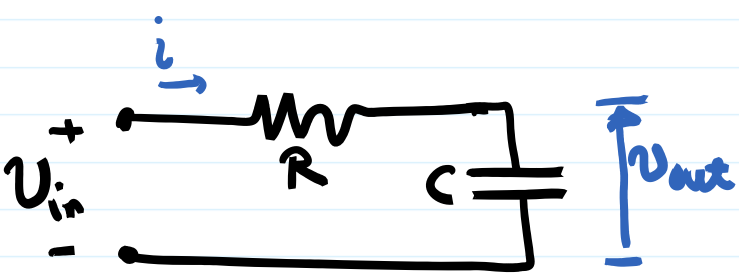
$$x_k \longrightarrow \boxed{x_k^f = 0.9\, x_{k-1}^f + 0.1\, x_k} \longrightarrow x_k^f$$

$$x_k^f = \alpha\, x_{k-1}^f + (1-\alpha)\, x_k$$

If α is chosen close enough to 1 it would filter the high freq component of the signal



* Analog filter (RC circuit)



Claim: v_{out} will filter high. freq component of v_{in}

$$v_{in} - iR - \frac{1}{C}\int_0^t i\, d\tau = 0 \quad ①$$

$$v_{out} = \frac{1}{C}\int_0^t i\, d\tau \quad ②$$

Let's eliminate "i" from ① & ②

$$\dot{v}_{out} = \frac{1}{C} i \Rightarrow i = C \dot{v}_{out} \quad (3)$$

Replacing (3) & (2) into (1):

$$\underbrace{v_{in} - RC\,\dot{v}_{out} - v_{out} = 0}_{\text{input - output equ.}} \quad (4)$$

Discretize (4): Approximate derivatives:

(1) $$\dot{x} = \lim_{\Delta t \to 0} \frac{x_K - x_{K-1}}{\Delta t} \cong \frac{x_K - x_{K-1}}{\Delta t} \quad \text{if } \Delta t \text{ is small}$$

Backward difference

(2) $$\dot{x} \cong \frac{x_{K+1} - x_K}{\Delta t}$$ Forward difference

(3) $$\dot{x} \cong \frac{x_{K+1} - x_{K-1}}{2\Delta t}$$ Central difference

(4) $$\ddot{x} \cong \frac{x_{K+1} + x_{K-1} - 2x_K}{\Delta t^2} \;?$$

Let's apply Backward difference method to

$$v_{in} - RC\,\dot{v}_{out} - v_{out} = 0$$

$$\Rightarrow v_{in}^K - RC\,\frac{v_{out}^K - v_{out}^{K-1}}{T_s} - v_{out}^K = 0$$

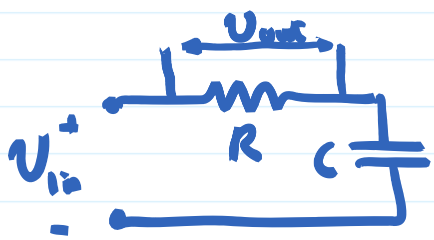
$$\Rightarrow -v_{out}^{k}\left(\frac{RC}{T_s}+1\right)+\frac{RC}{T_s}v_{out}^{k-1}+v_{in}^{k}=0$$

$$\Rightarrow v_{out}^{K}=\underbrace{\frac{RC}{RC+T_s}}_{\alpha}v_{out}^{K-1}+\underbrace{\frac{T_s}{RC+T_s}}_{1-\alpha}v_{in}^{K}$$

$$v_{out}^{K}=\alpha\, v_{out}^{k-1}+(1-\alpha)\, v_{in}^{K}$$

where $\alpha=\frac{RC}{RC+T_s}$

High-pass RC filter



if we pick voltage of the resistor as output we have a high-pass filter