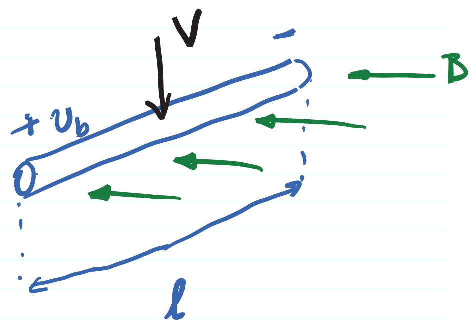
Moving wire in a magnetic field:

B: Magnetic field

V: Wire velocity

ℓ: Wire length



$$\Rightarrow v_b = (\vec{V} \times \vec{B})\ell$$

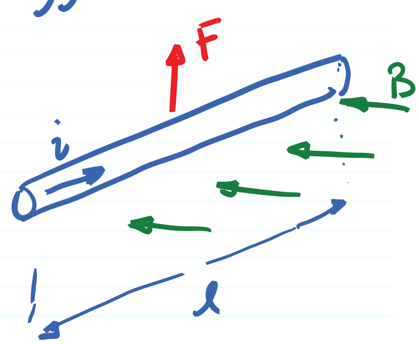
v_b: Induced voltage

$$= V B \ell \sin\theta$$

θ: Angle between magnetic field and velocity

This is conversion from mechanical to Electrical Energy!

Conversion from electrical energy to mechanical energy.

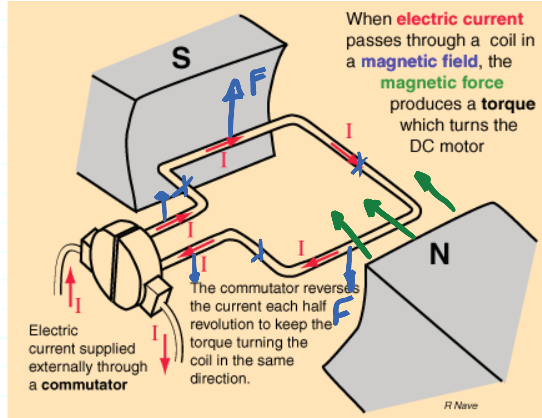


$$\vec{F} = (\vec{I} \times \vec{B})\ell$$

$$= i B \ell \sin\theta$$

θ: angle between current and magnetic field.

Modeling DC Motors



$F = B\,i\,l_{eff}$

$\underset{\text{Torque}}{\tau_e} = F\cdot r = (\underbrace{B r l_{eff}}_{\text{torque Const. } K_\tau})\, i$

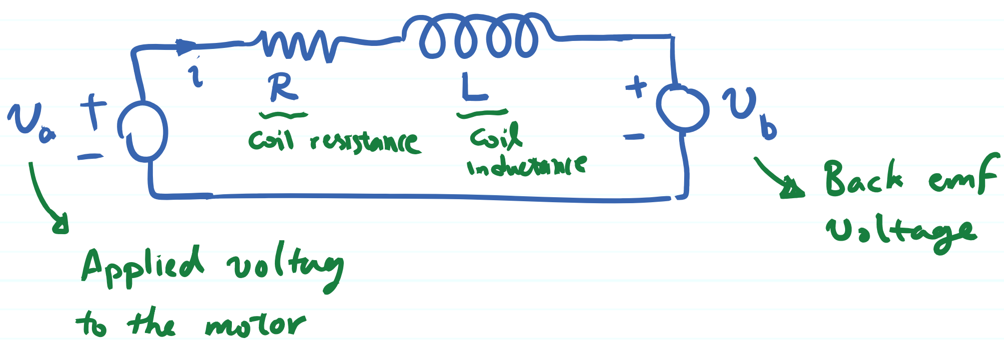
$$\boxed{\tau_e = K_\tau\, i}$$

When the wire moves it creates electrical potential so called Back Electromotive Force (Back emf)

$v_b = V B l_{eff} = r \omega B l_{eff} = (\underbrace{r B l_{eff}}_{K_b \text{ (Back emf const.)}})\, \omega$

$$\boxed{v_b = k_b\, \omega}$$

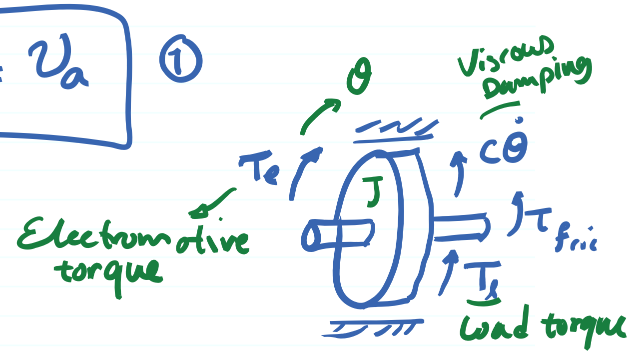
Circut of the coil:



$v_a - Ri - L\dfrac{di}{dt} - v_b = 0$

$$\boxed{L\frac{di}{dt} + Ri + K_b\dot{\theta} = v_a} \quad ①$$

Mechanical System



Newton's Law for rotational mech system:

$$\sum M = J\ddot{\theta}$$

$$\tau_e - c\dot{\theta} - T_l - T_{fric} = J\ddot{\theta}$$

$$\boxed{J\ddot{\theta} + c\dot{\theta} = K_\tau i - T_l - T_{fric}} \quad (2)$$

Let's put the system in the state-space:

$$\begin{cases} x_1 = i & \Rightarrow \dot{x}_1 = \frac{di}{dt} = \frac{1}{L}\left(-R\underbrace{i}_{x_1} - k_b\underbrace{\dot{\theta}}_{x_3} + \underbrace{U_a}_{u}\right) \\ x_2 = \theta & \rightarrow \dot{x}_2 = \dot{\theta} = x_3 \\ x_3 = \dot{\theta} & \Rightarrow \dot{x}_3 = \ddot{\theta} = \frac{1}{J}\left(-c\underbrace{\dot{\theta}}_{x_3} + K_\tau \underbrace{i}_{x_1} - T_l - T_{fric}\right) \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & 0 & -\frac{K_b}{L} \\ 0 & 0 & 1 \\ \frac{K_\tau}{J} & 0 & -\frac{c}{J} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{1}{J} & -\frac{1}{J} \end{bmatrix}}_{B} \begin{bmatrix} u \\ T_l \\ T_{fric} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

y_1 = current

y_2 = Angle

y_3 = Ang. velocity

Transfer function:

$$L\frac{di}{dt} + Ri + k_b\dot{\theta} = V_a \xrightarrow[I.C.=0]{\text{Laplace}} Ls\,I(s) + R\,I(s) + k_b s\,\theta(s) = V_a(s)$$

$$J\ddot{\theta} + c\dot{\theta} = K_T i - T_{fric} - T_l \Rightarrow Js^2\theta(s) + cs\,\theta(s) = K_T I(s) - T_{fric}(s) - T_l(s)$$

let's calculate $G(s) = \dfrac{\theta(s)}{V_a(s)}$

$$I(s) = \frac{k_b s}{Ls+R}\theta(s) + \frac{V_a(s)}{Ls+R}$$

$$\theta(s) = \frac{k_T I(s)}{Js^2+Cs} + \cdots T_{fric} - \cdots T_\ell(s)$$

$$\Rightarrow \theta(s) = \frac{k_b k_T s}{(Ls+R)(Js^2+Cs)}\theta(s) + \frac{k_T}{(Js^2+Cs)(Ls+R)}V_a(s)$$

$$\theta(s)\left(1 - \frac{k_b k_T s}{(Ls+R)(Js^2+Cs)}\right) = \frac{k_T}{(Js^2+Cs)(Ls+R)}V_a(s)$$

$$\rightarrow \frac{\theta(s)}{V_a(s)} = \underline{\qquad\qquad\qquad\qquad}$$