Section 1.3

1.

b) Let $x \in$ (All stones)
Let $P(x) = x$ is precious
Let $B(x) = x$ is beautiful
Symbolically, the statement is:
$(\forall x)(P(x) \Rightarrow \sim B(x))$

c) Let $x \in$ (All triangles).
Let $R(x) = x$ is a right triangle
Let $I(x) = x$ is an isosceles triangle
Symbolically, the statement is:
$(\exists x)(P(x) \wedge R(x))$

d) Note: Using definitions from part **c**
Symbolically, the statement is:
$(\forall x)(P(x) \Rightarrow \sim I(x))$

e) Note: Using definitions from part **c**
Symbolically, the statement is:
$(\forall x)(\sim I(x) \Rightarrow R(x))$

f) Let $x \in$ (All people)
Let $H(x) = x$ is honest.
Symbolically, the statement is:
$[(\forall x)(H(x))] \vee [(\forall x)(\sim H(x))]$

g) Note: Using definitions from part **f**
Let $y \in$ (All people)
Symbolically, the statement is:
$[(\exists x)(H(x))] \wedge [(\exists y)(\sim H(y))]$

h) Let $x \in \mathbb{R}$
Symbolically, the statement is:
$(\forall x)(x \neq 0 \Rightarrow (x > 0 \vee x < 0))$

j) Let $x, y \in \mathbb{Z}$
Symbolically, the statement is:
$(\forall x)(\exists y)(x > y)$

l) Let $x, y \in \mathbb{Z}$, let $m \in \mathbb{R}$
Symbolically, the statement is:
$(\forall x)(\forall y)(\exists m)[(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$

o) Let $x, y \in$ (All people)
Let $L(x, y) = x$ loves y
Symbolically, the statement is:
$(\forall x)(\exists y)(L(x, y))$

p) Let $x, y \in \mathbb{R}$
Symbolically, the statement is:
$(\forall x)(\exists! y)((x > 0) \Rightarrow (2^y = x))$

2.

b) $\sim (\forall x)(P(x) \Rightarrow \sim B(x)) \equiv (\exists x) \sim (P(x) \Rightarrow \sim B(x)) \equiv (\exists x)(P(x) \wedge B(x))$
"Some stones are precious and beautiful."

c) $\sim (\exists x)(P(x) \wedge R(x)) \equiv (\forall x) \sim (P(x) \wedge R(x)) \equiv (\forall x)(\sim P(x) \vee \sim R(x))$
"All trianges are either not right or not isosceles."

d) $\sim (\forall x)(P(x) \Rightarrow \sim I(x)) \equiv (\exists x) \sim (P(x) \Rightarrow \sim I(x)) \equiv (\exists x)(P(x) \vee I(x))$
"Some triangles are either right or isosceles."

e) $\sim (\forall x)(\sim I(x) \Rightarrow R(x)) \equiv (\exists x) \sim (\sim I(x) \Rightarrow R(x)) \equiv (\exists x)(\sim I(x) \vee \sim R(x))$
"Some triangles are neither right nor isosceles."

f) $\sim \{[(\forall x)(H(x))] \vee [(\forall x)(\sim H(x))]\} \equiv \sim [(\forall x)(H(x))] \wedge \sim [(\forall x)(\sim H(x))]$
$\equiv [(\exists x)(\sim H(x))] \vee [(\exists x)(H(x))]$
"Some people are not honest or some people are honest."

g) $\{[(\exists x)(H(x))] \wedge [(\exists y)(\sim H(y))]\} \equiv \sim [(\exists x)(H(x))] \vee \sim [(\exists y)(\sim H(y))]$
$\equiv [(\forall x)(\sim H(x))] \vee [(\forall y)(H(y))]$
"All people are not honest or everyone is honest."

h) $\sim (\forall x)(x \neq 0 \Rightarrow (x > 0 \vee x < 0)) \equiv (\exists x) \sim (x \neq 0 \Rightarrow (x > 0 \vee x < 0))$
$\equiv (\exists x)(x \neq 0 \wedge \sim (x > 0 \vee x < 0)) \equiv (\exists x)(x \neq 0 \ \wedge (x \leq 0 \wedge x \geq 0))$
"There exists a nonzero real number that is greater than or equal to zero and is less than or equal to zero."

j) $\sim (\forall x)(\exists y)(x > y) \equiv (\exists x) \sim (\exists y)(x > y) \equiv (\exists x)(\forall y) \sim (x > y) \equiv (\exists x)(\forall y)(x \leq y)$
"There exests an integer x such that for every integer y, x $\leq$ y."

l) $\sim (\forall x)(\forall y)(\exists m)[(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$
$\equiv (\exists x) \sim (\forall y)(\exists m)[(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$
$\equiv (\exists x)(\exists y) \sim (\exists m)[(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$
$\equiv (\exists x)(\exists y)(\forall m) \sim [(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$
$\equiv (\exists x)(\exists y)(\forall m)[(x \neq y) \wedge \sim (x > m > y) \wedge \sim (y > m > x)]$
"There exist nonequal reals x and y such that every real number cannot be between them."

o) $\sim (\forall x)(\exists y)(L(x,y)) \equiv (\exists x) \sim (\exists y)(L(x,y)) \equiv (\exists x)(\forall y)(\sim L(x,y))$
"Someone does not love everyone."

p) Let $A(x,y) = ((x > 0) \Rightarrow (2^y = x))$
$\sim (\forall x)(\exists! y)A(x,y) \equiv (\exists x) \sim (\exists! y)A(x,y)$
$\equiv (\exists x) \sim [(\exists y)A(x,y) \wedge (\forall a)(\forall b)(A(x,a) \wedge A(x,b) \Rightarrow a = b)]$
$\equiv (\exists x)[\sim (\exists y)A(x,y) \vee \sim (\forall a)(\forall b)(A(x,a) \wedge A(x,b) \Rightarrow a = b)]$
$\equiv (\exists x)[(\forall y) \sim A(x,y) \vee (\exists a)(\exists b) \sim (A(x,a) \wedge A(x,b) \Rightarrow a = b)]$
$\equiv (\exists x)[(\forall y) \sim A(x,y) \vee (\exists a)(\exists b)(A(x,a) \wedge A(x,b) \wedge a \neq b)]$
Substituting back in $A(x,y)$:
$\equiv (\exists x)[(\forall y) \sim ((x > 0) \Rightarrow (2^y = x)) \vee (\exists a)(\exists b)(((x > 0) \Rightarrow (2^a = x)) \wedge ((x > 0) \Rightarrow (2^b = x)) \wedge a \neq b)]$
$\equiv (\exists x)[(\forall y)((x > 0) \wedge (2^y \neq x)) \vee (\exists a)(\exists b)(((x > 0) \Rightarrow (2^a = x)) \wedge ((x > 0) \Rightarrow (2^b = x)) \wedge a \neq b)]$
$\equiv (\exists x)[(\forall y)((x > 0) \wedge (2^y \neq x)) \vee (\exists a)(\exists b)(((x > 0) \Rightarrow (2^a = x \wedge 2^b = x)) \wedge a \neq b)]$
"There exists a real number x such that for every real y x is positive and $2^y \neq x$, or such that for some real numbers a and b, x being positive implies that $2^a = x$ and $2^b = x$, and a is not equal to b."

6.

b) True for ***T*** False for ***U, V, W***

c) True for ***T, U, V*** False for ***W***

d) True for ***T*** False for ***U, V, W***

Section 1.3

7.

a) ***Proof:***
$\sim (\exists x)A(x)$ is true in U
iff $(\exists x)A(x)$ is false in U
iff truth set of $A(x)$ is empty
iff truth set of $\sim A(x)$ is U
iff $(\forall x) \sim A(x)$ is true in U

8.

a) **F**

c) **F**

d) **F**

f) **T**

g) **F**

i) **T**

j) **F**

k) **T**

l) **T**

12.

b) Not always true. Two polynomails are not equall iff the coefficents are not equal at one index i. It does not need to be true for every index.

c) Similar to the last one, just said differently. This does not need to be true for every pair of coefficents.

d) This is true, as it is the definition for two polynomails not being equal. Only one pair of coefficents need be different.

e) True, since it simplifies to the same as as part **d**.

f) Not always true since it simplifies to the same as part **c**.

Section 1.3

g) This is true, since it only requires one index of coefficents to be unequal.

h) The is true, since the premise is false for all forms of inequality other than the one described by the conclusion. Again, only one set of coefficents is required.