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1.

b) Let $x \in$ (All stones)
Let $P(x) = x$ is precious
Let $B(x) = x$ is beautiful
Symbolically, the statement is:
$(\forall x)(P(x) \Rightarrow \sim B(x))$

c) Let $x \in$ (All triangles).
Let $R(x) = x$ is a right triangle
Let $I(x) = x$ is an isosceles triangle
Symbolically, the statement is:
$(\exists x)(P(x) \wedge R(x))$

d) Note: Using definitions from part **c**
Symbolically, the statement is:
$(\forall x)(P(x) \Rightarrow \sim I(x))$

e) Note: Using definitions from part **c**
Symbolically, the statement is:
$(\forall x)(\sim I(x) \Rightarrow R(x))$

f) Let $x \in$ (All people)
Let $H(x) = x$ is honest.
Symbolically, the statement is:
$[(\forall x)(H(x))] \vee [(\forall x)(\sim H(x))]$

g) Note: Using definitions from part **f**
Let $y \in$ (All people)
Symbolically, the statement is:
$[(\exists x)(H(x))] \wedge [(\exists y)(\sim H(y))]$

h) Let $x \in \mathbb{R}$
Symbolically, the statement is:
$(\forall x)(x \neq 0 \Rightarrow (x > 0 \vee x < 0))$

j) Let $x, y \in \mathbb{Z}$
Symbolically, the statement is:
$(\forall x)(\exists y)(x > y)$

l) Let $x, y \in \mathbb{Z}$, let $m \in \mathbb{R}$
Symbolically, the statement is:
$(\forall x)(\forall y)(\exists m)[(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$

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o) Let $x, y \in$ (All people)
Let $L(x, y) = x$ loves y
Symbolically, the statement is:
$(\forall x)(\exists y)(L(x, y))$

p) Let $x, y \in \mathbb{R}$
Symbolically, the statement is:
$(\forall x)(\exists! y)((x > 0) \Rightarrow (2^y = x))$

2.

b) $\sim (\forall x)(P(x) \Rightarrow \sim B(x)) \equiv (\exists x) \sim (P(x) \Rightarrow \sim B(x)) \equiv (\exists x)(P(x) \wedge B(x))$
"Some stones are precious and beautiful."

c) $\sim (\exists x)(P(x) \wedge R(x)) \equiv (\forall x) \sim (P(x) \wedge R(x)) \equiv (\forall x)(\sim P(x) \vee \sim R(x))$
"All trianges are either not right or not isosceles."

d) $\sim (\forall x)(P(x) \Rightarrow \sim I(x)) \equiv (\exists x) \sim (P(x) \Rightarrow \sim I(x)) \equiv (\exists x)(P(x) \vee I(x))$
"Some triangles are either right or isosceles."

e) $\sim (\forall x)(\sim I(x) \Rightarrow R(x)) \equiv (\exists x) \sim (\sim I(x) \Rightarrow R(x)) \equiv (\exists x)(\sim I(x) \vee \sim R(x))$
"Some triangles are neither right nor isosceles."

f) $\sim \{[(\forall x)(H(x))] \vee [(\forall x)(\sim H(x))]\} \equiv \sim [(\forall x)(H(x))] \wedge \sim [(\forall x)(\sim H(x))]$
$\equiv [(\exists x)(\sim H(x))] \vee [(\exists x)(H(x))]$
"Some people are not honest or some people are honest."

g) $\{[(\exists x)(H(x))] \wedge [(\exists y)(\sim H(y))]\} \equiv \sim [(\exists x)(H(x))] \vee \sim [(\exists y)(\sim H(y))]$
$\equiv [(\forall x)(\sim H(x))] \vee [(\forall y)(H(y))]$
"All people are not honest or everyone is honest."

h) $\sim (\forall x)(x \neq 0 \Rightarrow (x > 0 \vee x < 0)) \equiv (\exists x) \sim (x \neq 0 \Rightarrow (x > 0 \vee x < 0))$
$\equiv (\exists x)(x \neq 0 \wedge \sim (x > 0 \vee x < 0)) \equiv (\exists x)(x \neq 0 \;\wedge (x \leq 0 \wedge x \geq 0))$
"There exists a nonzero real number that is greater than or equal to zero and is less than or equal to zero."

j) $\sim (\forall x)(\exists y)(x > y) \equiv (\exists x) \sim (\exists y)(x > y) \equiv (\exists x)(\forall y) \sim (x > y) \equiv (\exists x)(\forall y)(x \leq y)$
"There exests an integer x such that for every integer y, x $\leq$ y."

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l) $\sim (\forall x)(\forall y)(\exists m)[(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$
$\equiv (\exists x) \sim (\forall y)(\exists m)[(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$
$\equiv (\exists x)(\exists y) \sim (\exists m)[(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$
$\equiv (\exists x)(\exists y)(\forall m) \sim [(x \neq y) \Rightarrow (x > m > y) \vee (y > m > x)]$
$\equiv (\exists x)(\exists y)(\forall m)[(x \neq y) \wedge \sim (x > m > y) \wedge \sim (y > m > x)]$
"There exist nonequal reals x and y such that every real number cannot be between them."

o) $\sim (\forall x)(\exists y)(L(x,y)) \equiv (\exists x) \sim (\exists y)(L(x,y)) \equiv (\exists x)(\forall y)(\sim L(x,y))$
"Someone does not love everyone."

p) $letA(x) = ((x > 0) \Rightarrow (2^y = x))$
$\sim (\forall x)(\exists! y)A(x) \equiv (\exists x) \sim (\exists! y)A(x)$
$\equiv (\exists x) \sim [(\exists x)A(X)]$ FIX THIS "There exists some real number x such that there is no unique real y for which $(x > 0)$ implies $(2^y = x)$"

6.

b) True for ***T*** False for ***U, V, W***

c) True for ***T, U, V*** False for ***W***

d) True for ***T*** False for ***U, V, W***

7.

a) ***Proof:***
$\sim (\exists x)A(x)$ is true in U
iff $(\exists x)A(x)$ is false in U
iff truth set of $A(x)$ is empty
iff truth set of $\sim A(x)$ is U
iff $(\forall x) \sim A(x)$ is true in U

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8.

- **a)** **F**
- **c)** **F**
- **d)** **F**
- **f)** **T**
- **g)** **F**
- **i)** **T**
- **j)** **F**
- **k)** **T**
- **l)** **T**

12.

- **b)** Not always true. Two polynomails are not equall iff the coefficents are not equal at one index i. It does not need to be true for every index.
- **c)** Similar to the last one, just said differently. This does not need to be true for every pair of coefficents.
- **d)** This is true, as it is the definition for two polynomails not being equal. Only one pair of coefficents need be different.
- **e)** True, since it simplifies to the same as as part **d**.
- **f)** Not always true since it simplifies to the same as part **c**.
- **g)** This is true, since it only requires one index of coefficents to be unequal.
- **h)** The is true, since the premise is false for all forms of inequality other than the one described by the conclusion. Again, only one set of coefficents is required.