

1.

- a) This is not a proposition because it does not have a set truth value.
- b) This is a proposition. Let:
 $P = \pi \text{ is a rational number} \rightarrow \text{False}$
This statement can then be written symbolically as:
 $\sim (\sim P) \equiv \sim (\sim F) \equiv \sim T \equiv F$
So this statement is false.
- c) This is not a proposition, since the truth value depends on what set x is an element of, which is undefined here.
- d) This is not a proposition, again because of the undefined nature of x .
- e) This is a proposition. Let:
 $P = \pi \text{ is rational} \rightarrow \text{False}$
 $Q = 17 \text{ is prime} \rightarrow \text{True}$
 $R = 7 < 13 \rightarrow \text{True}$
 $S = 81 \text{ is a perfect square} \rightarrow \text{True}$
So, this statement can be written symbolically as:
 $(P \wedge Q) \vee (R \wedge S) \equiv (T \wedge F) \vee (T \wedge T) \equiv F \vee T \equiv T$
So this statement is true.
- f) This is a proposition. Let:
 $P = 2 \text{ is rational} \rightarrow \text{True}$
 $Q = \pi \text{ is irrational} \rightarrow \text{True}$
 $R = 2\pi \text{ is rational} \rightarrow \text{False}$
So, this statement can be written symbolically as:
 $(P \wedge Q) \vee R \equiv (T \wedge T) \vee F \equiv T \vee F \equiv T$
So this statement is true.
- g) This is a proposition. Let:
 $P = 5\pi \text{ is rational} \rightarrow \text{False}$
 $Q = 4.9 \text{ is rational} \rightarrow \text{True}$
 $R = \text{There are exactly four primes less than 10} \rightarrow \text{True}$
So, this statement can be written symbolically as:
 $(P \wedge Q) \vee R \equiv (F \wedge T) \vee T \equiv F \vee T \equiv T$
So this statement is true.

h) This is a proposition. Let:

$P = -3.7 \text{ is rational} \rightarrow \text{True}$

$Q = 3\pi < 10 \rightarrow \text{True}$

$R = 3\pi > 15 \rightarrow \text{False}$

So, this statement can be written symbolically as:

$P \wedge (Q \vee R) \equiv T \wedge (T \vee F) \equiv T \wedge T \equiv T$

So this statement is true.

i) This is a proposition. Let:

$P = 39 \text{ is prime} \rightarrow \text{False}$

$Q = 64 \text{ is a power of } 2 \rightarrow \text{True}$

So, this statement can be written symbolically as:

$\sim P \vee Q \equiv \sim F \vee T \equiv T \vee T \equiv T$

So this statement is true.

j) This statement is not a proposition, since there are more than three false statements in the book but the truth of the last part of the statement changes the truth of the entire statement in a contradictory manner.

10.

a)

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$\sim P \wedge \sim Q$	$(P \wedge Q) \vee (\sim P \wedge \sim Q)$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

The output varies for different values for P and Q, so it is neither a tautology nor a contradiction.

b)

P	$\sim P$	$P \wedge \sim P$	$\sim (P \wedge \sim P)$
T	F	F	T
F	T	F	T

This is a tautology since it evaluates to true for all possible values of P

c)

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$\sim P \vee \sim Q$	$(P \wedge Q) \vee (\sim P \vee \sim Q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

This is a tautology since it evaluates to true for all possible combinations of values of P and Q.

d)

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$P \wedge \sim Q$	$\sim P \wedge Q$	$\sim P \wedge \sim Q$
T	T	F	F	T	F	F	F
T	F	F	T	F	T	F	F
F	T	T	F	F	F	T	F
F	F	T	T	F	F	F	T
$(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$							T
							T
							T
							T

This is a tautology since it evaluates to true for all possible combinations of values of P and Q.

e)

P	Q	R	$\sim P$	$Q \wedge \sim P$	$P \wedge R$	$\sim (P \wedge R)$	$(Q \wedge \sim P) \wedge \sim (P \wedge R)$
T	T	T	F	F	T	F	F
T	T	F	F	F	F	T	F
T	F	T	F	F	T	F	F
T	F	F	F	F	F	T	F
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	T	F
F	F	F	T	F	F	T	F

The output varies for different values for P, Q, and R, so it is neither a tautology nor a contradiction.

f)

P	Q	R	$\sim Q$	$\sim Q \wedge P$	$R \vee Q$	$(\sim Q \wedge P) \wedge (R \vee Q)$	$P \vee [(\sim Q \wedge P) \wedge (R \vee Q)]$
T	T	T	F	F	T	F	T
T	T	F	F	F	T	F	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	F	T
F	T	T	F	F	T	F	F
F	T	F	F	F	T	F	F
F	F	T	T	F	T	F	F
F	F	F	T	F	F	F	F

The output varies for different values for P, Q, and R, so it is neither a tautology nor a contradiction.